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THE
ELEMENTS
OF
ALGEBRA

IN A
New and Eaſy Method;
WITH THEIR
USE and APPLICATION,
IN THE
Solution of a great Variety of Arithmetical
and Geometrical Queſtions;
By general and univerſal RULES.
To which is prefixed an
INTRODUCTION,
CONTAINING
A Succinct HISTORY of this SCIENCE.

By *NATHANIEL HAMMOND*,
Of the BANK.

The SECOND EDITION, Corrected.

LONDON:
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Hulls

A SYSTEM OF Practical Mathematics:

CONTAINING

Vulgar and Decimal Fractions; the Extraction of the Square and Cube Roots; Multiplication of Feet, Inches, and Parts; the Menſuration of Superficies and Solids, and all Sorts of Artificers Work.

Plain Geometry and Trigonometry, with their Application in meaſuring the Altitude of Objects and Diſtance of Places; Surveying; Spherical Geometry and Trigonometry; Aſtronomy, and Dialling.

WITH

A plain Account of the *Gregorian* or New STYLE, ſettled by Act of Parliament; the Method of finding the Epact, Moon's Age, Tides, *&c.*

With neceſſary TABLES:

Particularly the Table calculated by the Right Honourable GEORGE Earl of *Macclesfield*, for finding *Eaſter*.

TABLES of Logarithms, Sines, and Tangents; with their Deſcription and Uſe.

The Whole in a moſt eaſy and plain Method; deſigned for the Uſe of Schools, and the Benefit of Carpenters, Joyners, Bricklayers, and other Artificers, to whom the Practical Mathematics are of great and immediate Uſe.

To which are added, by Way of APPENDIX, Cycles of the primary Planets, never before publiſhed: With Mr. *Whiſton*'s Cycle for the Calculation of Solar Eclipſes without Parallaxes.

By *JOHN POTTER.*

I have peruſed Mr. Potter's Syſtem of Practical Mathematics, *and eſteem it the moſt uſeful Book of the Kind I have yet ſeen publiſhed.*

Bank of England,
Feb. 4, 1752.

Nath. Hammond.

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THE

PREFACE.

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THE Practical Mathematics *being of great and general Uſe, I have endeavoured to form a Syſtem of them, particularly adapted for the Benefit of ſuch Perſons to whom they are of Service—in making the Connexion of the Parts the moſt natural and eaſy, and the Directions ſo diſtinct and copious, as to be ſufficient to convey the Knowledge of any Part of the enſuing Treatiſe to ſuch who have not the Benefit of an Opportunity to be taught them, if they underſtand the Rules of Addition, Subtraction, Multiplication, and Diviſion, in Arithmetic.*

The common Rules of Arithmetic in whole Numbers being univerſally known, I have begun with Vulgar Fractions; *where I have ſtudied the moſt natural* *and eaſy Order, and given the Reader full and ample Directions how he is to manage this Part of Arithmetic. Then follow* Decimal Fractions, *of more general Uſe, and a much more neat and elegant Method of Computation. The Doctrine of Decimal Fractions differs little from that of whole Numbers, except in the Article of* Diviſion, *the Rule for which I have copiouſly explained in ſeveral Operations. Both Vulgar and Decimal Fractions have their particular Advantages.*

The Reader having acquired the Knowledge of Vulgar and Decimal Fractions, the Extraction of the Square Root *follows, an Operation but little harder than common Diviſion. Then follows the Extraction of the* Cube Root, *by a Method which, I think, is the beſt I have yet ſeen publiſhed. The Extraction of theſe Roots are illuſtrated by a great Variety of Examples, connected, for the Eaſe of the Learner, in a natural and eaſy Manner. I am obliged to Mr.* Triplett, *Teacher of the Mathematics at* Kingſton, *for his communicating to me the Rule for the Cube Root.*

Artificers, in computing their Work, may uſe either Decimal Fractions, or the Multiplication of Feet, Inches and Parts, by Feet, Inches and Parts; the laſt is moſtly uſed, and commonly called Croſs-Multiplication; *for which Mr.* Triplett *has likewiſe been ſo kind as to favour me with a new and* *eaſy Rule, and which the Reader will find ſufficiently illuſtrated by Examples. To this follows the Menſuration of* Superficies *and* Solids, *the Meaſuring* Boards *and* Timber, *the Explanation of the* five regular or *Platonic* Bodies, *the Deſcription and Uſe of the* Sliding-Rule, *with a few Queſtions to exerciſe the Learner in the foregoing Rules.*

Theſe Things being learnt, it naturally follows to teach the Reader the meaſuring the Works of Carpenters, Bricklayers, Joyners, Plaiſterers, Painters, Glaziers, Maſons, &c. *which are particularly explained; and, in the difficult Parts, I have conſulted ſome of the moſt eminent Surveyors.*

Practical Geometry *next follows, the uſeful Problems of which are here taught. To this ſucceeds the Uſe of* Logarithms, *that great Diſcovery in Mathematics, which does its noble Inventor immortal Honour. The Uſe of the Logarithms, with the Table of* Sines *and* Tangents, *I have endeavoured to explain in an eaſy Manner. The Knowledge of theſe Tables being acquired, the Reader is ſhown their excellent Uſe in* Plain Trigonometry, *both right and oblique-angled; where there are particular Directions how to conſtruct the Figures and ſolve the Problems. After this, the Doctrine of plain Triangles is, in the ſame ample Manner, applied to the meaſuring of acceſſible and inacceſſible Heights and Diſtances. Then follows the Method of* Surveying: *And, to give the Reader* *a further Pleaſure, he is now ſhown the Conſtruction of the* Line of Chords, Sines, Tangents *and* Secants.

In the Spherical Geometry *I have attentively ſtudied the moſt eaſy Order in connecting the ſeveral Problems of this Part of Science; and, however new and different it may appear from the preceding Parts of the Book, I have taken ſuch particular Care in the Directions, that, I am perſuaded, the Conſtruction of theſe Problems will not be difficult to the Reader. In* Spherical Trigonometry *there are Directions how to conſtruct the Problems, and the Solutions fully explained by Lord* Nepier's Catholic Propoſition. *In the oblique Part of Spherical Trigonometry all Care was taken to render it as eaſy as may be.*

By theſe moſt natural and eaſy Steps, in which every Thing that is ſuperfluous has been induſtriouſly avoided, is the Reader led to Aſtronomy, *a Science which has its particular Charms. Here he will find a Collection of the moſt uſeful and practical Problems; with ſome Idea of the Grandeur of the* Solar Syſtem, *in the Magnitude, Diſtances, and other Affections of the Planets; and of the ſurpriſing Comet which appeared in* 1680, *and the intenſe Heat it ſuſtained in its neareſt Approach to the Sun. After this follows* Dialling.

Then the Explication of the Gregorian *or* New Style, *with the Method of finding the* Golden Number, Epact, Age and Change of the Moon, *&c. follows. Yet the Reader may begin and learn theſe firſt, or when he pleaſes, the Rules having no Connexion with any other Parts of the Book.*

The Appendix, *containing the* Cycles of the Planets, *is a Diſcovery I made many Years ſince; and have here added it together with Mr.* Whiſton's Cycle for Eclipſes, *as I thought it would be acceptable to the Curious.*

I have nothing more to add, but that I do not ſee any Objection why an induſtrious Reader may not, by himſelf, acquire all the Knowledge of the ſeveral Parts of this Book; the Directions and Inſtructions are deſigned for that Purpoſe, in regard to Individuals; and the whole Collection to be a regular Syſtem for the Uſe of School.

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A
SYSTEM
OF
Practical Mathematics.

Of Vulgar Fractions.

ARITHMETIC is the Art of numbering. Every Thing, therefore, which may be numbered; is the Subject of Arithmetic. Hence, not only Integers, or whole Things, but their ſeveral Parts, are brought into arithmetical Computation; ſince all Things are, or may be ſuppoſed, diviſible into any Number of equal Parts.

Some Things are, for Convenience in Commerce, by Cuſtom divided into Parts, which have a particular Denomination: As a Pound Sterling is divided into 20 Shillings, a Shilling into 12 Pence, and a Penny into 4 Farthings: A Foot is divided into Inches, and an Inch into Quarters and Half-Quarters; and the like of other Things. But it often happens, that Integers are divided into Parts, which have no particular Denomination, but are denominated only from the Number, by which the Integer is divided. Thus any Integer (as a Pound Sterling, for Example) may be ſuppoſed to be divided into 5, 13, 28, 57, or any other Number of equal Parts; which can only be denominated from the Number by which the Integer is divided, and called a fifth, a thirteenth, a twenty-eighth, a fifty-ſeventh Part of a Pound, or whatever is thus divided.

In arithmetical Calculations, it often happens, that Integers are divided into different Parts; and a Number of thoſe Parts are required to be taken, which generally are leſs than the Number of Parts into which the Integer is divided. Thus a Pound Sterling may be divided into 36 Parts, and it may be required to take 19 of thoſe Parts; which are called *nineteen Thirty-ſixths* of a Pound. Again, if a Pound Sterling, or any other Integer, be divided into 17 Parts, and 11 of thoſe Parts are required to be taken, this is called *eleven Seventeenths* of a Pound, or whatever elſe is ſuppoſed to be thus divided. Theſe are what are uſually called Fractions, or the fractional Parts of Things. But, as it is frequently neceſſary to bring thoſe fractional Parts into Calculations, a Method has been invented to expreſs them in Figures: The Way of doing which is next to be ſhewn.

Of the Notation of Fractions, and their ſeveral Kinds.

A *Fraction*, otherwiſe called a *broken Number*, is that by which the Parts of Things are expreſſed. Thus, if we were to expreſs three Farthings, a Farthing being the Fourth of a Penny, three Farthings are three Fourths of a Penny; which muſt be wrote thus, $\frac{3}{4}$. A vulgar Fraction always conſiſts of two Parts, that is, two Numbers, ſet one over the other, with a Line drawn betwixt them. To expreſs any Number of Inches in Foot-Meaſure, as ſeven Inches, you muſt put down 7, and 12 under it, with a Line betwixt, as $\frac{7}{12}$; which is to be read, *ſeven Twelfths* of a Foot, an Inch being the twelfth Part of a Foot. If a Foot, a Pound Sterling, or any other Thing, were to be divided into 100 equal Parts, in order to expreſs 32 of thoſe Parts by a Fraction, write them thus, $\frac{32}{100}$, that is, *thirty-two Hundredth-Parts* of a Foot, Pound, or whatever was ſuppoſed to be divided. And the Number above the Line is called the *Numerator*, that below the *Denominator*: Thus, in the Fraction above-mentioned, $\frac{7}{12}$, the 7 is the Numerator, and the 12 the Denominator: So in this, $\frac{5}{17}$, *five Seventeenths*, 5 is the Numerator, and 17 the Denominator.

The Denominator is ſo called, becauſe it denominates or ſhews into how many Parts the Integer is divided; and the Numerator is ſo called, becauſe it fixes the Number of Parts ſignified by the Fraction: Thus, in the Fraction $\frac{32}{100}$, the Denominator 100 ſhews that the Integer is divided into 100

Parts; and the Numerator 32 ſhews that 32 of thoſe Parts are ſignified by the Fraction, or to be taken for its Value.

There are two Sorts of Fractions common in Practice; namely, Vulgar, of which we are now treating; and Decimal, of which hereafter.

Vulgar Fractions are of ſeveral Sorts; that is, either *proper* or *improper*, *ſimple* or *compound.* A proper Fraction is that whoſe Numerator is leſs than the Denominator, as $\frac{3}{4}$, $\frac{4}{9}$, $\frac{7}{12}$, or the like. A Fraction is called improper, when the Numerator is equal to, or greater than, the Denominator, as $\frac{6}{6}$, $\frac{9}{4}$, $\frac{15}{3}$, or the like. Theſe are called improper Fractions, becauſe they contain the Whole of an Integer when the Numerator is equal to the Denominator, and more than an Integer when the Numerator is greater than the Denominator.

A *ſimple* or ſingle Fraction is that which has but one Numerator and one Denominator, as $\frac{5}{6}$, or any of the Fractions above ſet down. A *compound* Fraction is that which hath more than one Numerator and Denominator, and hath the Word *of* interpoſed betwixt the ſeveral Parts of which it conſiſts: Thus $\frac{1}{2}$ of $\frac{3}{4}$ is a compound Fraction, expreſſing one Half of three Fourths of an Integer: So $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{2}{9}$ is a compound Fraction, ſignifying three Fourths of five Sevenths of two Ninths of an Integer, or whole Thing. Which Kind of Fractions are uſually called *Fractions of Fractions:* Thus a Farthing is the one Fourth of a Penny, which is the one Twelfth of a Shilling, which is the one Twentieth of a Pound; and by Fractions is wrote thus, $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a Pound Sterling.

Proper ſimple Fractions often ariſe in Diviſion of whole Numbers; for, when the Diviſor will not equally divide the Dividend, but there is a Remainder, this Remainder is the Numerator of a Fraction, whoſe Denominator is the Diviſor: This Fraction, annexed to the integral Part of the Quotient, gives the true Quotient of the Work. Thus, if 26 Pounds were to be divided among 3 Men, the Quotient will be 8; and there will be a Remainder of 2 Pounds; which, being placed over the Diviſor 3, makes the Fraction $\frac{2}{3}$ of a Pound, to be annexed to the 8; ſo that the true Anſwer is 8 $\frac{2}{3}$ of a Pound, that is, 8 Pounds, and $\frac{2}{3}$ of a Pound, for each Man. The third Part of a Pound being 6*s.* 8*d.* two Thirds are 13*s.* 4*d.* ſo that the true Anſwer is 8*l.* 13. 4*d.*

When a Fraction is annexed to a whole Number, this is called a *mixed Number.* Thus, in the Example above, 8 $\frac{2}{3}$ is a mixed Number, 8 being the integral Part, and $\frac{2}{3}$ the fractional

fractional Part; that is, it is 8 Pounds, and two Thirds of a Pound more. Alſo, if you would expreſs 18 *l.* 15 *s.* it is to be wrote thus, 18 $\frac{15}{20}$, or, which is the ſame, 18 $\frac{3}{4}$, that is, 18 Pounds, and three Fourths of a Pound, which is 15 *s.* for, the $\frac{1}{4}$ of a Pound being 5 *s.* three Fourths will be 15 *s.* This, therefore, is a mixed Number, 18 being the integral Part, and $\frac{15}{20}$, or $\frac{3}{4}$, the fractional Part.

Having ſhewn the Method of the Notation of Fractions, and the ſeveral Kinds, I proceed to

The Reduction of Vulgar Fractions.

Article 1. To reduce a whole Number to an improper Fraction.

Rule. Place the given whole Number as a Numerator, and under it 1, or Unity, for a Denominator.

Exam. 1.	*Exam.* 2.	*Exam.* 3.
Reduce 8 to an improper Fraction.	Reduce 13 to an improper Fraction.	Reduce 5 to an improper Fraction.
Anſwer $\frac{8}{1}$.	Anſwer $\frac{13}{1}$.	Anſwer $\frac{5}{1}$.

Exam. 1. Under the 8, given to be reduced to an improper Fraction, place 1, with a Line drawn betwixt, which makes $\frac{8}{1}$, and is to be read, 8 Units, or 8 Wholes. The other Examples are done in the ſame Manner, and do not require Explanation. But it will be proper to obſerve, if a whole Number be given to be reduced to an improper Fraction, whoſe Denominator is given, that, in ſuch Caſe, the Rule is, multiply the given Integer by ſuch aſſigned Denominator, and place the Product over the given Denominator; ſo will that be an improper Fraction, equal to the given whole Number.

Exam. 4.	*Exam.* 5.	*Exam.* 6.
Reduce 6 to an improper Fraction, whoſe Denominator is 4.	Reduce 8 to an improper Fraction, whoſe Denominator is 2.	Reduce 12 to an improper Fraction, whoſe Denominator is 15.
6	8	15
4	2	12
24 Anſ. $\frac{24}{4}$.	16 Anſ. $\frac{16}{2}$.	180 Anſ. $\frac{180}{15}$.

Exam. 4. The Integer 6 being given to be reduced to an improper Fraction, whoſe Denominator ſhall be 4; multiply 6 by 4, the Product is 24; which placed over 4 gives $\frac{24}{4}$, equal to 6; as appears hereafter by the Rule for finding the Value of a Vulgar Fraction.

Exam. 5. The Integer 8 being given to be reduced to an improper Fraction, whoſe Denominator is 2; multiply 8 by 2, placing the Product 16 over the given Denominator 2; ſo is $\frac{16}{2}$ the improper Fraction ſought.

Exam. 6. Multiply the given Integer 12 by the given Denominator 15 (or 15 by 12, which is the ſame); the Product 180 placed over 15 is $\frac{180}{15}$, the improper Fraction ſought.

Article 2. To reduce a mixed Number to an improper Fraction.

Rule. Multiply the integral Part of the mixed Number by the Denominator of the fractional Part, and then add to the Product the Numerator of the Fraction; this Sum is the new Numerator; under which place the Denominator in the given mixed Number, and this will be the Fraction ſought.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
	Reduce 10 $\frac{4}{5}$ to an improper Fraction.	Reduce 18 $\frac{3}{4}$ to an improper Fraction.	Reduce 3 $\frac{2}{3}$ to an improper Fraction.
Integ.	10	18	3
Mult.	5	4	3
	—	—	—
	50	72	9
Add	4	3	2
	—	—	—
	54 Anſ. $\frac{54}{5}$.	75 Anſ. $\frac{75}{4}$.	11 Anſ. $\frac{11}{3}$.

Exam. 1. Multiply 10 by 5 the Denominator, and to the Product 50 add 4 the Numerator; the Sum 54 is the new Numerator, to be placed over 5 the Denominator of the given Fraction; ſo is $\frac{54}{5}$ the Fraction ſought.

Exam. 2. Multiply 18 by 4, and to the Product 72 add 3; the Sum 75 placed over 4 gives $\frac{75}{4}$, the Fraction ſought.

The third Example is done in the ſame Manner.

Exam. 4.

Reduce $13\frac{5}{7}$ to an improper Fraction.

$$\begin{array}{r}13\\7\\\hline 91\\5\\\hline 96\end{array}$$

Anſ. $\frac{96}{7}$.

Exam. 5.

Reduce $97\frac{2}{3}$ to an improper Fraction.

$$\begin{array}{r}97\\3\\\hline 291\\2\\\hline 293\end{array}$$

Anſ. $\frac{293}{3}$.

Exam. 6.

Reduce $127\frac{16}{27}$ to an improper Fraction.

$$\begin{array}{r}127\\27\\\hline 889\\254\\\hline 3429\\16\\\hline 3445\end{array}$$

Anſ. $\frac{3445}{27}$.

Art. 3. To reduce compound Fractions to ſimple Fractions.

Rule. Multiply all the Numerators together for a new Numerator, and all the Denominators for a new Denominator; the Product of the Numerators, being placed over the Product of the Denominators, will be the ſimple Fraction ſought.

Exam. 1.

Reduce $\frac{1}{2}$ of $\frac{3}{4}$ to a ſimple Fraction.

$$\begin{array}{rr}1&2\\3&4\\\hline 3&8\end{array}$$

Anſ. $\frac{3}{8}$.

Exam. 2.

Reduce $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ to a ſimple Fraction.

$$\begin{array}{rr}1&4\\1&12\\\hline 1&48\\1&20\\\hline 1&960\end{array}$$

Anſ. $\frac{1}{960}$.

Exam. 1. The two Numbers 1 and 3 multiplied give 3 for the new Numerator; then multiply the two Denominators 2 and 4, which makes 8, for the new Denominator; ſo is $\frac{3}{8}$ the ſimple Fraction ſought.

Exam. 2. The Numerators, being all Units, in multiplying produce 1 for the new Numerator. The Denominators 4, 12, and 20, multiplied together, give 960 for a new Denominator: Thus is $\frac{1}{960}$ the Fraction ſought.

The following Examples are done in the ſame Manner.

Exam. 3.

Reduce $\frac{3}{5}$ of $\frac{7}{10}$ to a ſimple Fraction.

$$\begin{array}{rr} 7 & 10 \\ 3 & 5 \\ \hline 21 & 50 \end{array}$$

Anſ. $\frac{21}{50}$.

Exam. 4.

Reduce $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{6}{13}$ to a ſimple Fraction.

$$\begin{array}{rr} 2 & 3 \\ 4 & 7 \\ \hline 8 & 21 \\ 6 & 13 \\ \hline 48 & 63 \\ & 21 \\ \hline & 273 \end{array}$$

Anſ. $\frac{48}{273}$.

Exam. 5.

Reduce $\frac{17}{19}$ of $\frac{43}{66}$ to a ſimple Fraction.

$$\begin{array}{rr} 43 & 66 \\ 17 & 19 \\ \hline 301 & 594 \\ 43 & 66 \\ \hline 731 & 1254 \end{array}$$

Anſ. $\frac{731}{1254}$.

Exam. 6.

Reduce $\frac{5}{12}$ of $\frac{3}{7}$ of $\frac{11}{12}$ to a ſimple Fraction.

$$\begin{array}{rr} 5 & 12 \\ 3 & 7 \\ \hline 15 & 84 \\ 11 & 12 \\ \hline 15 & 1008 \\ 15 & \\ \hline 165 & \end{array}$$

Anſ. $\frac{165}{1008}$.

Article 4. To reduce ſimple Fractions having different Denominators to Fractions of the ſame Denominator.

Rule. Multiply each Numerator into all the Denominators except its own; theſe Products are the new Numerators for each Fraction whoſe Numerator was ſo multiplied: Then multiply all the Denominators together; this Product is the common Denominator to all the Fractions.

Exam. 1.

Reduce $\frac{1}{2}$ and $\frac{3}{4}$ to Fractions of the ſame Denominator.

$$\begin{array}{rrr} 1 & 3 & 4 \\ 4 & 2 & 2 \\ \hline 4 & 6 & 8 \end{array}$$

Anſ. $\frac{4}{8}$ and $\frac{6}{8}$.

Exam. 2.

Reduce $\frac{2}{3}$ and $\frac{4}{5}$ to the ſame Denominator.

$$\begin{array}{rrr} 2 & 4 & 5 \\ 5 & 3 & 3 \\ \hline 10 & 12 & 15 \end{array}$$

Anſ. $\frac{10}{15}$ and $\frac{12}{15}$.

Exam. 3.

Exam. 3.

Reduce $\frac{3}{4}$ and $\frac{2}{3}$ and $\frac{4}{5}$ to Fractions of the ſame Denominators.

3	2	4	4	
3	4	3	3	
9	8	12	12	
5	5	4	5	Anſwer
45	40	48	60	$\frac{45}{60}$ and $\frac{40}{60}$ and $\frac{48}{60}$.

Exam. 1. The Numerator 1 multiplied by the Denominator 4 is 4, for a new Numerator to the Fraction $\frac{1}{2}$: The Numerator 3 multiplied by the Denominator 2 is 6, for the new Numerator to the Fraction $\frac{3}{4}$: Then the Denominator 2 multiplied by the Denominator 4 is 8, for the common Denominator to both the Fractions.

The ſecond Example is done in the ſame Manner.

Exam. 3. The Numerator 3 multiplied by the Denominator 3, and that Product by the Denominator 5, is 45, the new Numerator for the Fraction $\frac{3}{4}$: The Numerator 2 multiplied by the Denominator 4, and that Product by the Denominator 5, is 40, the new Numerator to the Fraction $\frac{2}{3}$: Then the Numerator 4 multiplied by the Denominator 3, and that Product by the other Denominator 4, is 48, the new Numerator to the Fraction $\frac{4}{5}$: Laſtly, multiply all the Denominators together, and the Product is 60, the common Denominator; which, being put under each of the Numerators before found, gives the Fractions $\frac{45}{60}$, $\frac{40}{60}$, and $\frac{48}{60}$, equal to the given Fractions.

The following Examples are done in the ſame Manner.

Exam. 4.

Reduce $\frac{9}{10}$ and $\frac{7}{12}$ and $\frac{5}{6}$ to Fractions of the ſame Denominator.

9	7	5	10
12	10	12	12
108	70	60	120
6	6	10	6
648	420	600	720

Anſ. $\frac{648}{720}$ and $\frac{420}{720}$ and $\frac{600}{720}$.

Exam. 5.

Reduce $\frac{11}{12}$ and $\frac{9}{13}$ to Fractions of the ſame Denominator.

11	9	13
13	12	12
33	108	156
11		
143		

Anſ. $\frac{143}{156}$ and $\frac{108}{156}$.

Article 5. To reduce compound Fractions and ſimple Fractions to Fractions of the ſame Denominator.

Rule. Firſt reduce the compound Fraction to a ſimple Fraction, by Art. 3; then reduce that and the given ſimple Fraction, to Fractions of the ſame Denominator, by Art. 4.

Exam. 1.

Reduce $\frac{1}{2}$ of $\frac{3}{4}$ and $\frac{5}{7}$ to Fractions of the ſame Denominator.

$$\begin{array}{cc} 1 & 2 \\ 3 & 4 \\ \hline 3 & 8 \end{array}$$

$\frac{3}{8}$ ſimple Fraction for the compound Fraction.

$\frac{3}{8}$ and $\frac{5}{7}$ to be reduced to the ſame Denominator.

$$\begin{array}{ccc} 3 & 5 & 8 \\ 7 & 8 & 7 \\ \hline 21 & 40 & 56 \end{array}$$

Anſwer $\frac{21}{56}$ and $\frac{40}{56}$.

Exam. 2.

Reduce $\frac{3}{7}$ of $\frac{5}{8}$ and $\frac{2}{3}$ to Fractions of the ſame Denominator.

$$\begin{array}{cc} 3 & 8 \\ 5 & 7 \\ \hline 15 & 56 \end{array}$$

$\frac{15}{56}$ compound Fraction reduced to a ſimple Fraction.

$\frac{15}{56}$ and $\frac{2}{3}$ to be reduced to the ſame Denominator.

$$\begin{array}{ccc} 15 & 56 & 56 \\ 3 & 2 & 3 \\ \hline 45 & 112 & 168 \end{array}$$

Anſwer $\frac{45}{168}$ and $\frac{112}{168}$.

Exam. 1. The Numerators 1 and 3 of the compound Fraction multiplied together give 3, for a new Numerator; and the Denominators 2 and 4 multiplied together produce 8, for a new Denominator; which reduces it to the ſimple Fraction $\frac{3}{8}$, by Art. 3: Then the Numerator 3 of this new Fraction multiplied into the Denominator 7 of the given ſimple Fraction is 21, for a new Numerator to the Fraction $\frac{3}{8}$; and the Numerator 5 multiplied into the Denominator 8 is 40, for the other new Numerator: Laſtly, the Denominators 8 and 7 multiplied make 56, for the common Denominator; over which placing the Numerators before found, theſe are the two new Fractions ſought.

Exam. 2. Multiplying 3 and 5, the Numerators of the compound Fraction, the Product is 15; multiplying 7 and 8, the Denominators of the compound Fraction, the Product is 56; and $\frac{15}{56}$ is the ſimple Fraction: Hence the two ſimple Fractions are $\frac{15}{56}$ and $\frac{2}{3}$. Now, 15 multiplied by 3 is 45, one of the new Numerators; and 2 multiplied by 56 is 112, the other new Numerator; and 56 multiplied by 3 is 168, the common Denominator: Hence $\frac{45}{168}$ and $\frac{112}{168}$ are the two new Fractions ſought.

The two following Examples are done in the ſame Manner.

Exam. 3. Reduce $\frac{2}{3}$ of $\frac{1}{2}$ and $\frac{3}{4}$ to Fractions of the ſame Denomination.

$$\begin{array}{cc}2 & 3\\ 1 & 2\\ \hline 2 & 6\end{array}$$

$\frac{2}{6}$ ſimple Fraction for the compound Fraction.

$\frac{2}{6}$ and $\frac{3}{4}$ to be reduced to the ſame Denomination.

$$\begin{array}{ccc}2 & 3 & 6\\ 4 & 6 & 4\\ \hline 8 & 18 & 24\end{array}$$

Anſwer $\frac{8}{24}$ and $\frac{18}{24}$.

Exam. 4. Reduce $\frac{3}{5}$ and $\frac{2}{3}$ of $\frac{4}{5}$ to Fractions of the ſame Denomination.

$$\begin{array}{cc}2 & 3\\ 4 & 5\\ \hline 8 & 15\end{array}$$

$\frac{8}{15}$ ſimple Fraction for the compound Fraction.

$\frac{3}{5}$ and $\frac{8}{15}$ to be reduced to the ſame Denomination.

$$\begin{array}{ccc}15 & 8 & 15\\ 3 & 5 & 5\\ \hline 45 & 40 & 75\end{array}$$

Anſwer $\frac{45}{75}$ and $\frac{40}{75}$.

Article 6. To reduce mixed Numbers and ſimple Fractions to Fractions of the ſame Denomination.

Rule. Firſt reduce the mixed Number to an improper Fraction, by Art. 2; then reduce that improper Fraction and the given ſimple Fraction to a common Denominator, by Art. 4.

Exam. 1. Reduce 9 $\frac{4}{5}$ and $\frac{7}{10}$ to the ſame Denominator.

$$\begin{array}{c}9\\ 5\\ \hline 45\\ 4\\ \hline 49\end{array}$$

$\frac{49}{5}$ the mixed Number reduced to an improper Fraction.

$\frac{49}{5}$ and $\frac{7}{10}$ to be reduced to the ſame Denominator.

$$\begin{array}{ccc}49 & 7 & 10\\ 10 & 5 & 5\\ \hline 490 & 35 & 50\end{array}$$

Anſwer $\frac{490}{50}$ and $\frac{35}{50}$.

Ex. 2. Reduce 7 $\frac{3}{4}$ and 3 $\frac{1}{2}$ and $\frac{6}{7}$ to a common Denominator.

$$\begin{array}{cccc}7 & & 3 & \\ 4 & & 2 & \\ \cline{1-1}\cline{3-3} 28 & \frac{31}{4} & 6 & \frac{7}{2}\\ 3 & & 1 & \\ \cline{1-1}\cline{3-3} 31 & & 7 & \end{array}$$

Whence the two mixed Numbers are reduced to the improper Fractions $\frac{31}{4}$ and $\frac{7}{2}$; which Fractions, and $\frac{6}{7}$, are to be reduced to a common Denominator.

$$\frac{31}{4} \quad \frac{7}{2} \quad \frac{6}{7}$$

$$\begin{array}{cccc}31 & 7 & 6 & 4\\ 2 & 4 & 2 & 2\\ \hline 62 & 28 & 12 & 8\\ 7 & 7 & 4 & 7\\ \hline 434 & 196 & 48 & 56\end{array}$$

Anſwer $\frac{434}{56}$ and $\frac{196}{56}$ and $\frac{48}{56}$.

Exam. 1. The Integer 9 multiplied by 5 the Denominator of the Fraction is 45; to which 4 the Numerator being added, is 49, for a new Numerator; under which placing the Denominator 5, the improper Fraction is $\frac{49}{5}$, by Art. 2. Then multiplying the Numerator 49 by the Denominator 10 of the given ſimple Fraction, the Product is 490; and the Numerator 7 multiplied by the Denominator 5 of the improper Fraction $\frac{49}{5}$ is 35; theſe are the new Numerators. Laſtly, the two Denominators 5 and 10 multiplied give 50 for the common Denominator. Whence $\frac{490}{50}$ and $\frac{35}{50}$ are the Fractions ſought.

Exam. 2. The mixed Number $7\frac{3}{4}$ being reduced to the improper Fraction $\frac{31}{4}$, and the other mixed Number $3\frac{1}{2}$ to $\frac{7}{2}$, by Art. 2; multiply the Numerator 31 into the two Denominators 2 and 7, which makes 434, for a new Numerator to the Fraction $\frac{31}{4}$; and the Numerator 7 multiplied into the Denominators 4 and 7 is 196, for a new Numerator to the Fraction $\frac{7}{2}$; alſo the Numerator 6 multiplied by the Denominators 4 and 2 is 48, for a new Numerator to the Fraction $\frac{6}{7}$: Laſtly, the Denominators 4, 2, and 7, multiplied, give 56 for a common Denominator to the Fractions $\frac{434}{56}$, $\frac{196}{46}$, and $\frac{48}{56}$.

Exam. 3.

Reduce $\frac{2}{3}$ and $5\frac{1}{2}$ to Fractions of the ſame Denomination.

$$\begin{array}{r} 5\\ 2\\ \hline 10\\ 1\\ \hline 11 \end{array}$$

$\frac{11}{2}$ the mixed Number reduced to an improper Fraction.

$\frac{2}{3}$ $\frac{11}{2}$

$$\begin{array}{rrr} 2 & 11 & 3\\ 2 & 3 & 2\\ \hline 4 & 33 & 6 \end{array}$$

Anſwer $\frac{4}{6}$ and $\frac{33}{6}$.

Exam. 4.

Reduce $3\frac{1}{4}$ and $\frac{4}{5}$ to Fractions of the ſame Denomination.

$$\begin{array}{r} 3\\ 4\\ \hline 12\\ 1\\ \hline 13 \end{array}$$

$\frac{13}{4}$ the mixed Number reduced to an improper Fraction.

$\frac{13}{4}$ $\frac{4}{5}$

$$\begin{array}{rrr} 13 & 4 & 5\\ 5 & 4 & 4\\ \hline 65 & 16 & 20 \end{array}$$

Anſwer $\frac{65}{20}$ and $\frac{16}{20}$.

Theſe Examples are done as the firſt and ſecond.

Article 7. To reduce a mixed Number and a compound Fraction to the ſame Denomination.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2; then reduce the compound Fraction to a ſimple Fraction, by Art. 3; laſtly, reduce theſe two Fractions to the ſame Denomination, by Art. 4, and they will be the new Fractions ſought.

Exam. 1.

Reduce $3\frac{3}{7}$ and $\frac{2}{3}$ of $\frac{3}{4}$ to the ſame Denomination.

$$\begin{array}{r} 3 \\ 7 \\ \hline 21 \\ 3 \\ \hline 24 \end{array} \quad \frac{24}{7} \qquad \begin{array}{r} 2 \\ 3 \\ \hline 6 \end{array} \quad \begin{array}{r} 3 \\ 4 \\ \hline 12 \end{array} \quad \frac{6}{12}$$

Whence the two Fractions in the Room of them that were given are $\frac{24}{7}$ and $\frac{6}{12}$.

$$\begin{array}{r} 24 \\ 12 \\ \hline 288 \end{array} \quad \begin{array}{r} 6 \\ 7 \\ \hline 42 \end{array} \quad \begin{array}{r} 12 \\ 7 \\ \hline 84 \end{array}$$

Anſwer $\frac{288}{84}$ and $\frac{42}{84}$.

Exam. 2.

Reduce $96\frac{7}{36}$ and $\frac{1}{3}$ of $\frac{3}{5}$ to the ſame Denomination.

$$\begin{array}{r} 96 \\ 36 \\ \hline 576 \\ 288 \\ \hline 3456 \\ 7 \\ \hline 3463 \end{array} \quad \frac{3463}{36} \qquad \begin{array}{r} 1 \\ 3 \\ \hline 3 \end{array} \quad \begin{array}{r} 3 \\ 5 \\ \hline 15 \end{array} \quad \frac{3}{15}$$

Hence the two Fractions in the Room of thoſe in the Example are $\frac{3463}{36}$ and $\frac{3}{15}$.

$$\begin{array}{r} 3463 \\ 15 \\ \hline 17315 \\ 3463 \\ \hline 51945 \end{array} \quad \begin{array}{r} 3 \\ 36 \\ \hline 18 \\ 9 \\ \hline 108 \end{array} \quad \begin{array}{r} 36 \\ 15 \\ \hline 180 \\ 36 \\ \hline 540 \end{array}$$

Anſwer $\frac{51945}{540}$ and $\frac{108}{540}$.

Exam. 1. The Integer 3 multiplied by 7 is 21; to which add 3, and it makes 24; thus is $3\frac{3}{7}$ reduced to $\frac{24}{7}$. Then the Numerators 2 and 3 of the compound Fraction multiplied give 6, and the Denominators 3 and 4 multiplied give 12; therefore $\frac{6}{12}$ is the ſimple Fraction to which the compound Fraction is reduced. The Numerator 24 multiplied into the Denominator 12, and the Numerator 6 into the Denominator 7, 7, give the new Numerators 288 and 42. Laſtly, the Denominators 7 and 12 multiplied produce 84, for the common Denominator; over which placing the Numerators before found, we have $\frac{288}{84}$ and $\frac{42}{84}$, the Fractions ſought.

Exam. 2. The mixed Number 96 $\frac{7}{36}$ is reduced, by Art. 2, to $\frac{3463}{36}$; and the compound Fraction $\frac{1}{3}$ of $\frac{3}{5}$ to the ſimple Fraction $\frac{3}{15}$, by Art. 3. Then reduce theſe two Fractions to a common Denominator, by Art. 4, and we have $\frac{51945}{540}$ and $\frac{108}{540}$, the Fractions ſought.

Article 8. To reduce ſimple Fractions and Integers to Fractions of the ſame Denominator.

Rule. Multiply the Integer by the Denominator of the Fraction; make this Product a Numerator by writing under it the Denominator of the Fraction; which Fraction will be inſtead of the Integers, and will have the ſame Denominator with the given Fraction.

Exam. 1.	*Exam.* 2.	*Exam.* 3.
Reduce 6 to a Fraction of the ſame Denominator with $\frac{3}{4}$.	Reduce 35 to a Fraction of the ſame Denominator with $\frac{2}{7}$.	Reduce 14 to a Fraction of the ſame Denominator with $\frac{3}{9}$.
6	35	14
4	7	9
24 Anſwer $\frac{24}{4}$ and $\frac{3}{4}$.	245 Anſwer $\frac{245}{7}$ and $\frac{2}{7}$.	126 Anſwer $\frac{126}{9}$ and $\frac{3}{9}$.

Exam. 1. The Integer 6 multiplied by 4 the Denominator of the given Fraction is 24, for a Numerator; under which placing 4 the Denominator of the given Fraction, we have the improper Fraction $\frac{24}{4}$, having the ſame Denominator with $\frac{3}{4}$ the given Fraction.

The other two Examples are done in the ſame Manner.

Article 9. To reduce a compound Fraction and Integers to the ſame Denominator.

Rule. Reduce the compound Fraction to a ſimple Fraction, by Art. 3; then reduce the Integer to an improper Fraction of the ſame Denominator with the ſimple Fraction before found, by Art. 8: Theſe will be the Fractions ſought.

Exam. 1.

Reduce $\frac{1}{2}$ of $\frac{3}{4}$ and 5 to Fractions of the ſame Denominator.

$$\begin{array}{cccc} 1 & 2 & & 5 \\ 3 & 4 & & 8 \\ \hline 3 & 8 & \frac{3}{8} & 40 \end{array}$$

Anſwer $\frac{3}{8}$ and $\frac{40}{8}$.

Exam. 2.

Reduce $\frac{3}{4}$ of $\frac{3}{16}$ and 17 to Fractions of the ſame Denominator.

$$\begin{array}{cccc} 3 & 16 & & 64 \\ 3 & 4 & & 17 \\ \hline 9 & 64 & \frac{9}{64} & 448 \\ & & & 64 \\ \hline & & & 1088 \end{array}$$

Anſwer $\frac{9}{64}$ and $\frac{1088}{64}$.

Exam. 1. The Numerators 1 and 3 of the compound Fraction multiplied give 3, and the Denominators 2 and 4 multiplied give 8; the compound Fraction is thus reduced to the ſimple Fraction $\frac{3}{8}$: Then multiply the Integer 5 by the Denominator 8, and it gives 40 for the Numerator; under which place the Denominator 8: So are $\frac{3}{8}$ and $\frac{40}{8}$ the Fractions ſought.

The ſecond Example is done in the ſame Manner.

Exam. 3.

Reduce $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{7}{10}$ and 16 to Fractions of the ſame Denominator.

$$\begin{array}{cccc} 3 & 5 & & 150 \\ 2 & 3 & & 16 \\ \hline 6 & 15 & & 900 \\ 7 & 10 & & 150 \\ \hline 42 & 150 & \frac{42}{150} & 2400 \end{array}$$

Anſwer $\frac{42}{150}$ and $\frac{2400}{150}$.

Exam. 4.

Reduce 7 and $\frac{2}{3}$ of $\frac{1}{2}$ to Fractions of the ſame Denominator.

$$\begin{array}{ccc} 2 & 2 & \\ 1 & 3 & \\ \hline 2 & 6 & \frac{2}{6} \end{array}$$

$$7 \quad \frac{2}{6}$$

$$\begin{array}{c} 7 \\ 6 \\ \hline 42 \end{array}$$

Anſwer $\frac{42}{6}$ and $\frac{2}{6}$.

Exam. 3. The three Numerators 3, 2, and 7, multiplied together, give 42; and the three Denominators 5, 3, and 10, multiplied together, make 150, for the Denominator; ſo is the compound Fraction reduced to the ſimple Fraction $\frac{42}{150}$. Multiplying the Integer 16 by 150, the Product is 2400, for the Numerator of the improper Fraction, anſwering to 16; under which place the Denominator 150: Whence $\frac{42}{150}$ and $\frac{2400}{150}$ are the Fractions ſought.

The fourth Example is done in the ſame Manner.

Article 10. To reduce mixed Numbers and Integers to the ſame Denomination.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2; then reduce the whole Number to an improper Fraction of the ſame Denominator with the former Fraction, by Art. 8; and theſe will be the Fractions ſought.

Exam. 1.	*Exam.* 2.	*Exam.* 3.
Reduce 2 $\frac{3}{4}$ and 6 to Fractions of the ſame Denominator.	Reduce 3 $\frac{1}{4}$ and 11 to Fractions of the ſame Denominator.	Reduce 12 $\frac{1}{2}$ and 62 to Fractions of the ſame Denominator.

2	6		3	11		12	62	
4	4		4	4		2	2	
8	24	$\frac{24}{4}$	12	44	$\frac{44}{4}$	24	124	$\frac{124}{2}$
3			1			1		
11	$\frac{11}{4}$		13	$\frac{13}{4}$		25	$\frac{25}{2}$	

Anſ. $\frac{11}{4}$ and $\frac{24}{4}$.	Anſ. $\frac{13}{4}$ and $\frac{44}{4}$.	Anſ. $\frac{25}{2}$ and $\frac{124}{2}$.

Exam. 1. The Integer 2 multiplied by the Denominator 4, to which Product adding 3, makes the improper Fraction $\frac{11}{4}$, equal to 2 $\frac{3}{4}$; then the Integer 6 multiplied by the Denominator 4 is 24, for a new Numerator to the other Fraction $\frac{24}{4}$, equal to 6: Whence $\frac{11}{4}$ and $\frac{24}{4}$ are the Fractions ſought.

The other Examples are done in the ſame Manner.

Addition of Vulgar Fractions.

Article 11. To add ſimple Fractions together.

Rule. If the given Fractions have the ſame Denominator, add the Numerators together, and place that Sum as a Numerator over the common Denominator; but, if the Denominators of the Fractions are different, reduce them to the ſame Denominator, by Art. 4, and then add their Numerators together, and under this Sum place the common Denominator; and this Fraction will be the Sum of the given Fractions.

Exam. 1.

Add $\frac{2}{3}$ and $\frac{3}{4}$ together.

$$\begin{array}{ccc}2&3&4\\ \underline{4}&\underline{3}&\underline{3}\\ 8&9&12\end{array}$$

$\frac{8}{12}$ and $\frac{9}{12}$

$$\begin{array}{r}9\\ \underline{8}\\ \frac{17}{12}\end{array}$$ Anſ.

Exam. 2.

Add $\frac{1}{2}$ and $\frac{3}{5}$ and $\frac{2}{7}$ together.

$$\begin{array}{cccc}1&3&2&2\\ \underline{5}&\underline{2}&\underline{5}&\underline{5}\\ 5&6&10&10\\ \underline{7}&\underline{7}&\underline{2}&\underline{7}\\ 35&42&20&70\end{array}$$

$\frac{35}{70}$ and $\frac{42}{70}$ and $\frac{20}{70}$

$$\begin{array}{r}35\\ 42\\ \underline{20}\\ \frac{97}{70}\end{array}$$ Anſ.

Exam 1. The given Fraction $\frac{2}{3}$ and $\frac{3}{4}$ being reduced, by Art. 4, to $\frac{8}{12}$ and $\frac{9}{12}$, add the Numerators 8 and 9 together; and their Sum 17 placed over the common Denominator 12 gives $\frac{17}{12}$, for the Sum of the given Fractions.

Exam. 2. The three Fractions reduced to a common Denominator, by Art. 4, are $\frac{35}{70}$, $\frac{42}{70}$, and $\frac{20}{70}$; therefore, adding the three Numerators 35, 42, and 20 together, and placing their Sum 97 over the common Denominator 70, we have the Fraction $\frac{97}{70}$, the Sum of the three given Fractions.

The two following Examples are done in the ſame Manner.

Exam. 3.

Add $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{1}{7}$ together.

$$\begin{array}{cccc}3&2&1&4\\ \underline{5}&\underline{4}&\underline{5}&\underline{5}\\ 15&8&5&20\\ \underline{7}&\underline{7}&\underline{4}&\underline{7}\\ 105&56&20&140\end{array}$$

Now the three Fractions to be added are $\frac{105}{140}$, $\frac{56}{140}$, and $\frac{20}{140}$.

$$\begin{array}{r}105\\ 56\\ \underline{20}\end{array}$$

Anſwer $\frac{181}{140}$.

Exam. 4.

Add $\frac{1}{6}$, $\frac{2}{5}$, and $\frac{3}{8}$ together.

$$\begin{array}{cccc}1&2&3&8\\ \underline{5}&\underline{6}&\underline{5}&\underline{5}\\ 5&12&15&40\\ \underline{8}&\underline{8}&\underline{6}&\underline{6}\\ 40&96&90&240\end{array}$$

Now the three Fractions to be added are $\frac{40}{240}$, $\frac{96}{240}$, and $\frac{90}{240}$.

$$\begin{array}{r}90\\ 96\\ \underline{40}\end{array}$$

Anſwer $\frac{226}{240}$.

Article 12. To add compound and ſimple Fractions together.

Rule. Reduce the compound Fraction to a ſimple Fraction, by Art. 3; then reduce that and the given ſimple Fraction to the ſame Denominator, by Art. 4; laſtly, add the Numerators together, under which Sum place the common Denominator, as in the laſt Article.

Exam. 1.

Add $\frac{3}{4}$ of $\frac{5}{9}$ and $\frac{2}{7}$ together.

$$\begin{array}{ccc} 3 & 4 & \\ 5 & 9 & \\ \hline 15 & 36 & \frac{15}{36} \end{array}$$

Now the two Fractions to be added are $\frac{15}{36}$ and $\frac{2}{7}$.

$$\begin{array}{ccc} 15 & 36 & 36 \\ 7 & 2 & 7 \\ \hline 105 & 72 & 252 \end{array}$$

Add $\frac{105}{252}$ to $\frac{72}{252}$, and $\frac{177}{252}$ is the Anſwer.

Exam. 2.

Add $\frac{2}{3}$ of $\frac{3}{8}$ and $\frac{4}{13}$ together.

$$\begin{array}{ccc} 2 & 3 & \\ 3 & 8 & \\ \hline 6 & 24 & \frac{6}{24} \end{array}$$

Now the two Fractions to be added are $\frac{6}{24}$ and $\frac{4}{13}$.

$$\begin{array}{ccc} 6 & 24 & 24 \\ 13 & 4 & 13 \\ \hline 78 & 96 & 72 \\ & & 24 \\ \hline & & 312 \end{array}$$

Add $\frac{78}{312}$ to $\frac{96}{312}$, and $\frac{174}{312}$ is the Anſwer.

Exam. 1. The compound Fraction $\frac{3}{4}$ of $\frac{5}{9}$ is reduced, by Art. 3, to $\frac{15}{36}$; reduce that and the given Fraction $\frac{2}{7}$ to the ſame Denominator, by Art. 4, and we have $\frac{105}{252}$ and $\frac{72}{252}$; then add the Numerators 105 and 72 together, and their Sum 177 placed over the common Denominator 252 is $\frac{177}{252}$, the Sum of the given Fractions.

Exam. 2. The compound Fraction $\frac{2}{3}$ of $\frac{3}{8}$, by Art. 3, is reduced to $\frac{6}{24}$; then reduce $\frac{6}{24}$ and $\frac{4}{13}$ to $\frac{78}{312}$ and $\frac{96}{312}$, by Art. 4; laſtly, add the Numerators together, placing their Sum over the Denominator 312, and they give $\frac{174}{312}$ for the Sum of the given Fractions.

The following Examples are done by the ſame Rule.

Exam. 3.

Add $\frac{1}{2}$ of $\frac{7}{9}$ of $\frac{5}{8}$ and $\frac{3}{4}$ together.

$$\begin{array}{ccc} 1 & 2 & \\ \underline{7} & \underline{9} & \\ 7 & 18 & \\ \underline{5} & \underline{8} & \\ 35 & 144 & \frac{35}{144} \end{array}$$

Now the two Fractions to be added are $\frac{35}{144}$ and $\frac{3}{4}$.

$$\begin{array}{ccc} 35 & 144 & 144 \\ \underline{4} & \underline{3} & \underline{4} \\ 140 & 432 & 576 \end{array}$$

Add $\frac{140}{576}$ to $\frac{432}{576}$, and $\frac{572}{576}$ is the Anſwer.

Exam. 4.

Add $\frac{9}{17}$ of $\frac{3}{10}$ and $\frac{11}{15}$ and $\frac{3}{8}$ together.

$$\begin{array}{ccc} 9 & 17 & \\ \underline{3} & \underline{10} & \\ 27 & 170 & \frac{27}{170} \end{array}$$

Now the three Fractions to be added are $\frac{27}{170}$, $\frac{11}{15}$, and $\frac{3}{8}$.

$$\begin{array}{cccc} 27 & 11 & 170 & 170 \\ \underline{15} & \underline{170} & \underline{3} & \underline{15} \\ 135 & 770 & 510 & 850 \\ \underline{27} & \underline{11} & \underline{15} & \underline{170} \\ 405 & 1870 & 2550 & 2550 \\ \underline{8} & \underline{8} & \underline{510} & \underline{8} \\ 3240 & 14960 & 7650 & 20400 \end{array}$$

Add $\frac{3240}{20400}$, $\frac{14960}{20400}$, $\frac{7650}{20400}$, and $\frac{25850}{20400}$ is the Sum ſought.

Article 13. To add mixed Numbers and ſimple Fractions together.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2; reduce this improper Fraction and the given ſimple Fraction to the ſame Denominator, by Art. 4; then add the new Numerators together, and place their Sum over the common Denominator, as in the two preceding Articles.

Exam. 1. Add $7\frac{3}{10}$ and $\frac{4}{9}$ together.

$$\begin{array}{c} 7 \\ \underline{10} \\ 70 \\ \underline{3} \\ 73 \end{array}$$

$\frac{73}{10}$ the improper Fraction in the Room of the mixed Number.

$$\begin{array}{ccc} \frac{73}{10} & \frac{4}{9} & \\ 73 & 4 & 10 \\ \underline{9} & \underline{10} & \underline{9} \\ 657 & 40 & 90 \end{array}$$

Add $\frac{657}{90}$ to $\frac{40}{90}$, and $\frac{697}{90}$ is the Anſwer.

Exam. 2. Add $3\frac{1}{7}$ and $\frac{12}{27}$ together.

3		$\frac{22}{7}$	$\frac{12}{27}$	
7				
—	$\frac{22}{7}$ the improper	22	12	27
21	Fraction in	27	7	7
1	the Room of	—	—	—
—	the mixed	154	84	189
22	Number.	44		
		—		
		594		

Add $\frac{594}{189}$ to $\frac{84}{189}$, and $\frac{678}{189}$ is the Anſwer.

Exam. 1. The mixed Number $7\frac{3}{10}$ is reduced, by Art 2, to the improper Fraction $\frac{73}{10}$; reduce this and the given Fraction $\frac{4}{9}$ to Fractions of a common Denominator, by Art. 4, and we have $\frac{657}{90}$ and $\frac{40}{90}$; then adding the two Numerators 657 and 40, it makes 697, to be placed over 90 the common Denominator: Thus is $\frac{697}{90}$ the Sum of the given Fraction and mixed Number.

The ſecond Example is done in the ſame Manner.

Article 14. To add mixed Numbers and compound Fractions together.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2; then reduce the compound Fraction to a ſimple Fraction, by Art. 3; reduce theſe two Fractions to the ſame Denominator, by Atr. 4; then add their Numerators together for a new Numerator, placing it over the common Denominator, and this Fraction will be the Sum of the given Fractions.

Exam. 1. Add $2\frac{5}{7}$ and $\frac{3}{4}$ of $\frac{7}{12}$ together.

2	3	12		19	21	48
7	7	4		48	7	7
—	—	—		—	—	—
14	21	48	$\frac{21}{48}$	152	147	336
5				76		
—				—		
19	$\frac{19}{7}$			912		

Hence the two ſimple Fractions are $\frac{19}{7}$ and $\frac{21}{48}$.

Add $\frac{912}{336}$ to $\frac{147}{336}$, and $\frac{1059}{336}$ is the Anſwer.

Exam. 2. Add 7 $\frac{9}{16}$ and $\frac{5}{8}$ of $\frac{3}{11}$ together.

16	5	11	121	15	88
7	3	8	88	16	16
112	15	88 $\frac{15}{88}$	968	90	528
9			968	15	88
121 $\frac{121}{16}$			10648	240	1408

Hence the two ſimple Fractions are $\frac{121}{16}$ and $\frac{15}{88}$.

Add $\frac{10648}{1408}$ to $\frac{240}{1408}$, and $\frac{10888}{1408}$ is the Anſwer.

Exam. 1. The mixed Number 2 $\frac{5}{7}$ is reduced, by Art. 2, to the improper Fraction $\frac{19}{7}$; and the compound Fraction $\frac{3}{4}$ of $\frac{7}{12}$ is reduced, by Art. 3, to the ſimple Fraction $\frac{21}{48}$; then reducing theſe two Fractions to Fractions of the ſame Denominator, by Art. 4, gives $\frac{912}{336}$ and $\frac{147}{336}$, whoſe Numerators added together make 1059, which being placed over the common Denominator 336 is $\frac{1059}{336}$, for the Sum of the given mixed Number and compound Fraction.

The ſecond Example and the two following are done in the ſame Way.

Exam. 3.

Add 2 $\frac{3}{8}$ and $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{4}{9}$ together.

2	2	9
8	5	6
16	10	54
3	4	3
19 $\frac{19}{8}$	40	162 $\frac{40}{162}$

Hence the two ſimple Fractions are $\frac{19}{8}$ and $\frac{40}{162}$.

162	40	162
19	8	8
1458	320	1296
162		
3078		

Add $\frac{3078}{1296}$ to $\frac{320}{1296}$, and $\frac{3398}{1296}$ is the Anſwer.

Exam. 4.

Add 3 $\frac{2}{5}$ and $\frac{1}{2}$ of $\frac{3}{4}$ together.

3	1	4
5	3	2
15	3	8 $\frac{3}{8}$
2		
17 $\frac{17}{5}$		

Hence the two ſimple Fractions are $\frac{17}{5}$ and $\frac{3}{8}$.

17	3	8
8	5	5
136	15	40

Add $\frac{136}{40}$ to $\frac{15}{40}$, and $\frac{151}{40}$ is the Anſwer.

Article 15. To add ſimple Fractions and Integers together.
This is only to annex the Fraction to the Integer, which makes it a mixed Number.

Exam. 1.	*Exam.* 2.
Add 7 and $\frac{5}{9}$ together.	Add 512 and $\frac{3}{4}$ together.
Anſwer 7 $\frac{5}{9}$.	Anſwer 512 $\frac{3}{4}$.

This wants no farther Explanation.

Article 16. To add compound Fractions and Integers together.

Rule. Reduce the compound Fraction to a ſimple Fraction, by Art. 3; then annex this ſimple Fraction to the Integer, as in the laſt Article.

Exam. 1.	*Exam.* 2.
Add $\frac{3}{5}$ of $\frac{7}{12}$ of $\frac{9}{20}$ and 16 together.	Add 200 and $\frac{3}{4}$ of $\frac{6}{7}$ together.

$$\begin{array}{rrl} 3 & 5 & \\ 7 & 12 & \\ \hline 21 & 60 & \\ 9 & 20 & \\ \hline 189 & 1200 & \frac{189}{1200} \end{array} \qquad \begin{array}{rrl} 3 & 4 & \\ 6 & 7 & \\ \hline 18 & 28 & \frac{18}{28} \end{array}$$

Anſwer 16 $\frac{189}{1200}$. Anſwer 200 $\frac{18}{28}$.

Exam. 1. The compound Fraction $\frac{3}{5}$ of $\frac{7}{12}$ of $\frac{9}{20}$ is reduced to $\frac{189}{1200}$, by Art. 3; then joining to this Fraction the given Integer 16, we have 16 $\frac{189}{1200}$, a mixed Number, for the Sum ſought.

The other Example is done in the ſame Manner.

Article 17. To add mixed Numbers and Integers together.

Rule. This is no more than to add the Integer to the integral Part of the Fraction, and then to annex the Fraction to this Sum, as at Art. 15.

Exam. 1.	*Exam.* 2.
Add 30 $\frac{3}{5}$ and 45 together.	Add 19 $\frac{5}{8}$ and 11 together.
Anſwer 75 $\frac{3}{5}$.	Anſwer 30 $\frac{5}{8}$.

Subtraction of Vulgar Fractions.

Article 18. To ſubtract ſimple Fractions.

Rule. If the given Fractions have the ſame Denominators ſubtract the leſſer Numerator from the greater, and place the Remainder over the common Denominator; this new Fraction is the Difference ſought. But, if the Fractions have different Denominators, reduce them to the ſame Denominator, by ſome of the foregoing Rules for reducing of Fractions; then ſubtract the leſſer Numerator from the greater, placing the Remainder over the common Denominator; this will be the Difference ſought.

Exam. 1. Subtract $\frac{2}{3}$ from $\frac{5}{6}$. *Exam.* 2. Subtract $\frac{1}{2}$ from $\frac{19}{23}$.

2	5	6	23	19	23
6	3	3	1	2	2
12	15	18	23	38	46

$\frac{12}{18}$ from $\frac{15}{18}$ leaves $\frac{3}{18}$ Anſwer. $\frac{23}{46}$ from $\frac{38}{46}$ leaves $\frac{15}{46}$ Anſwer.

Exam. 1. The given Fractions are reduced, by Art. 4, to $\frac{12}{18}$ and $\frac{15}{18}$; ſubtracting 12 from 15 there remains 3; which being placed over the Denominator 18 gives $\frac{3}{18}$ for the Difference, for ſo much $\frac{5}{6}$ is more than $\frac{2}{3}$.

The other Example is done in the ſame Manner.

Article 19. To ſubtract ſimple Fractions from compound Fractions, or compound from ſimple.

Rule. Reduce the compound Fraction to a ſimple Fraction, by Art. 3; then reduce this and the given ſimple Fraction to the ſame Denominator, and proceed as in the laſt Article.

Exam. 1. Subtract $\frac{1}{4}$ of $\frac{7}{8}$ from $\frac{4}{5}$.

1	4	
7	8	
7	32	$\frac{7}{32}$

Hence $\frac{7}{32}$ is in the Room of the compound Fraction.

$\frac{7}{32}$	$\frac{4}{5}$	
7	32	32
5	4	5
35	128	160

$\frac{35}{160}$ from $\frac{128}{160}$ leaves $\frac{93}{160}$ Anſ.

Exam. 2. Subtract $\frac{1}{3}$ of $\frac{5}{9}$ from $\frac{2}{7}$.

$$\begin{array}{ccc} 1 & 9 & \\ 5 & 3 & \\ \hline 5 & 27 & \frac{5}{27} \end{array}$$

Hence $\frac{5}{27}$ is in the Room of the compound Fraction.

$$\begin{array}{ccc} \frac{5}{27} & \frac{2}{7} & \\ 5 & 27 & 27 \\ 7 & 2 & 7 \\ \hline 35 & 54 & 189 \end{array}$$

$\frac{35}{189}$ from $\frac{54}{189}$ leaves $\frac{19}{189}$ Anſ.

Exam. 1. The compound Fraction $\frac{1}{4}$ of $\frac{7}{8}$ is reduced to $\frac{7}{32}$, by Art. 3; reduce that and the ſimple Fraction $\frac{4}{5}$ to the ſame Denominator, by Art. 4, and they are $\frac{35}{160}$ and $\frac{128}{160}$; then ſubtracting the Numerator 35 from the Numerator 128, the Remainder is 93, to be placed over the common Denominator 160: Thus is $\frac{93}{160}$ the Difference ſought.

The ſecond Example is done in the ſame Manner.

Exam. 3.

Subtract $\frac{1}{8}$ from $\frac{2}{3}$ of $\frac{1}{2}$.

$$\begin{array}{cc} 2 & 2 \\ 1 & 3 \\ \hline 2 & 6 \end{array}$$

$\frac{2}{6}$ in the Room of the compound Fraction.

Now we are to take $\frac{1}{8}$ from $\frac{2}{6}$.

$$\begin{array}{ccc} 1 & 8 & 8 \\ 6 & 2 & 6 \\ \hline 6 & 16 & 48 \end{array}$$

$\frac{6}{48}$ from $\frac{16}{48}$ leaves $\frac{10}{48}$ Anſwer.

Exam. 4.

Subtract $\frac{2}{9}$ from $\frac{3}{4}$ of $\frac{2}{3}$.

$$\begin{array}{cc} 3 & 4 \\ 2 & 3 \\ \hline 6 & 12 \end{array}$$

$\frac{6}{12}$ in the Room of the compound Fraction.

Now we are to take $\frac{2}{9}$ from $\frac{6}{12}$.

$$\begin{array}{ccc} 2 & 6 & 12 \\ 12 & 9 & 9 \\ \hline 24 & 54 & 108 \end{array}$$

$\frac{24}{108}$ from $\frac{54}{108}$ leaves $\frac{30}{108}$ Anſ.

These are done by the ſame Rule.

Article 20. To ſubtract ſimple Fractions from mixed Numbers.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2; and reduce this and the given Fraction to the ſame Denominator, by Art. 4; then ſubtract the leſſer Numerator from the greater, and place the Difference over the common Denominator, as at Art. 18.

Exam. 1.

Subtract $\frac{2}{5}$ from $13\frac{3}{4}$.

$$\begin{array}{r} 13 \\ 4 \\ \hline 52 \\ 3 \\ \hline 55 \end{array} \quad \frac{55}{4}.$$

Now the two Fractions are $\frac{2}{5}$ and $\frac{55}{4}$.

$$\begin{array}{r} 2 \\ 4 \\ \hline 8 \end{array} \qquad \begin{array}{r} 55 \\ 5 \\ \hline 275 \end{array} \qquad \begin{array}{r} 5 \\ 4 \\ \hline 20 \end{array}$$

$\frac{8}{20}$ from $\frac{275}{20}$ leaves $\frac{267}{20}$ Anſ.

Exam. 2.

Subtract $\frac{3}{4}$ from $10\frac{2}{7}$.

$$\begin{array}{r} 10 \\ 7 \\ \hline 70 \\ 2 \\ \hline 72 \end{array} \quad \frac{72}{7}$$

Now the two Fractions are $\frac{3}{4}$ and $\frac{72}{7}$.

$$\begin{array}{r} 3 \\ 7 \\ \hline 21 \end{array} \qquad \begin{array}{r} 72 \\ 4 \\ \hline 288 \end{array} \qquad \begin{array}{r} 7 \\ 4 \\ \hline 28 \end{array}$$

$\frac{21}{28}$ from $\frac{288}{28}$ leaves $\frac{267}{28}$ Anſ.

Exam. 1. The mixed Number $13\frac{3}{4}$ is reduced to the improper Fraction $\frac{55}{4}$, by Art. 2; then this Fraction and the given ſimple Fraction $\frac{2}{5}$ are reduced to $\frac{8}{20}$ and $\frac{275}{20}$, Fractions of the ſame Denominator, by Art. 4: Subtract 8 from 275 and there remains 267, to be placed over the common Denominator 20; ſo is $\frac{267}{20}$ the Difference ſought.

Exam. 2. The mixed Number $10\frac{2}{7}$ is reduced to the improper Fraction $\frac{72}{7}$, by Art. 2; then the Fraction $\frac{72}{7}$, and $\frac{3}{4}$ the given ſimple Fraction, reduced to the ſame Denominator, by Art. 4, are $\frac{21}{28}$ and $\frac{288}{28}$: The Difference of the Numerators 21 and 288 is 267, which placed over the Denominator 28 is $\frac{267}{28}$, the true Difference ſought.

The two following Examples are done in the ſame Manner.

Exam. 3. Subtract $\frac{3}{4}$ from $3\frac{2}{3}$.

$$\begin{array}{r} 3 \\ 3 \\ \hline 9 \\ 2 \\ \hline \end{array}$$

$\frac{11}{3}$ the improper Fraction.

$\frac{3}{4}$ and $\frac{11}{3}$ to be reduced to the ſame Denominator.

$$\begin{array}{r} 3 \\ 3 \\ \hline 9 \end{array} \qquad \begin{array}{r} 11 \\ 4 \\ \hline 44 \end{array} \qquad \begin{array}{r} 4 \\ 3 \\ \hline 12 \end{array}$$

$\frac{9}{12}$ from $\frac{44}{12}$ leaves $\frac{35}{12}$ Anſwer.

Exam. 4. Subtract $\frac{2}{5}$ from 2 $\frac{3}{7}$.

$$\begin{array}{l} 2 \\ \underline{7} \\ 14 \\ \underline{3} \\ \frac{17}{7} \text{ the improper Fraction.} \end{array}$$

$\frac{2}{5}$ and $\frac{17}{7}$ to be reduced to the ſame Denominator.

$$\begin{array}{ccc} 2 & 17 & 7 \\ \underline{7} & \underline{5} & \underline{5} \\ 14 & 85 & 35 \end{array}$$

$\frac{14}{35}$ from $\frac{85}{35}$ leaves $\frac{71}{35}$ Anſwer.

Article 21. To ſubtract compound Fractions from mixed Numbers.

Rule. Reduce the compound Fraction to a ſimple Fraction, by Art. 3, and the mixed Number to an improper Fraction, by Art. 2; then reduce theſe two new Fractions to the ſame Denominator, by Art. 4: Laſtly, ſubtract the leſſer Numerator from the greater, and place the Remainder over the common Denominator, for the Difference ſought.

Exam. 1.

Subtract $\frac{4}{5}$ of $\frac{7}{12}$ from 6 $\frac{2}{3}$.

$$\begin{array}{cccc} 4 & 5 & & 6 \\ \underline{7} & \underline{12} & & \underline{3} \\ 28 & 60 & \frac{28}{60} & 18 \\ & & & \underline{2} \\ & & & 20 \quad \frac{20}{3} \end{array}$$

Hence the two ſingle Fractions are $\frac{28}{60}$ and $\frac{20}{3}$.

$$\begin{array}{ccc} 28 & 20 & 60 \\ \underline{3} & \underline{60} & \underline{3} \\ 84 & 1200 & 180 \end{array}$$

$\frac{84}{180}$ from $\frac{1200}{180}$ leaves $\frac{1116}{180}$ Anſ.

Exam. 2.

Subtract $\frac{1}{3}$ of $\frac{2}{4}$ of $\frac{2}{7}$ from 19 $\frac{3}{5}$.

$$\begin{array}{cccccc} 1 & 2 & & 19 & \\ \underline{3} & \underline{4} & & \underline{5} & \\ 3 & 8 & & 95 & \\ \underline{2} & \underline{7} & & \underline{3} & \\ 6 & 56 & \frac{6}{56} & 98 & \frac{98}{5} \end{array}$$

Hence the two ſingle Fractions are $\frac{6}{56}$ and $\frac{98}{5}$.

$$\begin{array}{ccc} 6 & 98 & 56 \\ \underline{5} & \underline{56} & \underline{5} \\ 30 & 588 & 280 \\ & \underline{490} & \\ & 5488 & \end{array}$$

$\frac{30}{280}$ from $\frac{5488}{280}$ leaves $\frac{5458}{280}$ Anſ.

Exam. 1. The compound Fraction $\frac{4}{5}$ of $\frac{7}{12}$ is reduced to the ſimple Fraction $\frac{28}{60}$, by Art. 3, and the mixed Number 6 $\frac{2}{3}$ to the improber Fraction $\frac{20}{3}$, by Art. 2: Reduce theſe to the ſame Denominator, by Art, 4, and they make $\frac{84}{180}$ and $\frac{1200}{180}$: Laſtly, ſubtracting the leſſer Numerator from the greater, and placing the Remainder over the common Denominator, $\frac{1116}{180}$ is the Difference ſought.

The other Example is done in the ſame Manner.

Article 22. To ſubtract ſimple Fractions from Integers.

Rule. Subtract the Numerator of the given Fraction from the Denominator, place the Remainder over the Denominator, and join this Fraction to the Integers leſſened by 1: This is the Anſwer ſought.

Exam. 1.

Subtract $\frac{2}{5}$ from 7.

$$\begin{array}{cccl} 5 & & 7 & \\ 2 & & 1 & \\ \hline 3 & \frac{3}{5} & 6 & .\ \text{Anſ. } 6\frac{3}{5}. \end{array}$$

Exam. 2.

Subtract $\frac{9}{17}$ from 3.

$$\begin{array}{cccl} 17 & & 3 & \\ 9 & & 1 & \\ \hline 8 & \frac{8}{17} & 2 & \text{Anſ. } 2\frac{8}{17}. \end{array}$$

Exam. 1. The Numerator 2 being ſubtracted from the Denominator 5, there remains 3; which placed over the ſame Denominator makes $\frac{3}{5}$, the Remainder of the Fraction $\frac{2}{5}$ from an Unit: Then taking 1 from the Integer 7, for the 1 borrowed in ſubtracting the Fraction $\frac{2}{5}$ from 1 or Unity, the Remainder is 6, to be annexed to the aforeſaid $\frac{3}{5}$: So is $6\frac{3}{5}$ the Remainder.

The other Example is done in the ſame Manner.

The Reaſon of the Rule may be beſt underſtood by an Example taken from common Arithmetick: If it be required to ſubtract 3*l.* 6*s.* 8*d.* from 7*l.* as there are no Shillings or Pence in the Subtrahend, or Sum given for the other to be ſubtracted from, it is performed in the following Manner, *viz.*

l.	*s.*	*d.*
7	0	0
3	6	8
3	13	4

In this Example, there being no Pence in the Sum given to be ſubtracted from, we borrow 12 Pence, or 1 Shilling, to ſubtract the 8 from, and the Remainder is 4; this borrowed Shilling is then carried to the Shillings, which increaſes the

6 Shillings to 7 Shillings; then, there being no Shillings to ſubtract the 7 from, we again borrow 1 from the next Article of Pounds; from which 1 Pound, or 20 Shillings, borrowed, the 7 is ſubtracted; after which, that borrowed Pound is carried to the Place of Pounds. Now this is the ſame that is done in this Rule, for 8 Pence is $\frac{8}{12}$ of a Shilling; and this 8 ſubtracted from the Denominator 12 leaves $\frac{4}{12}$, *i. e.* 4 Pence; but then 1 is carried to the Place of Shillings, which is ſubtracting 1 from the Integer, of which the Fraction was a Part. So here $\frac{2}{5}$ of an Integer were to be ſubtracted from 7 Integers: Now, $\frac{2}{5}$ of an Integer ſubtracted from 1 of thoſe Integers, the Remainder muſt be $\frac{3}{5}$; becauſe $\frac{2}{5}$ and $\frac{3}{5}$ added together make $\frac{5}{5}$, or 1 Integer; then, as this 1 was borrowed to ſubtract the $\frac{2}{5}$ from, it muſt be carried as in common Subtraction; the Operation being in Effect thus, $\frac{2}{5}$ from nothing I cannot, but $\frac{2}{5}$ from $\frac{5}{5}$, or 1, there remains $\frac{3}{5}$; then I ſay, 1 that borrowed from 7, there remains 6; to which I annex the $\frac{3}{5}$ before remaining, and find the whole Remainder to be 6 $\frac{3}{5}$.

Exam. 3.

Subtract $\frac{3}{4}$ from 5.

$$\begin{array}{cccc} 4 & & 5 & \\ 3 & & 1 & \\ \hline 1 & \frac{1}{4} & 4 & \text{Anſ. } 4\frac{1}{4}. \end{array}$$

Exam. 4.

Subtract $\frac{1}{2}$ from 10.

$$\begin{array}{cccc} 2 & & 10 & \\ 1 & & 1 & \\ \hline 1 & \frac{1}{2} & 9 & \text{Anſ. } 9\frac{1}{2}. \end{array}$$

Article 23. To ſubtract compound Fractions from Integers.

Rule. Reduce the compound Fraction to a ſimple Fraction, by Art. 3; then proceed as in the laſt Article.

Exam. 1.

Subtract $\frac{2}{3}$ of $\frac{5}{9}$ from 29.

$$\begin{array}{cccccc} 2 & 3 & & 27 & & 29 \\ 5 & 9 & & 10 & & 1 \\ \hline 10 & 27 & \frac{10}{27} & 17 & \frac{17}{27} & 28 \end{array}$$

Anſwer 28 $\frac{17}{27}$.

Exam. 2.

Subtract $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{7}{8}$ from 5.

$$\begin{array}{ccccc} 1 & 2 & & 64 & 5 \\ 3 & 4 & & 21 & 1 \\ \hline 3 & 8 & & 43 & 4 \\ 7 & 8 & & & \\ \hline 21 & 64 & \frac{21}{64} & & \end{array}$$

Anſwer 4 $\frac{43}{64}$.

Exam. 1. The compound Fraction $\frac{2}{3}$ of $\frac{5}{9}$ is reduced to the ſimple Fraction $\frac{10}{27}$, by Art. 3; ſubtract the Numerator of this ſimple Fraction from the Denominator, and under the Remainder place the Denominator, and we have the Fraction $\frac{17}{27}$, to be annexed to the whole Number 29, leſs by the Unit that was borrowed: So is 28 $\frac{17}{27}$ the Remainder ſought.

The other Example is done in the ſame Manner.

Article 24. To ſubtract a mixed Number from an Integer greater than the Integer of the mixed Number.

Rule. Subtract the Numerator of the fractional Part of the mixed Number from the Denominator, and place the Remainder over the Denominator, as in Art. 22; then carrying the 1 which was borrowed to the integral Part of the mixed Number, ſubtract that from the Integer from which the mixed Number was to be taken, and annex the former remaining Fraction to this laſt Remainder; this is the Anſwer ſought.

Ex. 1. Subtract 3 $\frac{2}{5}$ from 6. *Ex.* 2. Subtract 22 $\frac{7}{10}$ from 56.

$$\begin{array}{llll} 5 & & 6 & \\ \underline{2} & & \underline{4} & \\ 3 & \frac{3}{5} & 2 & \text{Anſ. } 2\frac{3}{5}. \end{array} \qquad \begin{array}{llll} 10 & & 56 & \\ \underline{7} & & \underline{23} & \\ 3 & \frac{3}{10} & 33 & \text{Anſ. } 33\frac{3}{10}. \end{array}$$

Exam. 1. The Numerator 2 ſubtracted from the Denominator 5, there remains 3; that is, the Fraction $\frac{2}{5}$ ſubtracted from 1 (which is borrowed) the Remainder is $\frac{3}{5}$: Then ſay, the 1 that I borrowed and 3 is 4 from 6, there remains 2; to which join the $\frac{3}{5}$; ſo is 2 $\frac{3}{5}$ the Remainder required.

The other Example is done by the ſame Method.

Multiplication of Vulgar Fractions.

Article 25. To multiply ſimple Fractions together.

Rule. Multiply all the Numerators together for a new Numerator; then multiply all the Denominators together for a new Denominator; ſo is this new Fraction the Product ſought.

Exam. 1. Multiply $\frac{3}{4}$ by $\frac{7}{8}$. *Exam.* 2. Multiply $\frac{9}{15}$ by $\frac{2}{3}$.

$$\begin{array}{lll} 3 & 4 & \\ \underline{7} & \underline{8} & \\ 21 & 32 & \text{Anſ. } \frac{21}{32}. \end{array} \qquad \begin{array}{lll} 9 & 15 & \\ \underline{2} & \underline{3} & \\ 18 & 45 & \text{Anſ. } \frac{18}{45}. \end{array}$$

Exam. 1. The Numerators 3 and 7 multiplied give 21; and the Denominators 4 and 8 multiplied are 32: Therefore the Fraction $\frac{21}{32}$ is the Product of $\frac{3}{4}$ by $\frac{7}{8}$. The ſame of the other Example. And, if there had been more than two Fractions given, the Numerators and Denominators muſt all have been multiplied together in the ſame Manner.

Article 26. To multiply compound Fractions and ſimple Fractions together, or compound with compound.

Rule. Multiply all the Numerators, both of the ſeveral Parts of which the compound Fraction conſiſts, and of the ſimple Fraction, and the laſt Product is a new Numerator; then multiplying all the Denominators in the ſame Manner, the laſt Product is a new Denominator. This new Fraction is the Product. The ſame Rule is to be obſerved, if both the Fractions given to be multiplied are compound Fractions.

Exam. 1. Multiply $\frac{1}{3}$ of $\frac{2}{5}$ by $\frac{7}{9}$.			*Exam.* 2. Multiply $\frac{5}{6}$ of $\frac{11}{14}$ by $\frac{2}{3}$ of $\frac{7}{8}$.		
1	3		11	14	
2	5		5	6	
2	15		55	84	
7	9		2	3	
14	135	Anſ. $\frac{14}{135}$.	110	252	
			7	8	
			770	2016	Anſ. $\frac{770}{2016}$.

Theſe Examples, I think, do not require any Explication. But the Learner may probably obſerve, that all Fractions decreaſe by Multiplication, the Product being leſs than either of the Fractions given to be multiplied; the Reaſon of which is, that, all proper Fractions being only Parts of Unity, and therefore leſs than Unity, multiplying by a Fraction is only taking ſo many Parts of the Thing multiplied, as are expreſſed by the Fraction by which it is multiplied: For, as, in multiplying by a whole Number, the Product is ſo many Times the Multiplicand as there are Units in the Multiplier, and therefore multiplying by 1 neither adds nor diminiſhes; ſo, in multiplying by a Fraction, the Product is but ſo many Parts of the Multiplicand as the Multiplier is Parts of an Unit: All Multiplication being in Geometrical Proportion, the firſt

Term of which is 1; thus, as 1 is to the Multiplier, ſo is the Multiplicand to the Product; therefore, if the Multiplier is leſs than 1 (as all proper Fractions are) the Product will be leſs than the Multiplicand.

Article 27. To multiply mixed Numbers and ſimple or compound Fractions together.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2; then proceed to multiply the Numerators together for a new Numerator, and the Denominators for a new Denominator, as in the two laſt Articles.

Exam. 1.

Multiply $12\frac{3}{4}$ by $\frac{5}{6}$.

$$\begin{array}{r} 12 \\ 4 \\ \hline 48 \\ 3 \\ \hline 51 \end{array} \qquad \frac{51}{4}$$

$$\frac{51}{4} \text{ by } \frac{5}{6}$$

$$\begin{array}{rr} 51 & 4 \\ 5 & 6 \\ \hline 255 & 24 \end{array} \qquad \text{Anſ. } \frac{24}{255}.$$

Exam. 2.

Multiply $3\frac{4}{7}$ by $\frac{2}{3}$ of $\frac{5}{6}$.

$$\begin{array}{r} 3 \\ 7 \\ \hline 21 \\ 4 \\ \hline 25 \end{array} \qquad \frac{25}{7}$$

$$\frac{25}{7} \text{ by } \frac{2}{3} \text{ of } \frac{5}{6}$$

$$\begin{array}{rr} 25 & 7 \\ 2 & 3 \\ \hline 50 & 21 \\ 5 & 6 \\ \hline 250 & 126 \end{array} \qquad \text{Anſ. } \frac{250}{126}.$$

Exam. 1. The mixed Number $12\frac{3}{4}$ is reduced to the improper $\frac{51}{4}$, by Art. 2; then multiplying 51 by 5, the Product is 255; and the Denominators 4 and 6 multiplied give 24: Therefore $\frac{255}{24}$ is the Product ſought.

Exam. 2. The mixed Number $3\frac{4}{7}$ is reduced to the improper Fraction $\frac{25}{7}$, by Art. 2; then multiplying the Numerators 25, 2, and 5 together gives 250 for a new Numerator; then the Denominators 7, 3, and 6 multiplied give 126 for the new Denominator: So is $\frac{250}{126}$ the Product ſought.

Article 28. To multiply ſimple Fractions and Integers together.

Rule. Reduce the Integer to an improper Fraction, by Art. 1, that is, by placing an Unit under it for a Denominator; then proceed as in the foregoing Articles of Multiplication.

Exam. 1.

Multiply $\frac{5}{7}$ by 8.

$\frac{5}{7}$ by $\frac{8}{1}$

$$\begin{array}{rrl} 5 & 1 & \\ 8 & 7 & \\ \hline 40 & 7 & \text{Anſ. } \frac{40}{7}. \end{array}$$

Exam. 2.

Multiply $\frac{17}{29}$ by 56.

$\frac{17}{29}$ by $\frac{56}{1}$

$$\begin{array}{rrl} 56 & 1 & \\ 17 & 29 & \\ \hline 392 & 29 & \\ 56 & & \\ \hline 952 & & \text{Anſ. } \frac{952}{29}. \end{array}$$

Exam. 1. Placing 1 for a Denominator under the Integer 8 it makes $\frac{8}{1}$; then the Numerators 8 and 5 multiplied is 40 for a new Numerator, and the Denominators 1 and 7 multiplied is 7 for a new Denominator: So is $\frac{40}{7}$ the Product ſought. The other Example is done in the ſame Manner. But it may be neceſſary to obſerve, that, if a compound Fraction was given to be multiplied by an Integer, it is to be done in the ſame Manner as at Art. 26, as may be ſeen by the following Examples.

Exam. 1.

Multiply 26 by $\frac{2}{3}$ of $\frac{1}{2}$.

$\frac{2}{3}$ of $\frac{1}{2}$ by $\frac{26}{1}$.

$$\begin{array}{rrl} 26 & 1 & \\ 2 & 3 & \\ \hline 52 & 3 & \\ 1 & 2 & \\ \hline 52 & 6 & \text{Anſ. } \frac{52}{6}. \end{array}$$

Exam. 2.

Multiply 19 by $\frac{3}{7}$ of $\frac{9}{16}$.

$\frac{3}{7}$ of $\frac{9}{16}$ by $\frac{19}{1}$.

$$\begin{array}{rrl} 19 & 1 & \\ 3 & 7 & \\ \hline 57 & 7 & \\ 9 & 16 & \\ \hline 513 & 42 & \\ & 7 & \\ \hline & 112 & \text{Anſ. } \frac{513}{112}. \end{array}$$

Exam. 1. The Integer 26 is reduced to the improper Fraction $\frac{26}{1}$, by Art. 1; then the Numerators 26, 2, and 1 multiplied is 52 for the new Numerator; and the Denominators 1, 3, and 2 multiplied is 6 for a new Denominator: Thus is $\frac{52}{6}$ the Product ſought.

The other Example is done in the ſame Manner.

Article 29. To multiply mixed Numbers and Integers together.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2, and the Integer to an improper Fraction, by Art. 1; then proceed to multiply the Numerators together, and alſo the Denominators, according to the preceeding Articles.

Exam. 1.

Multiply 12 $\frac{2}{3}$ by 19.

	$\frac{38}{3}$ by $\frac{19}{1}$	
12		
3	38	3
—	19	1
36	——	—
2	342	3
—	38	
38	——	
	722	Anſ. $\frac{722}{3}$.

Exam. 2.

Multiply 27 by 2 $\frac{9}{10}$.

	$\frac{29}{10}$ by $\frac{27}{1}$	
2		
10	29	10
—	27	1
20	——	—
9	203	10
—	58	
29	——	
	783	Anſ. $\frac{783}{10}$.

Theſe Examples being ſo plain, their Explication is omitted.

Diviſion of Vulgar Fractions.

Article 30. To divide ſimple Fractions by ſimple Fractions.

Rule. Multiply the Denominator of the Diviſor into the Numerator of the Dividend for the new Numerator; and the Numerator of the Diviſor multiplied into the Denominator of the Dividend will be the Denominator of the Quotient.

Exam. 1.

Divide $\frac{3}{4}$ by $\frac{1}{2}$.

$\frac{1}{2}$) $\frac{3}{4}$ ($\frac{6}{4}$ Quotient.

Exam. 2.

Divide $\frac{1}{2}$ by $\frac{3}{4}$.

$\frac{3}{4}$) $\frac{1}{2}$ ($\frac{4}{6}$ Quotient.

Exam. 1. The Denominator of the Diviſor 2 multiplied by by 3 the Numerator of the Dividend is 6, for the Numerator of the Quotient; and 1 the Numerator of the Diviſor multiplied by 4 the Denominator of the Dividend is 4, for the

Denominator of the Quotient: Therefore $\frac{6}{4}$ is the Quotient ſought.

Exam. 2. Multiplying 4 the Denominator of the Diviſor by 1 the Numerator of the Dividend, it makes 4, for the Numerator of the Quotient; and 3 the Numerator of the Diviſor multiplied by 2 the Denominator of the Dividend is 6, for the Denominator of the Quotient: So that the Quotient here is $\frac{4}{6}$.

The Learner will probably obſerve, that, as the Product in Multiplication was leſs than either of the given Fractions, the Quotient in Diviſion is greater than either of the given Fractions. The Reaſon is the ſame; for every Diviſion is a geometrical Proportion, or Rule of three; that is, as the Diviſor is to 1, ſo is the Dividend to the Quotient: If, therefore, the Diviſor, or firſt Term, is leſs than 1 (as all proper Fractions are) then muſt the Dividend be leſs than the Quotient.

Article 31. To divide compound Fractions and ſimple Fractions.

Rule. Reduce the compound Fraction to a ſimple Fraction; then proceed as at Art. 30.

Exam. 1.

Divide $\frac{1}{3}$ of $\frac{3}{5}$ by $\frac{4}{9}$.

$$\begin{array}{ccc} 1 & 3 & \\ \underline{3} & \underline{5} & \\ 3 & 15 & \frac{3}{15} \end{array}$$

$\frac{4}{9}$) $\frac{3}{15}$ ($\frac{27}{60}$ Quotient.

Exam. 2.

Divide $\frac{4}{5}$ of $\frac{2}{7}$ by $\frac{3}{8}$.

$$\begin{array}{ccc} 4 & 5 & \\ \underline{2} & \underline{7} & \\ 8 & 35 & \frac{8}{35} \end{array}$$

$\frac{3}{8}$) $\frac{8}{35}$) $\frac{64}{105}$ Quotient.

Exam. 1. The compound Fraction $\frac{1}{3}$ of $\frac{3}{5}$, reduced by Art. 3, is $\frac{3}{15}$; then 9 the Denominator of the Diviſor multiplied into 3 the Numerator of the Dividend is 27, for the Numerator of the Quotient; and 4 the Numerator of the Diviſor multiplied into 15 the Denominator of the Dividend is 60, for the Denominator of the Quotient.

The ſecond Example is done in the ſame Manner.

If a compound Fraction be given to be divided by a compound Fraction, reduce both the compound Fractions to ſimple Fractions, by Art. 3; then proceed as at Art. 30.

Exam. 1.

Divide $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{2}{3}$ of $\frac{4}{5}$.

$$\begin{array}{cccccc} 1 & 4 & & 2 & 3 & \\ 3 & 2 & & 4 & 5 & \\ \hline 3 & 8 & \frac{3}{8} & 8 & 15 & \frac{8}{15} \end{array}$$

$\frac{8}{15}$) $\frac{3}{8}$ ($\frac{45}{64}$ Quotient.

Exam. 2.

Divide $\frac{2}{5}$ of $\frac{1}{2}$ by $\frac{3}{4}$ of $\frac{5}{7}$.

$$\begin{array}{cccccc} 2 & 5 & & 3 & 7 & \\ 1 & 2 & & 5 & 4 & \\ \hline 2 & 10 & \frac{2}{10} & 15 & 28 & \frac{15}{28} \end{array}$$

$\frac{15}{28}$) $\frac{2}{10}$ ($\frac{56}{150}$ Quotient.

Exam. 1. The compound Fraction $\frac{1}{2}$ of $\frac{3}{4}$ is reduced to the ſimple Fraction $\frac{3}{8}$, and the compound Fraction $\frac{2}{3}$ of $\frac{4}{5}$ is reduced to $\frac{8}{15}$, by Art. 3; then proceed as in Art. 30, and we have $\frac{45}{64}$ for the Quotient ſought.

The ſecond Example is done in the ſame Manner.

Article 32. To divide a mixed Number by a ſimple Fraction, or a ſimple Fraction by a mixed Number.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2; then proceed according to the Directions at Art. 30.

Exam. 1.

Divide 10 $\frac{7}{9}$ by $\frac{5}{6}$.

$$\begin{array}{cc} 10 & \\ 9 & \\ \hline 90 & \\ 7 & \\ \hline 97 & \frac{97}{9} \end{array}$$

$\frac{5}{6}$) $\frac{97}{9}$ ($\frac{582}{45}$ Quotient.

Exam. 2.

Divide 3 $\frac{1}{4}$ by $\frac{2}{3}$.

$$\begin{array}{cc} 3 & \\ 4 & \\ \hline 12 & \\ 1 & \\ \hline 13 & \frac{13}{4} \end{array}$$

$\frac{2}{3}$) $\frac{13}{4}$ ($\frac{39}{8}$ Quotient.

Exam. 1. The mixed Number 10 $\frac{7}{9}$ is reduced to the improper Fraction $\frac{97}{9}$, by Art. 2; then the Denominator 6 multiplied by the Numerator 97 gives 582, for the Numerator of the Quotient; and the Numerator 5 multiplied by the Denominator 9 gives 45, for the Denominator of the Quotient: So is $\frac{582}{45}$ the Quotient ſought.

The ſecond Example is done in the ſame Manner.

Article 33. To divide mixed Numbers and compound Fractions.

Rule. Reduce the mixed Number to an improper Fraction, by Art. 2, and the compound Fraction to a ſimple Fraction, by Art. 3; then proceed as at Art. 30.

Exam. 1.

Divide 2 $\frac{1}{9}$ by $\frac{2}{3}$ of $\frac{5}{6}$.

$$\begin{array}{llll} 2 & 2 & 3 & \\ \underline{9} & \underline{5} & \underline{6} & \\ 18 & 10 & 18 & \frac{10}{18} \\ \underline{1} & & & \\ 19 & \frac{19}{9} & & \end{array}$$

$\frac{10}{18}$) $\frac{19}{9}$ ($\frac{342}{90}$ Quotient.

Exam. 2.

Divide $\frac{1}{3}$ of $\frac{3}{7}$ by 9 $\frac{2}{5}$.

$$\begin{array}{llll} 1 & 3 & & 9 \\ \underline{3} & \underline{7} & & \underline{5} \\ 3 & 21 & \frac{3}{21} & 45 \\ & & & \underline{2} \\ & & & 47 \quad \frac{47}{5} \end{array}$$

$\frac{47}{5}$) $\frac{3}{21}$ ($\frac{15}{987}$ Quotient.

Exam. 1. The mixed Number 2 $\frac{1}{9}$ is reduced to $\frac{19}{9}$, by Art. 2, and the compound Fraction $\frac{2}{3}$ of $\frac{5}{6}$ is reduced to the ſimple Fraction $\frac{10}{18}$, by Art. 3; then 18 the Denominator of the Diviſor multiplied into 19 the Numerator of the Dividend is 342, for the Numerator of the Quotient: Laſtly, 10 the Numerator of the Diviſor multiplied into 9 the Denominator of the Dividend is 90, for the Denominator of the Quotient $\frac{342}{90}$.

Exam. 2. The mixed Number 9 $\frac{2}{5}$ is reduced to $\frac{47}{5}$, by Art. 2, and the compound Fraction $\frac{1}{3}$ of $\frac{3}{7}$ to $\frac{3}{21}$, by Art. 3; multiply the Denominator of the Diviſor into the Numerator of the Dividend for the Numerator of the Quotient, and the Numerator of the Diviſor into the Denominator of the Dividend for the Denominator of the Quotient $\frac{15}{987}$.

Article 34. To divide ſimple Fractions, mixed Numbers, or compound Fractions, by Integers; or Integers by ſimple or compound Fractions, or mixed Numbers.

Rule. Make the Integers, whether they are the Diviſor or Dividend, an improper Fraction, by putting an Unit for a Denominator, by Art. 1. If the Diviſor or Dividend be a mixed Number, reduce it to an improper Fraction, by Art. 2: And, if the Diviſor or Dividend be a compound Fraction, reduce it to a ſimple Fraction, by Art. 3: After which, the Diviſion muſt be performed as directed at Art. 30.

Exam. 1.

Divide $\frac{7}{15}$ by 4.

$\frac{7}{15}$ by $\frac{4}{1}$

$\frac{4}{1}$) $\frac{7}{15}$ ($\frac{7}{60}$ Quotient.

Exam. 2.

Divide $\frac{1}{3}$ of $\frac{4}{5}$ by 9.

$$\begin{array}{cc} 1 & 3 \\ 4 & 5 \\ \hline 4 & 15 \end{array}$$

$\frac{4}{15}$ by $\frac{9}{1}$

$\frac{9}{1}$) $\frac{4}{15}$ ($\frac{4}{135}$ Quotient.

Exam. 1. The Integer 4 is reduced to $\frac{4}{1}$, by Art. 1; then multiplying the Denominator of the Diviſor $\frac{4}{1}$ into the Numerator of the Dividend $\frac{7}{15}$, it gives 7, for a new Numerator; and multiplying 4 the Numerator of the Diviſor into 15 the Denominator of the Dividend, it gives 60, for a new Denominator: So is $\frac{7}{60}$ the Quotient.

Exam. 2. The compound Fraction $\frac{1}{3}$ of $\frac{4}{5}$ is reduced to $\frac{4}{15}$, by Art. 2, and the Integer to $\frac{9}{1}$, by Art. 1; then multiply as before, and there is $\frac{4}{135}$ for the Quotient.

Exam. 3.

Divide $7\frac{1}{4}$ by 13.

$$\begin{array}{r} 7 \\ 4 \\ \hline 28 \\ 1 \\ \hline 29 \end{array}$$

$\frac{29}{4}$ by $\frac{13}{1}$

$\frac{13}{1}$) $\frac{29}{4}$ ($\frac{29}{52}$ Quotient.

Exam. 4.

Divide 11 by $3\frac{1}{5}$.

$$\begin{array}{r} 3 \\ 5 \\ \hline 15 \\ 1 \\ \hline 16 \end{array}$$

$\frac{16}{5}$) $\frac{11}{1}$ ($\frac{55}{16}$ Quotient.

Exam. 3. The mixed Number $7\frac{1}{4}$ is reduced to $\frac{29}{4}$, by Art. 2, and 13 to $\frac{13}{1}$, by Art. 1; multiply 1 the Denominator of the Diviſor into 29 the Numerator of the Dividend for a new Numerator, and multiply 13 the Numerator of the Diviſor into 4 the Denominator of the Dividend for a new Denominator, and the Quotient is $\frac{29}{52}$.

Exam. 4. The Diviſor $3\frac{1}{5}$ is reduced to $\frac{16}{5}$, and the Dividend 11 to $\frac{11}{1}$; and, proceeding as in the other Examples, the Quotient will be $\frac{55}{16}$.

Thus having gone thro' the ſeveral Rules of adding, ſubtracting, multiplying, and dividing Vulgar Fractions, we ſhall now ſhew the Learner how to find the Value of Fractions of Money, Weights, or Meaſures, and to reduce Fractions to their loweſt Terms; theſe Articles are not neceſſary to the learning the Rules of Addition, Subtraction, &c. but are neceſſary, after the Operations are performed, to diſcover the Value and Reſult of the Work.

Article 35. To find the Value of a Fraction in the cuſtomary Diviſions of Money, Weights, Meaſures, and the like: As if it was required to find the Value of $\frac{6}{10}$ of a Pound Sterling.

Rule. Multiply the Numerator by 20, the Shillings in 1 Pound, then divide the Product by the Denominator, and the Quotient is Shillings, and, if nothing remains, is the true Value of the Fraction: If any Thing remains, multiply it by 12, the Pence in 1 Shilling, then divide the Product by the Denominator, and the Quotient is Pence, and, if nothing remains, the Shillings before found, and the Pence now found, is the exact Value of the given Fraction: But, if any Thing remains, multiply it by 4, the Farthings in 1 Penny, then divide the Product by the Denominator, and the Quotient is Farthings, which, being added to the Shillings and Pence before found, is the Value of the given Fraction; the Remainder, after the Multiplication by the ſmalleſt Diviſion into which the Integer is divided, being generally neglected.

Let it be required to find the Value of the following Fractions of a Pound Sterling.

Exam. 1. Of $\frac{6}{10}$.	*Exam.* 2. Of $\frac{8}{10}$.	*Exam.* 3. Of $\frac{2}{8}$.
6	8	2
20	20	20
10) 120 (12 *s.* Anſ.	10) 160 (16 *s.* Anſ.	8) 40 (5 *s.* Anſ.
10	10	40
20	60	0
20	60	
0	0	

Exam. 1. The Numerator 6 being multiplied by 20, and the Product divided by the Denominator 10, the Quotient is 12 Shillings, the exact Value of $\frac{6}{10}$ of a Pound, as nothing remains. And this appears to be true; for the *one Tenth* of a Pound is 2 Shillings, and, conſequently, *ſix Tenths* muſt be 12 Shillings.

The ſame of the other two Examples.

Exam. 4. Of $\frac{15}{40}$.	*Exam.* 5. Of $\frac{25}{40}$.	*Exam.* 6. Of $\frac{35}{40}$.
15	25	35
20	20	20
40) 300 (7 *s.*	40) 500 (12 *s.*	40) 700 (17 *s.*
280	40	40
20	100	300
12	80	280
40) 240 (6 *d.*	20	20
240	12	12
0	40) 240 (6 *d.*	40) 240 (6 *d.*
Anſwer 7 *s.* 6 *d.*	240	240
	0	0
	Anſwer 12 *s.* 6 *d.*	Anſwer 17 *s.* 6 *d.*

Exam. 1. The Numerator 15 being multiplied by 20, and the Product divided by the Denominator 40, we have 7 Shillings in the Quotient, and a Remainder of 20; which being multiplied by 12, the Product is 240, and, this being ſtill divided by the Denominator 40, the Quotient is 6 Pence, and nothing remains: Whence the exact Value of $\frac{15}{40}$ of a Pound Sterling is 7 *s.* 6 *d.* And this is true; for the *one Fortieth* of a Pound is 6 Pence, and, conſequently, *fifteen Fortieths* muſt be 7 *s.* 6 *d.*

The ſame of the other two Examples.

Exam. 7. Of $\frac{457}{960}$.

457
20
96|0)914|0(9*s.*
864
50
12
96)600(6*d.*
576
24
4
96)96(1*q.*
96
0

Anſ. 9*s.* 6*d.* ¼.

Exam. 8. Of $\frac{877}{960}$.

877
20
96|0(1754|0(18*s.*
96
794
768
26
12
96)312(3*d.*
288
24
4
96)96(1*q.*
96
0

Anſ. 18*s.* 3*d.* ¼.

Exam. 9. Of $\frac{947}{960}$.

947
20
96|0(1894|0(19*s.*
96
934
864
70
12
96)840(8*d.*
768
72
4
96)288(3*q.*
288
0

Anſ. 19*s.* 8*d.* ¾.

Exam. 7. The Numerator 457 being multiplied by 20, and the Product 9140 divided by the Denominator 960, the Quotient is 9 Shillings; and there remains 50, which being multiplied by 12, and the Product 600 divided by 96, (the Cypher of the Diviſor being cut off at the laſt Operation) the Quotient is 6 Pence; and there remains 24, which being multiplied by 4, and the Product 96 divided by 96, the Quotient is 1 Farthing, and nothing remains: Whence the exact Value of $\frac{457}{960}$ of a Pound is 9*s.* 6*d.* ¼. Which is true; for the *nine-hundred-and-ſixtieth* Part of a Pound is 1 Farthing, and, conſequently, *four hundred and fifty-ſeven* ſuch Parts are ſo many Farthings, or 9*s.* 6*d.* ¼.

The ſame of the eighth and ninth Examples.

Exam. 10.

Exam. 10. Of $\frac{5}{7}$.	*Exam.* 11. Of $\frac{7}{17}$.	*Exam.* 12. Of $\frac{13}{21}$.
5	7	13
20	20	20
7) 100 (14 *s.*	17) 140 (8 *s.*	21) 260 (12 *s.*
7	136	21
30	4	50
28	12	42
2	17) 48 (2 *d.*	8
12	34	12
7) 24 (3 *d.*	14	21) 96 (4 *d.*
21	4	84
3	17) 56 (3 *q.*	12
4	51	4
7) 12 (1 *q.*	5	21) 48 (2 *q.*
7		42
5		6

Exam. 10. The Numerator 5 being multiplied by 20, and the Product 100 divided by the Denominator 7, the Quotient is 14 Shillings; and there remains 2, which being multiplied by 12, and the Product 24 divided by 7, the Quotient is 3 Pence; and there remains 3, which being multiplied by 4, and the Product 12 divided by 7, the Quotient is 1 Farthing; and 5 remains; but, there being in *Engliſh* Money nothing leſs than Farthings, the Remainder is neglected: Hence we call the Value of $\frac{5}{7}$ of a Pound, 14 *s.* 3 *d.* $\frac{1}{4}$, tho' it is ſtrictly $\frac{5}{7}$, or *five Sevenths*, of a Farthing more; that is, the Remainder is made a Numerator and the Diviſor a Denominator of a Fraction, which is a Part of the Integer laſt found. Thus, at Example 11, the true Anſwer is 8 *s.* 2 *d.* $\frac{3}{4}$, and $\frac{5}{17}$ of a Farthing; and, at Example 12, the true Anſwer is 12 *s.* 4 *d.* $\frac{1}{2}$, and $\frac{6}{21}$ of a Farthing; but the Fractions of the loweſt Denomination are generally neglected in Practice.

And, if it was required to find the Value of a Vulgar Fraction of a Yard, multiply the Numerator by 3, the Feet in 1 Yard, then divide the Product by the Denominator, and the Quotient is Feet, and is the true Value of the Fraction, if there is no Remainder: But, if there is a Remainder, multiply it by 12, the Inches in 1 Foot, then divide the Product by the Denominator, and the Quotient is Inches, and, if there is no Remainder, the Feet and Inches thus found is the true Value ſought: But, if there is ſtill a Remainder, multiply it by 4, the Quarters in 1 Inch, then divide the Product by the Denominator, and the Quotient is Quarters of an Inch, which, being added to the Feet and Inches before found, will be the Value of the Fraction in Feet, Inches, and Quarters of an Inch: After which, the Remainder may be neglected, unleſs greater Exactneſs is required.

To find the Value of the following Fractions of one Yard.

Exam. 1. Of $\frac{8}{12}$.	*Exam.* 2. Of $\frac{5}{15}$.	*Exam.* 3. Of $\frac{16}{24}$.
8	5	16
3	3	3
12) 24 (2 Feet.	15) 15 (1 Foot.	24) 48 (2 Feet.
24	15	48
0	0	0

The Operation in theſe Examples will appear very eaſy by the Directions.

Exam. 4. Of $\frac{18}{36}$.	*Exam.* 5. Of $\frac{54}{72}$.	*Exam.* 6. Of $\frac{11}{18}$.
18	54	11
3	3	3
36) 54 (1 Foot.	72) 162 (2 Feet.	18) 33 (1 Foot.
36	144	18
18	18	15
12	12	12
36) 216 (6 Inches.	72) 216 (3 Inches.	18) 180 (10 Inches.
216	216	18
0	0	00
Anſ. 1 F. 6 Inch.	Anſ. 2 F. 3 Inch.	Anſ. 1 F. 10 Inch.

Exam. 4. The Numerator 18 being multiplied by 3, and the Product 54 divided by 36, gives 1 Foot; and 18 remains, which being multiplied by 12, and the Product 216 divided by

36, gives 6 Inches, and nothing remains: So is 1 F. 6 Inch. the exact Value of $\frac{18}{36}$ of a Yard.

The ſame of the fifth and ſixth Examples.

Exam. 7. Of $\frac{5}{18}$.	*Exam.* 8. Of $\frac{1}{9}$.	*Exam.* 9. Of $\frac{11}{36}$.
5	1	11
3	3	3
18) 15 (0 Feet.	9) 3 (0 Feet.	36) 33 (0 Feet.
12	12	12
18) 180 (10 Inches.	9) 36 (4 Inches.	36) 396 (11 Inches.
18	36	36
00	0	36
		36
Anſ. 10 Inches.	Anſ. 4 Inches.	0
		Anſ. 11 Inches.

Exam. 7. The Numerator 5 being multipied by 3, and the Product 15 divided by the Denominator 18, it gives 0 Feet in the Quotient; then multiply the 15, which is now the Remainder, by 12, and it makes 180, which being divided by 18 gives 10 Inches, the exact Value of $\frac{5}{18}$ of a Yard.

And ſo of the eighth and ninth Examples.

Exam. 10. Of $\frac{5}{7}$.	*Exam.* 11. Of $\frac{4}{11}$.	*Exam.* 12. Of $\frac{3}{13}$.
5	4	3
3	3	3
7) 15 (2 Feet.	11) 12 (1 Foot.	13) 9 (0 Feet.
14	11	12
1	1	13) 108 (8 Inches.
12	12	104
7) 12 (1 Inch.	11) 12 (1 Inch.	4
7	11	4
5	1	13) 16 (1 Qr.
4	4	13
7) 20 (2 Qrs.	11) 4 (0 Qrs.	3
14		
6		

In the tenth Example, the Anſwer is 2 Feet, 1 Inch, and 2 Quarters, or 1 Half; or exactly 2 Feet, 1 Inch, 2 Quarters, and $\frac{6}{7}$ of 1 Quarter of an Inch. In the eleventh Example, the Anſwer is 1 Foot, 1 Inch, and $\frac{4}{11}$ of 1 Quarter of an Inch. And, in the twelfth Example, the Anſwer is 8 Inches, 1 Quarter, and $\frac{3}{11}$ of 1 Quarter of an Inch.

And, in general, to find the Value of any Vulgar Fraction, is only to multiply the Numerator into the ſeveral Denominations that compoſe the Integer of which the Fraction is a Part, beginning with the Denomination which comes next to the Integer, and dividing by the Denominator, as in the foregoing Examples, which will give the Value ſought.

When the Value of an improper Fraction is required, divide the Numerator by the Denominator, and the Quotient will be Integers, and is the true Value, if there is no Remainder; if there is any Remainder, multiply it, and divide by the Denominator, as before taught, by which Means we ſhall have the Value of the improper Fraction.

Find the Value of the following Fractions of a Pound Sterling.

Exam. 1. Of $\frac{27}{8}$.	*Exam.* 2. Of $\frac{34}{9}$.	*Exam.* 3. Of $\frac{13}{7}$.
8) 27 (3 *l.*	9) 34 (3 *l.*	7) 13 (1 *l.*
24	27	7
3	7	6
20	20	20
8) 60 (7 *s.*	9) 140 (15 *s.*	7) 120 (17 *s.*
56	9	7
4	50	50
12	45	49
8) 48 (6 *d.*	5	1
48	12	12
0	9) 60 (5 *d.*	7) 12 (1 *d.*
Anſ. 3 *l.* 7 *s.* 6 *d.*	45	7
	5	5
	4	4
	9) 20 (2 *q.*	7) 20 (2 *q.*
	18	14
	2	6
	Anſ. 3 *l.* 15 *s.* 5 *d.* $\frac{1}{2}$	Anſ. 1 *l.* 17 *s.* 1 *d.* $\frac{1}{2}$

Exam. 1. The Numerator 27 being divided by 8 gives 3 in the Quotient, which is 3 Pounds; after this the Operation is the ſame as before; and ſo we find the improper Fraction $\frac{27}{8}$ of a Pound, is 3*l.* 7*s.* 6*d.*

And ſo of the other two Examples; where, in the Anſwers, the Remainders 2 and 6 are neglected.

If the Value of a compound Fraction is required, reduce it to a ſimple Fraction, by Art. 3, and then proceed as before taught.

Required the Value of the following Fractions.

Exam. 1.	*Exam.* 2.
Of $\frac{1}{2}$ of $\frac{3}{8}$ of a Pound Sterling.	Of $\frac{3}{4}$ of $\frac{2}{3}$ of a Yard.
1 2	3 4
3 8	2 3
3 16 $\frac{3}{16}$	6 12 $\frac{6}{12}$
3	6
20	3
16) 60 (3*s.*	12) 18 (1 Foot.
48	12
12	6
12	12
16) 144 (9*d.*	12) 72 (6 Inches.
144	72
0	0
Anſwer 3*s.* 9*d.*	Anſwer 1 F. 6 Inch.

The compound Fraction $\frac{1}{2}$ of $\frac{3}{8}$ is reduced to the ſimple Fraction $\frac{3}{16}$, by Art. 3; afterwards its Value is found as before taught, and is 3*s.* 9*d.* Which is ſhewn to be the true Value thus: The *eighth* Part of a Pound is 2*s.* 6*d.* whence *three Eighths* are 7*s.* 6*d.* but the *one Half* of 7*s.* 6*d.* is 3*s.* 9*d.* as in the Anſwer. And, as to the other Example, the *two Thirds* of a Yard are 2 Feet, but the *three Fourths* of 2 Feet are 18 Inches, or 1 F. 6 Inch. as before found.

The great Variety in Vulgar Fractions makes them appear with ſome Difficulty to a Learner; but I would not have him be diſcouraged, if he does not at firſt remember every Rule; it will be ſufficient, for his preſent Purpoſe, if he ſees how the Operation is performed by the Rule, for then he will quickly recover the Manner of any Operation by having Recourſe to the Directions.

Article 36. To reduce a Fraction to its loweſt Terms.

Rule. Divide the Denominator by the Numerator; if any Thing remains, divide the Numerator by the Remainder; and, if any Thing yet remains, divide the laſt Diviſor by ſuch Remainder, 'till nothing remains; then is that Diviſor the greateſt common Meaſure which will divide both the Numerator and the Denominator without a Remainder. By the common Meaſure ſo found divide the Numerator, and the Quotient is a new Numerator; then divide the Denominator by the ſame Number, and the Quotient is a new Denominator; and this new Fraction is equal to the given Fraction.

Exam. 1. Reduce $\frac{252}{315}$ to its loweſt Terms.

252) 315 (1
252
63) 252 (4
252
0

63) 252 (4 a new Numerator.
252
0
63) 315 (5 a new Denominator.
315
0 Anſ. $\frac{4}{5}$.

Exam. 2. Reduce $\frac{189}{513}$ to its loweſt Terms.

189) 513 (2
378
135) 189 (1
135
54) 135 (2
108
27) 54 (2
54
0

27) 189 (7 a new Numerator.
189
0
27) 513 (19 a new Denominator.
27
243
243
0 Anſ. $\frac{7}{19}$.

Exam. 3. Reduce $\frac{171}{399}$ to its loweſt Terms.

```
171) 399 (2          57) 399 (7
     342                 399
     ———                 ———
  57) 171 (3               0
      171
      ———
        0        Anſwer 3/7.
```

Exam. 1. The Denominator 315 being divided by the Numerator 252, there remains 63; by which divide the Diviſor 252, and nothing remains; therefore 63 is the common Meaſure ſought: Then divide the Numerator 252 by this common Meaſure 63, and the Quotient is 4, for the new Numerator; and dividing the Denominator 315 by the ſame 63, the Quotient is 5, for a new Denominator: So is the Fraction $\frac{4}{5}$ equal to $\frac{252}{315}$, the loweſt Terms to which it can be reduced.

Exam. 2. The Denominator 513 being divided by the Numerator 189, the Remainder is 135; by which divide the Diviſor 189, and there remains 54; and dividing the laſt Diviſor by 54, there remains 27; then divide the laſt Diviſor 54 by 27, and there remains nothing: Therefore 27 is the greateſt common Meaſure; by which divide the Numerator 189, and the Quotient is 7, for the new Numerator; and dividing 513 the Denominator by the ſame 27, the Quotient is 19: Therefore $\frac{7}{19}$ is the loweſt Terms of the given Fraction $\frac{189}{513}$.

The third Example is done in the ſame Manner; only as the ſecond Diviſion, not having any Remainder, ſhews, that the Diviſor 57 is the common Meaſure; and, as the Dividend was the Numerator, it was not neceſſary to divide the Numerator again by it, as is done in the firſt Example, ſince it might there have been omitted: Whence 3 is the new Numerator; and dividing 399 by 57, it gives 7 for a new Denominator: And ſo is $\frac{171}{399}$ reduced to its loweſt Terms $\frac{3}{7}$.

Exam. 4. Reduce $\frac{255}{663}$ to its loweſt Terms.

255) 663 (2
510

153) 255 (1
153

102) 153 (1
102

51) 102 (2
102

0

51) 255 (5
255

0

51) 663 (13
51

153
153

0

Anſ. $\frac{5}{13}$.

Exam. 5. Reduce $\frac{481}{629}$ to its loweſt Terms.

481) 629 (1
481

148) 481 (3
444

37) 148 (4
148

0

37) 481 (13
37

111
111

0

37) 629 (17
37

259
259

0 Anſ. $\frac{13}{17}$.

But if, in dividing to find a common Meaſure, the Remainder happens to be an Unite, then the Fraction is already in its loweſt Terms.

There are other Methods of reducing Fractions to their loweſt Terms: For if the Numerator and Denominator of a Fraction have each one or more Cyphers on the right Hand, the ſame Number of Cyphers in the Numerator and Denominator may be cut off, and the remaining Fraction will ſtill be of equal Value: Thus $\frac{60}{90}$, in the firſt Example of Art. 35, is the ſame with $\frac{6}{9}$, the taking away the Cyphers not altering the Value of the Fraction. So again, in the fourth Example of the ſame Article, $\frac{60}{150}$ may be reduced to $\frac{6}{15}$. Alſo, if either the Numerator or Denominator have a Cypher, and the other end in 5, than may both Parts be reduced by dividing by 5: Thus this Fraction, $\frac{125}{450}$, each Part being divided by 5, will be reduced to $\frac{25}{90}$; which, being again divided by 5, will be reduced to $\frac{5}{18}$, which is its loweſt Terms.

Another Method of reducing a Fraction is, if the Numerator and Denominator are even Numbers, take the Half of each; and, if they are yet both even, halve them again; and by this Means a Fraction may often be reduced ſo low, that it may appear what Number will divide both the Numerator and Denominator, ſo as to reduce them to their loweſt Terms.

Exam. 1. Reduce $\frac{112}{168}$ by Halving.

Half 112 is 56.	Half 168 is 84.
Half 56 is 28.	Half 84 is 42.
Half 28 is 14.	Half 42 is 21.

As both theſe laſt Numbers will divide by 7, this will reduce them to $\frac{2}{3}$, their loweſt Terms.

Exam. 2. Reduce $\frac{192}{288}$ by Halving.

Half 192 is 96.	Half 288 is 144.
Half 96 is 48.	Half 144 is 72.
Half 48 is 24.	Half 72 is 36.
Half 24 is 12.	Half 36 is 18.
Half 12 is 6.	Half 18 is 9.

Here 6 and 9 are diviſible by 3, which reduces them to $\frac{2}{3}$, their loweſt Terms.

Sometimes it happens a Fraction may be reduced by dividing it by 3, 4, 5, or any of the other Integers, as in the following Examples; where the Figures that ſtand over each Partition are the Numbers which divide both the Numerator and Denominator.

Exam. 1. To reduce $\frac{384}{576}$ to lower Terms.

$$\begin{array}{c|c|c|c|c} 2 & 4 & 4 & 6 & \\ \frac{384}{576} & \frac{192}{288} & \frac{48}{72} & \frac{12}{18} & \frac{2}{3} \end{array}$$

Exam. 2. To reduce $\frac{810}{1080}$ to lower Terms.

$$\begin{array}{c|c|c|c|c|c} 2 & 3 & 5 & 3 & 3 & \\ \frac{810}{1080} & \frac{405}{540} & \frac{135}{180} & \frac{27}{36} & \frac{9}{12} & \frac{3}{4} \end{array}$$

In the firſt Example, the Numerator and Denominator of the Fraction $\frac{384}{576}$ may be divided exactly by 2, which gives $\frac{192}{288}$; and this may be divided by 4, which gives $\frac{48}{72}$; and dividing this by 4, it gives $\frac{12}{18}$; and dividing this by 6, it gives $\frac{2}{3}$ for a new Fraction, in the Room of $\frac{384}{576}$. In the ſame Manner is the other Operation done. And in this Method the Learner is at Liberty to try any of the Integers for a Diviſor; only he muſt carefully remember, that it muſt divide both the Numerator and Denominator exactly, without any Remainder. And, if he has an Inclination to ſatisfy his Curioſity, to know whether he has done his Operation right, ſay, As the given Denominator is to the given Numerator, ſo is the new Denominator to the new Numerator; then multiply the firſt and fourth Numbers, and the ſecond and third Numbers together; if theſe Products are equal, the Works is true. Let us take the laſt Example:

As 1080 is to 810, ſo is 4 to 3.

$$\begin{array}{cc} 1080 & 810 \\ 3 & 4 \\ \hline 3240 & 3240 \end{array}$$

In the ſame Manner may any of the other Examples be proved.

Article 37. To reduce a Fraction of a lower Denomination to a Fraction of a higher; as the Fraction of a Farthing to the Fraction of a Shilling or a Pound.

Rule. Make it a compound Fraction, thro' all the intermediate Diviſions between the given Fraction and that Denomination to which it is to be reduced, as in the following Examples; and then reduce this compound Fraction to a ſimple Fraction, by Art. 3.

Exam. 1.

Reduce $\frac{2}{3}$ of a Farthing to the Fraction of a Pound.

$\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$.

$$\begin{array}{cc} 2 & 3 \\ 1 & 4 \\ \hline 2 & 12 \\ 1 & 12 \\ \hline 2 & 144 \\ 1 & 20 \\ \hline 2 & 2880 \end{array}$$

Anſ. $\frac{2}{2880}$ of a Pound Sterling.

Exam. 2.

Reduce $\frac{9}{10}$ of an Inch to the Fraction of a Foot.

$\frac{9}{10}$ of $\frac{1}{12}$.

$$\begin{array}{cc} 9 & 10 \\ 1 & 12 \\ \hline 9 & 120 \end{array}$$

Anſ. $\frac{9}{120}$ of a Foot.

Exam. 1. A Farthing is $\frac{1}{4}$ of a Penny, whence $\frac{2}{3}$ of a Farthing are $\frac{2}{3}$ of $\frac{1}{4}$ of a Penny; and, becauſe a Penny is $\frac{1}{12}$ of a Shilling, therefore $\frac{2}{3}$ of a Farthing are $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of a Shilling; and, becauſe a Shilling is $\frac{1}{20}$ of a Pound, $\frac{2}{3}$ of a Farthing are $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a Pound: Which compound Fraction being reduced to a ſimple Fraction, by Art. 3, is $\frac{2}{2880}$ of a Pound Sterling, equal to $\frac{2}{3}$ of a Farthing.

Exam. 2. Becauſe an Inch is $\frac{1}{12}$ of a Foot, therefore $\frac{9}{10}$ of an Inch is $\frac{9}{10}$ of $\frac{1}{12}$ of a Foot: Which compound Fraction being reduced to a ſimple Fraction, by Art. 3, is $\frac{9}{120}$ of a Foot, equal to $\frac{9}{10}$ of an Inch.

Article 38. To reduce a Fraction of a greater Denomination to a Fraction of a leſs.

Rule. This is the Reverſe of the former; and is performed by multiplying the Numerator of the given Fraction, by all the intermediate Diviſions of the Integer, for the new Numerator; under which place the given Denominator, and this will be the Fraction ſought.

Exam. 1.

Reduce $\frac{3}{4}$ of a Pound Sterling to the Fraction of a Farthing.

$$\begin{array}{r} 3\\ 20\\ \hline 60\\ 12\\ \hline 720\\ 4\\ \hline 2880 \end{array}$$

Anſ. $\frac{2880}{4}$ of a Farthing.

Exam. 2.

Reduce $\frac{3}{5}$ of a Pound Sterling to the Fraction of a Penny.

$$\begin{array}{r} 3\\ 20\\ \hline 60\\ 12\\ \hline 720 \end{array}$$

Anſ. $\frac{720}{5}$ of a Penny.

Thus have we done with the moſt difficult Part of Arithmetic, the Vulgar Fractions; only it may be neceſſary to ſhew the Learner the Manner of working an Example or two in the Rule of Three direct, in Fractions.

Article 39. To reſolve Queſtions in the Rule of Three in Fractions.

Rule. After the Queſtion is ſtated according to the uſual Manner in the common *Rule of Three*, multiply the Denominator of the firſt Term into the Product of the Numerators of the ſecond and third Terms, for a new Numerator; then multiply the Numerator of the firſt Term into the Product of the Denominators of the ſecond and third, for a new Denominator; and this new Fraction is the Anſwer ſought.

Exam. 1.	*Exam.* 2.
If $\frac{3}{4}$ of a Yard of Lace coſt $\frac{7}{8}$ of a Pound, what will 5 Yards coſt?	If $\frac{3}{5}$ of a Pound of Silver coſt 2*l.* 6*s.* 8*d.* what will $\frac{5}{8}$ of a Pound coſt?
As $\frac{3}{4}$ is to $\frac{7}{8}$, ſo is $\frac{5}{1}$.	As $\frac{3}{5}$ is to $\frac{7}{3}$, ſo is $\frac{5}{8}$.
$\begin{array}{rr} 7 & 8 \\ 5 & 1 \\ \hline 35 & 8 \\ 4 & 3 \\ \hline 140 & 24 \end{array}$	$\begin{array}{rr} 7 & 8 \\ 5 & 3 \\ \hline 35 & 24 \\ 5 & 3 \\ \hline 175 & 72 \end{array}$
Anſwer $\frac{140}{24}$ of a Pound, or 5*l.* 16*s.* 8*d.*	Anſwer $\frac{175}{72}$ of a Pound, or 2*l.* 8*s.* 7*d.* $\frac{1}{4}$.

Exam. 1. The 5 Yards are reduced to the improper Fraction $\frac{5}{1}$, by Art. 1; then multiply the Numerators of the ſecond and third Terms, and that Product being multiplied by 4, the Denominator of the firſt Term, it gives 140 for the Numerator of the fourth Term; then the Product of the Denominators of the ſecond and third Terms being multiplied by 3, the Numerator of the firſt Term, it gives 24 for the Denominator of the fourth Term: So is $\frac{140}{24}$ the Anſwer ſought; the Value of which, by Art. 35, is 5*l.* 16*s.* 8*d.* If the Learner has the Curioſity to examine this Queſtion by common Arithmetic, he may turn it into whole Numbers thus; $\frac{7}{8}$ of a Pound is 17*s.* 6*d.* then if $\frac{3}{4}$ of a Yard coſt 17*s.* 6*d.* 5 Yards will coſt 5*l.* 16*s.* 8*d.*

Exam. 2. The 6*s.* 8*d.* are changed into the Fraction of a Pound Sterling thus; becauſe 20 Shillings make a Pound, therefore 6 Shillings are $\frac{6}{20}$ of a Pound; and 12 Pence making 1 Shilling, and 20 Shillings 1 Pound, therefore 8 Pence are $\frac{8}{12}$ of $\frac{1}{20}$ of a Pound, by Art. 37: Whence $\frac{6}{20}$ of a Pound, being added to $\frac{8}{12}$ of $\frac{1}{20}$ of a Pound, will be in Fractions the ſame Thing as 6*s.* 8*d.* But $\frac{8}{12}$ of $\frac{1}{20}$ being reduced to a ſimple Fraction, by Art. 3, are $\frac{8}{240}$; then $\frac{6}{20}$ and $\frac{8}{240}$ being reduced to Fractions of the ſame Denomination, by Art. 4, are $\frac{1440}{4800}$ and $\frac{160}{4800}$; and the Sum of theſe Fractions, by

Art. 11, is $\frac{1600}{4800}$: Laſtly, this Fraction being reduced to its loweſt Terms, by Art. 36, is $\frac{1}{3}$; whence 2*l.* 6*s.* 8*d.* is in Fractions 2 $\frac{1}{3}$; which being reduced to an improper Fraction, by Art. 2, is $\frac{7}{3}$. Now ſtate the Queſtion in common Arithmetic, and proceed according to the Directions, and the Anſwer will be $\frac{175}{72}$ of a Pound, or 2*l.* 8*s.* 7*d.* $\frac{1}{2}$.

Of Decimal Fractions.

DECIMAL Fractions are a modern Invention, tho' of great Uſe: It is ſaid to be the Invention of *Regiomontanus*, for the Calculation of his Tables. Some object againſt Decimal Arithmetic, becauſe, in Diviſion, it does not always give the true Quotient, only very near, which will be the Caſe while there is a Remainder; whereas the Vulgar Fractions give the Truth. But this Objection is of no real Force: For if it be conſidered, that, tho' you can't have the true Quotient in ſome Caſes, yet, in all Caſes, you may come as near the Truth as you pleaſe, ſo as to be within the 10000th Part of a Farthing, or a Grain, or whatever elſe the Integer is. Add to this, that, in a Decimal Fraction, we can ſee, by bare Inſpection, without any Trouble of Reduction, the Value of it to Half a Farthing in Money, to $\frac{1}{8}$ of an Inch in Foot-Meaſure, and to as ſmall a Part in other Things, as will be ſhewn hereafter; which, in moſt Caſes, is near enough. Whereas, in Vulgar Fractions, after you have been at the Trouble of reducing them to the known Parts of Coin, Weights, Meaſures, *&c.* by Art. 35, you often have a fractional Part of the leaſt Denomination into which the Integer is uſually divided, as of a Farthing in Fractions of Money. But the ſuperior Excellence of Decimal Fractions will be readily acknowledged, by all who are acquainted with the Facility of their Operation, compared with the Difficulty of Vulgar Fractions, tho' theſe have their Uſe in ſome Caſes.

The Definition and Notation of Decimal Fractions.

When Integers are divided, or ſuppoſed to be divided, into 10, 100, 1000, *&c.* equal Parts, any Number of thoſe Parts are what are called Decimal Fractions: A Decimal Fraction, therefore, is that which has for its Denominator an Unit, with a Cypher, or Cyphers, annexed to it, as 10, 100, 1000, *&c.* But the uſual Way of noting down Decimal Fractions, and which makes them more ready for Uſe, is to omit the Denominator, and prefix a Point or Comma before the Numerator to the left Hand, to diſtinguiſh them from whole Numbers.

As, in whole Numbers, Cyphers to the right Hand increaſe the Value or Power of the Figures in a ten-fold Proportion; ſo, in Decimal Fractions, Cyphers to the left Hand decreaſe their Value or Power in a ten-fold Proportion. Thus .5 ſtands for $\frac{5}{10}$, or five tenth Parts; .05 for $\frac{5}{100}$, or five hundredth Parts; .005 for $\frac{5}{1000}$, or five thouſandth Parts. And, as Cyphers on the left Hand of whole Numbers neither add or diminiſh, in the ſame Manner, Cyphers on the right Hand of Decimals are of no Signification; for .8 and .80 and .800 are all the ſame Fraction, ſince they are $\frac{8}{10}$, $\frac{80}{100}$, and $\frac{800}{1000}$. And it has been already ſhewn, at Art. 36, that, when there are Cyphers both in the Numerator and Denominator, the ſame Number in both may be cut off or omitted: By which may be ſeen, that the difficult Work of reducing Vulgar Fractions to the ſame Denominators, in order to their being added or ſubtracted, is performed in the Decimals without any Trouble.

The Order of Places in Decimals proceeds from the left Hand to the right: The firſt next the Dot is called the Place of tenth Parts, the ſecond the Place of hundredth Parts, the third the Place of thouſandth Parts, *&c.* becauſe whatever Figure ſtands in thoſe Places ſtands reſpectively for ſo many tenth, hundredth, thouſandth, *&c.* Parts of an Unit: As in this, .5473; here the 5, in the firſt Place, is $\frac{5}{10}$, the 4, in the ſecond Place, is $\frac{4}{100}$; the 7, in the third Place, is $\frac{7}{1000}$; and the 3, in the laſt Place, being the fourth, is $\frac{3}{10000}$; and may be ſeparately expreſſed, .5, .04, .007, .0003. The Denominator of a Decimal Fraction is always known by the Number of Places in the Decimal, the Denominator conſiſting of as many Cyphers as there are decimal Places, with an Unit prefixed; always obſerving, that Cyphers in the Decimal are reckoned as Places as well as Figures: Thus .05207 is a Decimal of five Places, whoſe Denominator is 100000.

For the better underſtanding the Notation of Decimals, the following Scheme of Numbers, increaſing in a ten-fold Proportion from Unity to the left Hand, and decreaſing in the ſame Proportion from Unity to the right Hand, will be of Service.

	&c.		
Seventh Place	4	Millions	of Units.
Sixth Place	7	Hundred Thouſands	
Fifth Place	3	Ten Thouſands	
Fourth Place	5	Thouſands	
Third Place	2	Hundreds	
Second Place	8	Tens	
Firſt Place	6	Units (under 10)	
	.		
Firſt Place	7	Tenth Parts	of 1 Unit.
Second Place	9	Hundredth Parts	
Third Place	2	Thouſandth Parts	
Fourth Place	4	Ten Thouſandth Parts	
Fifth Place	5	Hundred Thouſandth Parts	
Sixth Place	3	Millionth Parts	
	&c.		

In this Scheme, or Table, you may obſerve, that the Places of whole Numbers are ſeparated from the Places of decimal Parts by a Dot. The Number on the left Hand of the Dot is 4735286, which expreſſes *four Million, ſeven Hundred and thrity-ſive Thouſand, two Hundred and eighty-ſix* Units or Integers. The Number on the right Hand of the Dot expreſſes *ſeven Hundred and ninety-two Thouſand, four Hundred and fifty-three* Parts of an Unit, the whole being divided into one Million equal Parts; which is to be wrote thus, $\frac{792453}{1000000}$, according to the Method of Vulgar Fractions; or thus, .792453, according to the Decimal Notation. So that, if I were to read this Decimal Fraction, .7, I ſhould call it *ſeven Tenths* of an Unit, the Unit being divided into 10 Parts. If I were to read this Decimal Fraction, .79, I ſhould call it *ſeven Tenths and nine Hundredth* Parts of an Unit, or *ſeventy-nine Hundredth* Parts of an Unit, the Unit being now divided into 100 equal Parts. And .792 is *ſeven Tenths, nine Hundredths, and two Thouſandth* Parts, or *ſeven Hundred and ninety two Thouſandth* Parts of an Unit, the Unit being now divided into 1000 Parts. And .7924 is *ſeven Tenths, nine* *ndredths, two Thouſandths, and four Ten-thouſandth* Parts of an Unit, or *ſeven Thouſand, nine Hundred and twenty-four Ten-thouſandth* Parts, the Unit being now divided into 10000 Parts; and the ſame of any other Decimal Fraction.

Addition of Decimal Fractions.

Article 40. To add Decimal Fractions together.

Rule. Place them one under the other, as in Addition of whole Numbers, obſerving to place Tenths under Tenths, Hundredths under Hundredths, Thouſandths under Thouſandths, and ſo on; then add them, as in whole Numbers, ſetting down the odd ones, and carrying the Tens to the next Place. If the laſt Place, or Place of Tenths, next the Dots, amounts to any Number of Tens, they muſt be carried to the Place of Units, on the left Hand of the Dot.

Exam. 1. Add .127 and .244 and .348 together.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
·127	·3075	·415	·215
·244	·1468	·028	·4923
·348	·2579	·473	·0514
·719	·7122	·916	·7587

Exam. 1. Having placed the ſeveral Fractions one under another, according to their ſeveral Denominations, and added them, as in whole Numbers, the Sum is .719, or *ſeven Tenths, one Hundredth, and nine Thouſandth* Parts of an Integer or Unit, or *ſeven Hundred and nineteen Thouſandth* Parts of an Integer.

The other three Examples are added in the ſame Manner.

Exam. 5.	*Exam.* 6.	*Exam.* 7.
·4573	·784	·94561
·589	·9127	·923
·7124	·8352	·7465
1·7587	2·5319	2·61511

Exam. 5. There being no Fraction in the middle Row of the ſame Denomination with the 3 in the upper and the 4 in the lower Rows, I add the 4 and the 3 together, and then proceed as before; only, in adding the Place of Tenths, the Sum is 17, therefore I ſet down the 7 in the Place of Tenths, with the Dot before it; and the 1, which is to be carried, I place to the left of the Dot: So is the Sum compoſed of one Unit, and the Fraction .7587.

The ſixth and ſeventh Examples are added in the ſame Manner.

Exam. 8.	*Exam.* 9.	*Exam.* 10.
4.579	17.9571	.7568
2.486	4.95	14.3721
7.5402	6.901	5.01
14.6052	29.8081	20.1389

Exam. 8. The Fractions are added as before; but, when you come to the Place of Tenths, the Sum being 16, the 6 muſt be placed down, and a Dot before it; then carry the 1 to the Integers, and add them, placing them to the left of the Dot, as is done in the Example. In the ninth Example, there is 2 to be carried to the Integers. In the ſame Manner may any other Addition in the ſame Kind be done, only taking Care that the Figures, whether Integers or Fractions, be placed under the Figures of a like Denomination; for, if I am to add 47.52 to .42, the Abſurdity would be evident, were they to be placed and added in this Manner, *viz.*

47.52
.42
89.52

But, by taking Care to place the Figures of the ſame Denomination one under another, the Work will eaſily be obſerved to be true, thus,

47.52
.42
47.94

Subtraction of Decimal Fractions.

Article 41. To ſubtract Decimal Fractions.

Rule. Place the Numbers one under another, according to their Denomination, as in Addition; then ſubtract them, as in Subtraction of whole Numbers.

Exam. 1.	*Exam.* 2.	*Exam.* 3.
From .628	From 3.275	From .628
Subtract .209	Subtract .984	Subtract .059
Remains .419	Remains 2.291	Remains .569
Proof .628	Proof 3.275	Proof .628

This being done, in all Reſpects, as in Subtraction of whole Numbers, there can be no Neceſſity of multiplying Examples: But it may be proper to obſerve, that, when the Number to be ſubtracted has more decimal Places than the Number from which it is to be ſubtracted, you muſt take ſuch Figures from 10, for which 1 muſt be carried to the next Place, as will appear by the following Examples.

Exam. 4.	*Exam.* 5.	*Exam.* 6.
From 12.372	From .78	From 73.14
Subtract .9758	Subtract .6942	Subtract 26.526
Remains 11.3962	Remains .0858	Remains 46.614
Proof 12.3720	Proof .7800	Proof 73.140

Exam. 5. There being nothing to ſubtract the 2 from, I take it from 10, and the Remainder is 8; then I carry the 1 that I borrowed to the 4, which makes 5, and, there being nothing over the 4, I take 5 from 10, and the Remainder is 5; then I carry the 1 to the 9, which makes 10; now 10 from 8 I cannot, but 10 from 18, as in whole Numbers, and there remains 8; and the 1 muſt be carried to the 6, as in Subtraction of whole Numbers. The other Examples are performed in the ſame Manner. And the Proof is the ſame as in whole Numbers, that is, add the Remainder to the Sum ſubtracted, and it makes the ſame as the Number it was ſubtracted from.

Multiplication of Decimal Fractions.

Article 42. To multiply Decimal Fractions.

Rule. This is performed in the ſame Manner as in Multiplication of whole Numbers; only, when the Multiplication is ended, there muſt be as many Places cut off in the Product, for Decimals, as there are decimal Places in the Multiplier and Multiplicand.

Exam. 1.

Multiply 327.1058
By 5.735

16355290
9813174
22897406
16355290

Product 1875.9517630

Exam. 2.

Multiply .75
By .25

375
150

Product .1875

Exam. 1. The Multiplication being performed as in whole Numbers, ſeven Places muſt be marked off for Decimals, becauſe there are four decimal Places in the Multiplicand, and three in the Multiplier; therefore the Product is 1875.951763, the Cypher on the right Hand, or laſt Place, being of no Signification.

Exam. 2. Multiply as in whole Numbers, and the Product is 1875, which muſt be all marked off for Decimals, becauſe there are four decimal Places in the given Numbers, *viz.* two in the Multiplier and two in the Multiplicand.

Exam. 3.

Multiply 3.125
By .08

Product .25000

Exam. 4.

Multiply 72.04
By 26

43224
14408

Product 1873.04

Exam. 3. There are three decimal Places in the Multiplicand, and two in the Multiplier, therefore there muſt be five Places marked off in the Product for Decimals, which are all the Figures of the Product: Whence the Product is .25, rejecting the Cyphers as inſignificant.

In the fourth Example, there are but two decimal Places in the Multiplicand, and none in the Multiplier, therefore there are but two Places to be marked off for Decimals in the Product.

Exam. 5.

```
Multiply .13082
      By .22463
     -----------
          39246
         78492
        52328
       26164
      26164
     -----------
     .0293860966
```

Exam. 6.

```
Multiply .0725
      By .0083
     ----------
          2175
         5800
     ----------
     .00060175
```

Exam. 7.

```
Multiply .0125
      By   .03
     ---------
     .000375
```

In each of theſe Examples, the Number of Decimal Places in the reſpective Multipliers and Multiplicands exceed the Number of Places in the Products; therefore Cyphers are annexed to the left Hand, to make the Number of Places of Decimals in the ſeveral Products equal to the decimal Places in their given Multipliers and Multiplicands.

The Learner will ſee, from the next Examples, that, if Cyphers were not to be prefixed to the left Hand of the Product, to make the Number of Places of Decimal Fractions equal to thoſe in the Numbers multiplied, the Product will be erroneous.

Exam. 8.

```
Multiply 2.
      By .3
      -----
         .6
```

Exam. 9.

```
Multiply .2
      By .3
      -----
        .06
```

Now, if a Cypher was not prefixed to the ninth Example, the Product of both Multiplications would be the ſame, which is abſurd.

In multiplying whole Numbers with 3 or 4 decimal Places annexed both to the Multiplier and Multiplicand, the Work is ſomewhat tedious; and it often is not neceſſary to have the

Product nearer than to 3 or 4 Places of Decimals, a thouſandth or a ten-thouſandth Part of the Integer being near enough in moſt Caſes; it will be proper, therefore, to inſtruct the young Mathematician how to contract Multiplication, ſo as to have but juſt ſo many Places of Decimals in the Product as are thought neceſſary.

Article 43. To contract Multiplication.

Rule. Place the Units of the Multiplier under the ſame decimal Place of the Multiplicand, as is required to be the laſt Place in the Product, and invert all the other Figures of the Multiplier; then begin multiplying each Figure, at that which ſtands immediately over it, adding what is to be carried from the Figure that ſtands next to it on the right Hand, as will be ſhewn in the following Examples.

Exam. 1.

Multiply 579.83652
By 7.46539 to make four Places of Decimals in the Product.

579.83652
93564.7

40588556
2319346
347902
28992
1740
521

4328.7057

Exam. 2.

Multiply 385.73974
By 28.52734 to make four Places of Decimals in the Product.

385.73974
43725.82

77147948
30859179
1928699
77148
27001
1157
154

11004.1286

Exam. 1. The Figures of the Multiplier being inverted, and the Units ſet under the fourth Place of Decimals, multiply the 7 into the 5 which is over it, and the Product is 35; but, as there is a 2 to the right Hand, which being multiplied into 7 is 14, from which 1 ought to be carried, and, as the Exceſs above 10 is but 4, there muſt be only 1 carried, which makes 36, ſet down the 6 and carry the 3, proceeding to multiply the 7 thro' all the other Figures in the Multiplicand, as in common Multiplication: Then begin multiplying by the 4, firſt into the 5, which ſtands next to the

Figure that is over it, in order to know what muſt be carried, and the Product is 20; therefore 2 muſt be carried; then ſay, 4 times 6 is 24, and 2 that I carried is 26; now ſet down this 6 under the former 6, carrying 2 to the next, and proceed as in common Multiplication. In the ſame Manner every Figure muſt firſt be multiplied into that which ſtands next to the Figure that is over it, in order to know what is to be carried; and, if it makes 5 above any Number of Tens, one more muſt be carried for that 5, *i. e.* if it amounts to 15 you muſt carry 2, if to 25 you muſt carry 3, *&c.* becauſe, in Caſe all the Figures were multiplied, the odd ones in caſting-up increaſe to Tens. For which Reaſon, in multiplying by the 3, tho' that, being multiplied into the 8 which ſtands next to it, makes but 24, yet, as there was a 4 rejected in the firſt Line, *i. e.* I carried but 1 for 14, therefore I here carry 3, tho' it is not more than 24. And, if this be carefully obſerved, there will ſcarce be a ten-thouſandth Part of an Unit loſt in this Method; as is the Caſe here, for all the Figures being multiplied in the common Way, the Product is 4328,70575804281.

Exam. 2. This being done in the ſame Manner, the Product is 11004,1286; and, if done at full Length, is 11004.1287144916, little more than one hundred-thouſandth Part of an Unit more than comes out in the Contraction, which is nearer than is neceſſary in moſt common Affairs; where it will be ſufficiently near if contracted to two Places of Decimals. Inſtead of giving other Examples, I ſhall ſet down the two former contracted to two Places, by which the Learner will ſee that the Difference is ſo little, as not to be worth the Trouble in common Caſes.

Exam. 1.	*Exam.* 2.
579.83652	385.73974
93564.7	43725.82
405886	771479
23193	308592
3479	19287
290	771
17	270
5	12
	2
4328.70	11004.13

Exam. 1. In multiplying by the 7 it muſt be obſerved, that, tho' 7 times 6 is but 42, yet, as there is a 5 in the next Place, which being multiplied by 7 is 35, and from which, according to this Rule, 4 ſhould be carried, this will make it 46; ſo that 5 muſt be carried to the 21, ariſing from multiplying the 7 into the 3 that ſtands over it, which makes it 26; ſet down the 6, and proceed in the Multiplication as before; in ſuch ſhort Contractions always taking Care not to omit carrying to the full Extent of the Rule. The Product here falls ſhort but .005; which, if it were in Foot-Meaſure, is not the $\frac{1}{16}$ of an Inch; but, in the Fraction of a Pound Sterling, is ſomewhat more than one Penny.

Exam. 2. In multiplying by the 2 I carry 1, but in multiplying by the 8 I carry 8; becauſe the 7 being multiplied by 8 makes 56; then multiplying the 9 by 8, and carrying 5 from the 56, it makes 77, which is more than is required for carrying 8 by the Rule. The ſame Rule being obſerved in the other Figures, the Product is 11004.13, which is but .0013 more than before; and which, in the Fraction of a Foot, is one hundredth Part of an Inch, or little more; but, in the Fraction of a Pound, is ſomewhat more than one Farthing.

Art. 44. *Diviſion of Decimal Fractions.*

Rule. Divide, in all Reſpects, as in whole Numbers; only there is ſome Difficulty in aſcertaining the Number of Decimal Places in the Quotient, which ſhall be taught in the Examples.

Exam. 1. Divide 75.20764
By 6.384

```
6.384) 75.20764 (11.78
       6.384
       -----
       11367
        6384
       -----
        49836
        44688
        -----
         51484
         51072
         -----
           412
```

Exam. 2. Divide 836.1705
By 12.75

```
12.75) 836.1705 (65.582
       76.50
       -----
        7117
        6375
        -----
        7420
        6375
        -----
        10455
        10200
        -----
          2550
          2550
          ----
             0
```

Exam. 1. Divide according to the Rule in common Arithmetic, and the Quotient is 1178. In order to know how many Places muſt be marked off, by a Dot, for Decimals, cut off in the Product firſt ſubtracted from the Dividend, that is 6384, as many Places of Fractions as there are in the Diviſor, which in this Caſe is three. Amongſt theſe Figures obſerve which ſtands in the Place of Units, which is the 6; then obſerve the Denomination or Power of the Figure in the Dividend that ſtands over this 6, which is the 7, and that ſtands in the Place of Tens, for the 5 is in the Place of Units in the Dividend: Therefore, the Dot muſt be ſo placed in the Quotient, that the firſt Figure, 1, may be of the ſame Denomination or Power with the 7 in the Dividend, which ſtood over the 6, the Units Place in the firſt Product ſubtracted from the Dividend; that is, as the 7 in the Dividend is in the ſecond Place of Integers, or Place of Tens, ſo muſt the firſt Figure, 1, in the Quotient be in the ſecond Place of Integers, or Place of Tens; therefore the Dot muſt be placed between the ſecond and third Figures: Thus is 11.78 the Quotient ſought.

In the ſecond Example, the firſt Product, 7650, has two Places cut off for Decimal Fractions, there being two Places in the Diviſor; the 6 ſtands in the Units Place, over which in the Dividend is the 3, in the ſecond Place of Integers, or Place of Tens; therefore the 6, the firſt Figure in the Quotient, muſt be the Place of Tens: So is 65.582 the true Quotient ſought.

Exam. 3.	*Exam.* 4.
54.7) 99.465 (1.81	497.) 3265.79 (6.93
54.7	2802.
4476	4637
4376	4473
1005	1649
547	1491
458	158

Exam. 3. Cutting off one Place of Fractions in the Product 547, which is firſt ſubtracted from the Dividend, the 4 will be in the Place of Units; which ſtanding under the 9, in the Place of Units in the Dividend, the Dot muſt be ſo placed in the Quotient that the firſt Figure of it may be in the Place of Units; that is, the Dot muſt be placed in the Quotient between the firſt and ſecond Figures.

Exam. 4. There being no Fractions in the Diviſor, I place the Dot to the right Hand of the Product 2802, and the 2 will be then in the Place of Unites; which ſtanding under the 5, in the Place of Units in the Dividend, I place the Dot in the Quotient as in the laſt Example.

Exam. 5.
75·3) 42·8547 (·569
376·5
5204
4518
6867
6777
90

Exam. 6.
4·87) 3·57601 (·734
34·09
1670
1461
2091
1948
143

Exam. 5. The Diviſion being ended, and there being one Place of Fractions in the Diviſor, I cut off one Figure for Fractions in the Product 3765; then the 6 is in the Place of Units; and, it ſtanding under the 8 in the Dividend, which is the firſt Place of Fractions, place the Dot ſo that the 5, the firſt Figure in the Quotient, may be in the firſt Place of Fractions: Whence .569 is the Quotient.

The ſixth Example is done in the ſame Manner.

If the Diviſor conſiſts of more Places than the Dividend, annex Cyphers, at Pleaſure, to the right Hand of the Decimal Fraction of the Dividend, for theſe do not alter the Value of the Dividend; and then proceed with the Diviſion by the Rule already given, as in the following Examples.

Exam. 7.
3·9827) 24·750000 (6·21
23·8962
85380
79654
57260
39827
17433

Exam. 8.
41·756) 5·730000 (·137
41·756
155440
125268
301720
292292
9428

Exam. 7. Obſerving that there are not ſo many Figures in the Dividend as there are in the Diviſor, I annex Cyphers to the right Hand of the Fraction in the Dividend, after which the Operation proceeds as before: And, in annexing of Cyphers, I am at Liberty to annex as many as are thought convenient, and of which the Learner, from a little Experience, will be able to form a Judgment.

And, after all the Figures in the Dividend are taken down, if it is convenient to continue the Diviſion to a farther Degree of Exactneſs, it may be done by continuing to take down Cyphers, as if they had been annexed to the Dividend, as in the following Examples.

```
      Exam. 9.                    Exam. 10.

4.25) 79.18 (18.63          75.) 48.31 (.6441
      4.25                        450.
     ------                      ------
      3668                        331
      3400                        300
     ------                      ------
       2680                       310
       2550                       300
      ------                     ------
        1300                      100
        1275                       75
       ------                    ------
          25                       25
```

Exam. 9. After the Dividend has been all taken down, there remains 268, to which I take down a Cypher; and, proceeding with my Diviſion, there remains 130; to which I take down another Cypher, and continue the Diviſion: This may be done at Pleaſure; and, after the Diviſion is ſtopped, the Quotient is ſtill ſet off by the ſame Rule.

And ſo of the tenth Example.

But now, if, after the Diviſion is ended, it appears that the firſt Figure in the Quotient is in the ſecond, third, fourth, or any other Place of Fractions below the firſt, then prefix ſo many Cyphers to the left Hand of the Quotient as will depreſs the firſt Figure in the Quotient to the Denomination required, as in the following Examples.

Exam. 11.

45.7) .7893 (.017
45.7
3323
3199
124

Exam. 12.

245.) 1.3652 (.0055
1225.
1402
1225
177

Exam. 11. The Diviſion being ended, and there being one Fraction in the Diviſor, I cut off one Fraction in the Product 457; and then, the 5 in the Units Place ſtanding under the 8 in the ſecond Place of Fractions in the Dividend, place one Cypher to the left Hand of the 1 in the Quotient, and the Dot to the left Hand of the Cypher: So is .017 the Quotient ſought.

Exam. 12. The Diviſion being ended, and there being no Fractions in the Diviſor, I obſerve, that the 5 in the Units Place in the Product 1225 is under the 5 in the third Place of Fractions in the Dividend; therefore place two Cyphers to the left Hand of the 5 in the Quotient, and the Dot to the left Hand of the Cyphers: So is .0055 the Quotient ſought.

Exam. 13.

753.) .9265 (.0012
753.
1735
1506
229

Exam. 14.

476.) .3182 (.0006
2856.
326

Exam. 13. The Diviſion being ended, and there being no Fractions in the Diviſor, and the 3 in the Units Place in the Product ſtanding under the 6 in the third Place of Fractions in the Dividend, I place two Cyphers and the Dot to the left Hand of the Quotient, as in the Example.

In the fourteenth Example, I muſt place three Cyphers to the left Hand of the Quotient Figure: So is .0006 the Quotient ſought.

It may happen, that there may be no Integers in the Product that is firſt ſubtracted from the Dividend; in this Caſe, the Place of Units muſt be ſupplied in the Mind; or, to make it more eaſy to the Learner, after the Fractions are ſeparated in the Product which is firſt ſubtracted from the Dividend, place a Cypher to the left Hand of the Dot, which repreſenting the Place of Units, the Quotient may be marked off as before, as in theſe Examples.

Exam. 15.	*Exam.* 16.
.135) 7.2496 (53.7	.25) 78.94 (315.7
0.675	0.75
499	39
405	25
946	144
945	125
1	190
	175
	15

Exam. 15. There being three Fractions in the Diviſor, the firſt Product 675, which is to be ſubtracted, is to be ſet off for Fractions, and I ſupply the Units Place with a Cypher; which, if the Dividend had been continued to the left Hand, would have been under the Place of Tens, for the 6 ſtands under the Place of Units in the Dividend; therefore place the Dot in the Quotient ſo that its firſt Figure to the left Hand may be in the Place of Tens.

Exam. 16. Having placed a Cypher to repreſent the Place of Units in the Product firſt ſubtracted from the Dividend, and as the 7 in the Product ſtands under the 7 in the ſecond Place of Integers in the Dividend; therefore, if the Dividend had been continued to the left Hand, the Cypher in the Place of Units would have been under the third Place of Integers in the Dividend; and therefore I place the Dot in the Quotient accordingly.

Exam. 17.

```
,024) 7.35 (306.25
 0.072
 -----
   150
   144
   ---
    60
    48
   ---
   120
   120
   ---
     0
```

Exam. 18.

```
,051) .876 (17.17
  0.051
  -----
    366
    357
    ---
     90
     51
    ---
    390
    357
    ---
     33
```

Exam. 17. Here I place a Cypher to the left Hand of the 72, to have as many Fractions in the Product as there are in the Diviſor; then I place a Dot, and a Cypher to the left of that, for the Place of Units; which would have been under the third Place of Integers, if the Dividend had been continued to the left Hand; and therefore I place the Dot in the Quotient accordingly.

The eighteenth Example is done in the ſame Manner.

Now the Reader is a little familiar with theſe Operations, he may learn to fix the Quotient before the Diviſion is ended; which, in ſome Caſes, may ſave Labour; for, as ſoon as the firſt Subtraction is made, the Reader may diſtinguiſh what will be the Denomination of the firſt Figure in the Quotient; and ſo know to how many Figures he ſhould continue the Quotient to have two Places of it Decimals, which, in moſt Caſes, are ſufficient. As ſuppoſe,

```
24.7) 5498.6734 (2
      49.4
      ----
        5
```

I obſerve, that the 9 in the Place of Units ſtands under the Place of Hundreds in the Dividend; which being in the third Place of Integers, I muſt continue the Diviſion to five Figures in the Quotient, to have two of them Decimals; and, if all the Dividend ſhould not be taken down, the reſt may be neglected, if two Places of Fractions are near enough; and, if there are not Figures enough in the Dividend to continue the Quotient to ſo many Places as are wanted, then that muſt be ſupplied by taking down Cyphers, as in the former Examples.

There is another Way of determining the Number of Decimals in the Quotient; which is this, count the Number of Decimals in the Diviſor, and then the Number of Decimals in the Dividend; and, if there are more Places of Decimals in the Dividend than there are in the Diviſor, mark off ſo many Places for Decimals in the Quotient as the Number of Decimals in the Dividend exceed the Diviſor. Thus, in Example 1, there are five decimal Places in the Dividend, and but three in the Diviſor, therefore there muſt be two in the Quotient. In the ſecond Example, there are four Places of Decimals in the Dividend, and but two in the Diviſor, for which Reaſon there are two Places cut off in the Quotient. But, in this Caſe, it muſt be carefully obſerved, whether in the Operation any Cyphers have been added to the Dividend; for, if there have been Cyphers added, they muſt be reckoned as ſo many Places of Fractions,

```
    Exam. 19.                  Exam. 20.
.16) 432. (2700,          2.5) 18400 (7360
    0.32                        17.5
     ----                       ----
      112                         90
      112                         75
      ---                         --
        0                         150
                                  150
                                  ---
                                    0
```

Exam. 19. There being no Units Place in the Diviſor, if it was ſupplied by putting a Cypher in the Units Place, to the left Hand of the firſt Product 32, that Cypher would ſtand one Place higher than the firſt Figure, 4, in the Dividend, that is, in the Place of Thouſands; therefore the 2, the firſt Figure in the Quotient, muſt be in the Place of Thouſands; but, as there are but two Figures in the Quotient, and the Diviſion being ended, the Places of Tens and Units muſt be ſupplied with Cyphers: So is 2700. the Quotient ſought.

Exam. 20. The Units Place in the firſt Product ſtands under the 8 in the Dividend, which is the fourth Place of Integers, or Place of Thouſands; therefore the firſt Figure in the Quotient muſt be Thouſands; but, as there are no more than three Places of Figures in the Quotient, and the Diviſion being ended, a Cypher muſt be annexed to ſupply the Units Place.

To give the Learner a clearer Idea of Diviſion of Decimals, let 925 be propoſed to be divided by 25; firſt in whole Numbers, and afterwards by reducing firſt the Diviſor, and then the Dividend, gradually, to Decimal Fractions; by which the Reaſon of the Difference of the Quotients will appear.

25.) 925. (37.
75
175
175
0

2.5) 925. (370.
7.5
175
175
0

.25) 925. (3700.
.75
175
175
0

.025) 925. (37000.
.075
175
175
0

25.) 92.5 (3.7
75
175
175
0

25.) 9.25 (.37
75
175
175
0

25.) .925 (.037
75
175
175
0

25.) .0925 (.0037
75
175
175
0

By theſe eight Examples it may be ſeen, that, tho' the Figures in the Quotient are ſtill the ſame, yet they vary in their Denomination or Power, as the Diviſor or Dividend are made Fractions; increaſing, by making the Diviſor a Decimal, from 37 to 37000.; and decreaſing, by making the Dividend a Fraction, from 37 to .0037. The Truth of any of theſe Operations may be proved by multiplying the Diviſor and Quotient, and marking the Decimals in the Products according to the Rule for Multiplication; which will then agree with the Dividend, in Integers and Fractions, if the Work is true.

When there are many Figures in the Diviſor in Diviſion of Decimals, and the Quotient is required only to be very near the Truth, the Work may be contracted, ſo as to ſave Trouble, without any conſiderable Loſs.

Article 45. To contract Diviſion of Decimals.

Rule. Firſt cut off from the Diviſor all the Places beyond the fourth, at leaſt, if not to the third; then begin dividing the Dividend, and find the Denomination of the firſt Figure in the Quotient, by Art. 44: By this is known how many Figures are required in the Quotient to bring out the Number of Decimals deſigned: As for Example, if you deſign to have three Places of Decimals, and the firſt Figure in the Quotient is in the Units Place, then you muſt have four Figures in the Quotient; if it be Tens, there will be five Places; if it be Hundreds, there will be ſix, *&c.* that is, you muſt have ſo many Figures in the Quotient, that, after the required Number of Decimals in the Quotient are ſet off, the firſt Figure in the Quotient may be of that Denomination as is required by the Rule for Diviſion of Decimals. Now, knowing how many Places of Figures are required in the Quotient, obſerve whether the Diviſor has the ſame Number of Figures as are required in the Quotient; and, if it has, proceed to dividing, after having cut off the Figures in the Dividend (if there be any) beyond where the firſt Product reached. The reſt ſhall be ſhewn in the Examples.

Exam. 1.

Divide 3946.87593
By 4.2753, ſo as to have three Places of Decimals.

4.275|3) 3946.875|97 (9.
38.475

993

Exam. 1. Having reduced the Diviſor to three Places of Decimals, by cutting off the 3 in the fourth Place with a Daſh, I proceed to divide as in the uſual Manner; and, by placing the Quotient Figure, 9, in the Quotient, and multiplying the Diviſor by 9, I have 38475 for the firſt Product; three Places of which being marked off as Decimals, becauſe there are three decimal Places in the Diviſor, I find the 8 in the Place of Units in the Product ſtands under the 9 in the Place of Hundreds in the Dividend; therefore the firſt Figure, 9, in the Quotient muſt be in the Place of Hundreds; ſo that the Quotient muſt have ſix Figures, *viz.* three Places of Integers and three of Decimals; which being more Figures by two than there are in the Diviſor, there muſt be two Figures left in the Dividend beyond where the firſt Product reached, and but two cut off, *viz.* the 9 and the 7, at the End: But, if the Diviſor had conſiſted of as many Places as are required in the Quotient, the laſt four Figures might have been cut off: I ſhall, therefore, ſet it down excluſive of thoſe two Figures, and perform the Remainder of the Work.

```
4.275 ) 3946.875 ( 923.246
 ...     38.475 ..
         ---------
          9937
          8550
         ------
          13875
          12825
         -------
           1050
            855
          ------
            195
            171
           -----
             24
             24
           ----
              0
```

When the two remaining Figures of the Dividend are taken down, as in common Diviſion, I put a Dot under the [illegible] the laſt Figure in the Diviſor, to denote that it is not to be uſed in the future Part of the Work: Then aſking how many times 427, the other three Figures of the Diviſor, may be had in the Remainder 1050, which is now the Dividend, or how many times 4 can be had in 10, according to the uſual Method, I find I can have it twice; therefore, putting the 2 in the Quotient, I multiply the Diviſor 4.27 by 2, (having firſt multiplied the 2 into the 5 that is rejected, in order to know what Increaſe is to be carried, which is 1, for twice 5 is 10, from whence 1 muſt be carried) ſaying, twice 7 is 14, and the 1 that I carried is 15; therefore I ſet down the 5, and carry the 1; then ſay, twice 2 is 4, and the 1 that I carried is 5; laſtly, twice 4 is 8, which I ſet down; now ſubtract this Product 855 from 1050, and the Remainder is the next Dividend: Then putting a Dot under the 7 in the Diviſor, to denote that it is to be rejected, I aſk how many times 42 I can have in 195, or how many times 4 can be had in 19, according to the uſual Method, which I find is 4 times; then ſetting down the 4 in the Quotient, and multiplying the 7 which was laſt marked off, to know what Increaſe muſt be carried, I ſay, 4 times 7 is 28; which being more than 25, I carry 3, as was taught in *contracted Multiplication*; then ſay, 4 times 2 is 8, and the 3 that I carried is 11, from which 1 muſt be carried; laſtly, 4 times 4 is 16, and the 1 that I carried is 17, which I ſet down; now ſubtract this Product from 195, and the Remainder is 24, which is now the Dividend: Then, putting a Dot under the 2 in the Diviſor, I aſk how many times 4, the only remaining Figure of the Diviſor, I can have in 24, and the Anſwer is 6 times: So that the Quotient to three Places of Decimals, as was required, is 923.246. But, in regard that 6 times 4 is 24, which is all that is in the Dividend, it appears from hence, that this Quotient rather exceeds than falls ſhort of the true Quotient; becauſe, according to the Rule of carrying the Increaſe ariſing from the rejected Figures, it would make 25, if we regard only the multiplying the 6 into the 2; tho', the Figure beyond that being a 7, and, as 6 times 7 is 42, if that 4 is carried to the 6 times 2, it will make 16; from which carrying 2, according to the Strictneſs of the Rule, it will make 26, which is too much. By working this Example at Length, I find the Quotient is 923.24584, &c. which is but little more than one ten-thouſandth Part of an Unit leſs than the other. This I thought proper to remark, to ſhew the Learner how near the contracted Mehod comes to the Truth;

and, at the ſame Time, the Work itſelf ſhews whether it be too little or too much, tho' only by ſuch a Trifle.

Exam. 2.	*Exam.* 3.
Divide 417.865942 By 25.293, to have three decimal Places.	Divide 27483.84209 By 42.1763, to have three decimal Places.
25.293) 417.86|5942 (16.521	42.1763) 27483.84|209 (651.642
• • • • 25.293	• • • • • 2530578
16493	217806
15176	210882
1317	6924
1265	4217
52	2707
50	2530
2	177
2	168
0	9
	8
	1

Exam. 2. The firſt Figure in the Quotient appearing, by Art. 44, to be in the Place of Tens, there muſt be five Figures in the Quotient, in order to have three Places of Decimals, as required; and, there being five Places of Figures in the Diviſor, I cut off the other four Figures in the Dividend: Then, proceeding to mark off one Figure in the Diviſor each Time, I firſt inquire how many times 25.29 can be had in the Remainder 16493, which is the preſent Dividend, or how many times 2 can be had in 16, as uſual; and I find I can have it 6 times; therefore, I ſay 6 times 3, the Figure that is marked off, is 18, for which I carry 2, it being near 20; then ſay 6 times 9 is 54, and the 2 that I carried is 56; now put down the 6 and carry the 5, as in common Multiplication; and, the Product being obtained, ſubtract it from the Dividend, and the Remainder 1317 is the next Dividend: Then marking off the 9, the ſecond Figure in the Diviſor, I aſk how often 25.2 can be had in this Dividend, and I find I can have it 5 times;

ſaying 5 times 9 is 45, for which I carry 5; then ſay 5 times 2 is 10, and the 5 that I carried is 15; now ſet down the 5, and proceed to the reſt; and the Product is 1265, which, being ſubtracted from the Dividend, leaves 52 for the next Dividend: Then marking off the 2, the third Figure in the Diviſor, I aſk how many times 25 may be had in 52, which is twice; then multiply and ſubtract as before, and there remains 2: Laſtly, mark off the 5, and then the Diviſor is reduced to the 2, which may be had once in 2, and the Work is ended. The other Example is done the ſame Way; always obſerving to have Regard to what is to be carried on account of the Figures marked off in the Diviſor.

Thus having gone thro' the Addition, Subtraction, Multiplication, and Diviſion of Decimal Fractions, it will now be proper to inſtruct the Learner in the Articles of Reduction; which, not being neceſſary before, could not have been ſo well underſtood, as the Method of doing them requires the Knowledge of ſome of thoſe Rules.

Reduction of Decimal Fractions.

There are three Rules of Reduction of Decimals, *viz.* To reduce Vulgar Fractions to Decimal Fractions; To find the Value of Decimal Fractions in the accuſtomary Diviſions of Coin, Weights, Meaſures, *&c.* And to reduce the cuſtomary Diviſions of Coin, Weights, Meaſures, *&c.* into Decimals of the reſpective Integers.

Article 46. To reduce Vulgar Fractions to Decimals of the ſame Value.

Rule. Annex Cyphers, at Pleaſure, to the Numerator, marking them as Decimals; then divide by the Denominator, and ſet off the Quotient according to the Rule for Diviſion of Decimals, conſidering the Diviſor as Integers; and this will be the Decimal ſought in lieu of the Vulgar Fraction: And, if the Diviſion ſhould not be ended, the Quotient may be continued to any Degree of Exactneſs by taking down Cyphers, as has been taught in the Diviſion of Decimal Fractions.

Exam. 1.

Reduce $\frac{2}{5}$ to a Decimal Fraction.

5.) 2.0 (.4 Anſ.
20.

Exam. 2.

Reduce $\frac{1}{8}$ to a Decimal Fraction.

8.) 1.000 (.125
8.
20
16
40
40
0

Exam. 3.

Reduce $\frac{5}{16}$ to a Decimal Fraction.

16.)5.00000(.3125
4.8
20
16
40
32
80
80
0

Exam. 1. Annex a Cypher to the Numerator 2, and mark the Cypher for a Fraction; then divide by the Denominator 5, which muſt be conſidered as an Integer, and the Quotient is 4; which muſt be in the firſt Place of Decimals, as the 0, in the Units Place in the Product which is ſubtracted from the Dividend, ſtands under the firſt Place of Fractions in the Dividend.

Exam. 2. Here three Cyphers are annexed, but a greater or leſs Number might have been annexed; the Diviſion being ended, the Quotient is .125, according to the Rule for Diviſion of Decimals.

So, in the third Example, the Quotient is .3125, a Decimal Fraction in lieu of the given Vulgar Fraction $\frac{5}{16}$.

Exam. 4. Reduce $\frac{3}{20}$ to a Decimal Fraction.

20.) 3.000 (.15 Anſ.
20
100
100
0

Exam. 5. Reduce $\frac{1}{16}$ to a Decimal Fraction.

16.) 1.0000 (.0625 Anſ.
96
40
32
80
80
0

Exam. 4. The Diviſion is complete by the ſecond Subtraction; therefore the laſt Cypher needs not be taken down.

Exam. 5. The firſt Product 96 being placed under the Dividend, and the 6, which is in the Place of Units, ſtanding under the ſecond Place of Decimals in the Dividend, therefore the firſt Figure in the Quotient muſt be in the ſecond Place of Decimals, by Art. 44; for which Reaſon a Cypher muſt be prefixed, and then .0625 is the Decimal Fraction equal to $\frac{1}{16}$.

Exam. 6.

Reduce $\frac{1}{3}$ to a Decimal Fraction.

3) 1.000 (.333, &c.
9
10
9
10
9
1

Exam. 7.

Reduce $\frac{3}{176}$ to a Decimal Fraction.

176) 3.000 (.017
176.
1240
1232
8

Exam. 6. The Remainder being the ſame at every Subtraction, if Cyphers were continued to be annexed, the Quotient Figures would be 3, or the ſame, continually, however far continued: In all ſuch Caſes, it is uſual to ſet down as many Places of Fractions in the Quotient at once as may be thought neceſſary.

Exam. 7. After two Figures are obtained in the Quotient, (which are the ſecond and third Places of Decimals, by Art. 44) the Remainder is but 8; ſo that the next would be a Cypher; therefore the Diviſion is ſtopped, as it may not be neceſſary to continue the Diviſion by annexing more Cyphers.

Article 47. To find the Value of Decimal Fractions in the accuſtomary Diviſions of Coin, Weights, Meaſures, &c.

Rule. Multiply the Decimal Fraction by the next lower Denomination, which is to be conſidered in the Multiplication as Integers, and mark off the ſame Number of Places for Decimals, by a Dot, as taught in Multiplication of Decimals, at Art. 42; and the whole Numbers, if any, are of the ſame

Denomination with the Number by which the Multiplication was made: Then multiply the Decimal ſo cut off by the next lower Denomination, ſtill cutting off as many Fractions as there are in the Numbers multiplied, the Number by which the Multiplication was made being conſidered as Integers; and, if there are any whole Numbers cut off, they are of the ſame Denomination as the Number by which this Multiplication was made: Then continue the Operation 'till you come at the loweſt Denomination into which the Integer is divided; and the whole Numbers cut off at each Multiplication will be the Value ſought.

To find the Value of the following Fractions of a Pound Sterling.

Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Of .5	Of .6	Of .8	Of .9
20.	20.	20.	20.
10.0	12.0	16.0	18.0

Exam. 1. The Fraction being of a Pound Sterling, multiply it by 20, the Shillings in a Pound, which mark as Integers; then in the Product cut off one Fraction, by Art. 42, and the Integers are 10, which are Shillings, and nothing remains: Hence 10*s.* is the exact Value of .5 of a Pound. So, in the ſecond Example, the Value is 12*s.* exactly. And, in the third Example, the exact Value is 16*s*, And, in the fourth Example, it is 18*s.*

Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
Of .325	Of .475	Of .825	Of .975
20.	20.	20.	20.
6.500	9.500	17.500	19.500
12.	12.	12.	12.
6.000	6.000	6.000	6.000

Exam. 5. The Fraction .325 being multiplied by 20, the Shillings in a Pound, and that 20 being conſidered as Integers, there muſt be three Places of Fractions marked off in the Product, by Art. 42; whence the 6 Integers are 6

Shillings: But, as there is a remaining Fraction of .500, which is the Fraction of a Shilling, multiply it by 12, the Pence in a Shilling; then ſet off the Product, by Art. 42, and we have 6 in the Place of Integers, which are Pence, that being the Denomination by which the laſt Multiplication was made: And, there being no Fraction remaining, the exact Value of .325 of a Pound is 6*s*. 6*d*.

By the ſame Operation, the exact Value of the ſixth Example is 9*s*. Of the ſeventh Example, is 17*s*. 6*d*. And of the eighth Example, is 19*s*. 6*d*.

In the four laſt Examples, after the Multiplication by 20, there are in the Product two Cyphers in the right Hand of the Decimal Fraction, which, being of no Value, might have been neglected, as in the following Examples; but, in ſuch Caſes, the Learner muſt carefully remember to cut off ſo many Places of Fractions leſs in the next Product; and, this Method being practiſed, I would recommend it to him, tho', for his Eaſe in underſtanding the Operation, the Cyphers were uſed in the laſt four Examples.

Exam. 9.	*Exam.* 10.	*Exam.* 11.	*Exam.* 12.
Of .234	Of .572	Of .47	Of .77
20.	20.	20.	20.
4.680	11.440	9.40	15.40
12.	12.	12.	12.
8.16	5.28	4.8	4.8
4.	4.	4.	4.
.64	1.12	3.2	3.2

Exam. 9. The Fraction being multiplied by 20, cut off three Fractions in the Product, there being ſo many in the Multiplicand, and the 4 Integers are 4 Shillings: The Fraction .68 is the Fraction of a Shilling, multiply it therefore by 12, the Pence in a Shilling; now cut off but two Fractions in the Product, the Cypher to the right Hand of the Fraction being dropped, and the 8 in the Integers is 8 Pence: The Fraction .16 being the Fraction of a Penny, multiply it by 4, the Farthings in a Penny; then cut off two Fractions, and we have no Integers: Whence the Value of .234 of a Pound is 4*s*. 8*d*. the Fraction .64 of a Farthing may be neglected.

By the ſame Operation, the Anſwer to the tenth Example is found to be 11*s.* 5*d.* $\frac{1}{4}$; the Fraction .12 of a Farthing being neglected. The Anſwer to the eleventh Example is 9*s.* 4*d.* $\frac{3}{4}$; the Fraction .2 of a Farthing being neglected. And the Anſwer to the twelfth Example is 15*s.* 4*d.* $\frac{3}{4}$.

Exam. 13.	*Exam.* 14.	*Exam.* 15.	*Exam.* 16.
Of .474	Of .043	Of .049	Of .003
20.	20.	20.	20.
9.480	0.860	0.980	0.060
12.	12.	12.	12.
5.76	10.32	11.76	0.72
4.	4.	4.	4.
3.04	1.28	3.04	2.88
Anſ. 9*s.* 5*d.* $\frac{3}{4}$.	Anſ. 10*d.* $\frac{1}{4}$.	Anſ. 11*d.* $\frac{3}{4}$.	Anſ. $\frac{1}{4}$.

To find the Value of the following Fractions of a Yard.

Exam. 17.	*Exam.* 18.	*Exam.* 19.	*Exam.* 20.
Of .45	Of .79	Of .175	Of .236
3.	3.	3.	3.
1.35	2.37	.525	.708
12.	12.	12.	12.
4.20	4.44	6.300	8.496
	4.	4.	4.
	1.76	1.2	1.984

Exam. 17. This being the Fraction of a Yard, multiply it by 3, the Feet in a Yard; then cut off the Fractions, and we have 1 Foot, and .35 of a Foot: Multiply the .35 by 12, the Inches in a Foot; then cut off the Fractions, and we have 4 Inches: So is 1 Foot, 4 Inches and .2 the exact Value of .45 of a Foot.

In the ſame Manner are the Feet and Inches found in the eighteenth Example; but, there being a remaining Fraction of .44 of an Inch, to get your Value nearer the Truth, multiply it by 4, the Quarters in an Inch; then, neglecting the remaining Fraction, .76 of the Quarter of an Inch, the Anſwer is 2 Feet, 4 Inches, and 1 Quarter of an Inch. In the ſame Manner are found the Anſwers to the nineteenth and twentieth Examples, the former being 6 Inches and 1 Quarter, and the latter 8 Inches and 1 Quarter.

And, to find the Value of any Decimal Fraction, it is only to multiply it by the ſeveral Denominations which compoſe the Integers of which the Decimal is a Fraction, beginning with the higheſt Denomination next the Integer, and reckon theſe ſeveral Multipliers as whole Numbers; then mark off the Quotient according to Art. 42; and the Integers ſo found will be the Value of the Fraction ſought, and of the ſame Denomination reſpectively with that Number by which the Fraction was at that Time multiplied.

To ſhew the Reader, that, tho' Fractions are changed from Vulgar to Decimal, yet they are of the ſame Value, let us take the Vulgar Fraction $\frac{687}{960}$ of a Pound, and find its Value, by Art 35; then change the Vulgar Fraction into a Decimal, by Art. 46, and find the Value of this Decimal Fraction, by Art. 47, which will be the ſame with the Value before found,

687	960)687.0(.715625	.715625
20	6720	20.
96|0)1374|0(14*s.*	1500	*s.* 14.312500
96	960	12.
414	5400	*d.* 3.7500
384	4800	4.
30	6000	*q.* 3.00
12	5760	
96)360(3*d.*	2400	
288	1920	
72	4800	
4	4800	
96)288(3*q.*	0	
288		
0		

By this Operation, it appears that $\frac{687}{960}$ of a Pound, or .715625, the Decimal into which it is changed, are both of the ſame Value, that is, 14*s*. 3*d*. $\frac{3}{4}$.

Suppoſe $\frac{7}{9}$ of a Pound Sterling was given to find its Value as a Vulgar Fraction, and then to reduce it to a Decimal Fraction, and find its Value when in Decimals.

$$\begin{array}{r}
7\\
20\\
\hline
9)\,140\,(\,15\,s.\\
9\\
\hline
50\\
45\\
\hline
5\\
12\\
\hline
9)\,60\,(\,6\,d.\\
54\\
\hline
6\\
4\\
\hline
9)\,24\,(\,2\,q.\\
18\\
\hline
6
\end{array}$$

$$\begin{array}{r}
9)\,7.0\\
\hline
.777777\\
20.\\
\hline
s.\ 15.555540\\
12.\\
\hline
d.\ 6.66648\\
4.\\
\hline
q.\ 2.66592
\end{array}$$

By this Operation, it appears that $\frac{7}{9}$, being reduced to a Decimal, is .777777 continually: I continued the Diviſion at Pleaſure, by ſupplying in my Mind the Dividend with Cyphers: And, as the Diviſion will never end, we cannot have the exact Value of $\frac{7}{9}$ in Decimals; but, tho' we cannot have its exact Value, yet we have the ſame Value in Money, 15*s*. 6*d*. $\frac{1}{2}$, as in Vulgar Fractions; and the Error only affects the Remainder .66592, which is ſomething too little. In the ſame Manner the Reader may try any other Caſe, for his own Information.

The Operation of changing Vulgar Fractions into Decimals is the ſame as the common Rule of Three: Thus, to reduce $\frac{7}{9}$ to a Decimal: As 9 the Denominator is to 7 the Numerator, ſo is 10, or 100, or 1000, or 10000 (according to the

Number of Places you would have in the Fraction) to the Decimal ſought.

If the Reader thinks proper, he may omit the following Method of finding the Value of a Decimal Fraction, and proceed to the next Article.

But, in the Fraction of a Pound Sterling, and in Foot-Meaſure, there is a much readier Way to find their Value than by the common Method of Reduction: Thus, in the Fraction of a Pound, the firſt Figure next the Dot being doubled is Shillings: And, if the ſecond Figure is leſs than 5, reckon that and the next as ſo many Farthings; only, if they are more than 13, abate 1, or, more than 38, abate 2: And the Farthings thus found, being added to the Shillings before found, will be the Value of the Decimal Fraction.

But, if the ſecond Figure ſhould be more than 5, then take 5 from it, for which add 1 Shilling to the Shillings before found: And 5 being taken from the ſecond Figure, to the Remainder, if there is any, join the third Figure, and reckon them as Farthings; if they are above 13, abate 1, or, if above 38, abate 2, as before: And the Farthings thus found, being added to the Shillings before found, will be the Value of the Decimal Fraction.

If the Fraction is of four Places, the Value may be aſcertained a ſmall Matter nearer, by taking in the fourth Figure; but, as this is not ſo neceſſary to my preſent Deſign, I ſhall omit it, and proceed to Examples, to illuſtrate what has been ſaid.

The Value of .2 of a Pound is 4 *s.*

The Value of .5 of a Pound is 10 *s.*

The Value of .7 of a Pound is 14 *s.*

The Value of .9 of a Pound is 18 *s.* this being no more but to double the firſt Figure.

The Value of .25 of a Pound is 5 *s.* For the 2 being doubled is 4 *s.* and the ſecond Figure being a 5 is 1 *s.* and this being added to the 4 *s.* it makes 5 *s.* for the Value of .25 of a Pound.

The Value of .45 of a Pound is 9 *s.* For the 4 being doubled is 8 *s.* to which adding 1 for the 5, it makes 9 *s.*

The Value of .75 of a Pound is 15 *s.*

And the Value of .95 of a Pound is 19 *s.*

The Value of .412 of a Pound is 8 *s.* 3 *d.* For the 4 being doubled is 8 *s.* then, the ſecond Figure being under 5, and the ſecond and third Figures being leſs than 13, the 12 are reckoned as Farthings, which is 3*d.* Whence the Value is 8*s.* 3*d.*

The Value of .519 of a Pound is 10*s.* 4*d.* $\frac{1}{2}$: For the 5 being doubled is 10*s.* then abate 1 from the 19, and there remains 18, which are Farthings: Whence the Value is 10*s.* 4*d.* $\frac{1}{2}$.

The Value of .741 of a Pound is 14*s.* 9*d.* $\frac{3}{4}$: For the 7 being doubled is 14*s.* then from the 41 abate 2, and there remains 39, which are Farthings: So that the Value is 14*s.* 9*d.* $\frac{3}{4}$.

The Value of .561 of a Pound is 11*s.* 2*d.* $\frac{3}{4}$: For the 5 being doubled is 10*s.* then, the ſecond Figure being more than 5, ſubtract 5 from it, for which add 1 to the Shillings; and, when 5 is taken from the 6, there remains 1; to which join the third Figure, 1, and it makes 11, which are Farthings: Hence the Value is 11*s.* 2*d.* $\frac{3}{4}$.

To find the Value of .785 of a Pound: The 7 being doubled is 14*s.* then ſubtract 5 from the 8, and for it add 1 to the Shillings; and, 5 being ſubtracted from the 8, there remains 3; to which joining the third Figure, 5, it makes 35, from which abate 1, and there remains 34, which are Farthings: Whence the Value is 15*s.* 8*d.* $\frac{1}{2}$.

To find the Value of .895 of a Pound: The 8 being doubled is 16*s.* then ſubtract 5 from the 9, for which add 1 to the Shillings; and, 5 being ſubtracted from the 9, there remains 4; to which joining the third Figure, 5, it makes 45, from which abate 2, and there remains 43, which are Farthings: So that the Value is 17*s.* 10*d.* $\frac{3}{4}$.

If the Reader has an Inclination to ſee how near theſe, or any other Concluſions of this Sort, come to the Truth, he may find the Value of the Decimal Fraction by the former Part of this Article.

And, in the Fraction of a Foot, if the firſt Figure is under 5, reckon the two firſt Figures as Half-Quarters of an Inch, abating 1, if they are above 13, or 2, if they are more than 38: But, if the firſt Figure ſhould be above 5, ſubtract 5 from it, and for the 5 reckon 6 Inches; then, to what remains of the firſt Figure after the 5 is ſubtracted, join the ſecond Figure, and reckon theſe as Half-Quarters of an Inch; only, if they are above 13, abate 1, or, if above 38, abate 2, as in the Caſe of Farthings.

Thus, .12 of a Foot is 12 Half-Quarters of Inch, or 1 Inch and an Half.

.25 of a Foot is 24 Half-Quarters of an Inch, or 3 Inches: For, from the 25 abating 1, there remains 24, which are Half-Quarters.

.45 of a Foot is 43 Half-Quarters of an Inch: For, from the 45 abating 2, there remains 43, which are Half-Quarters, that is, 5 Inches and 3 Half-Quarters.

.65 of a Foot is 7 Inches and 6 Half-Quarters: For, ſubtracting 5 from the 6, there remains 1, to which joining the 5, it is 15: Now, for the 5 ſubtracted reckon 6 Inches; then from the 15 abate 1, and the remaining 14 are Half-Quarters, or 1 Inch and 6 Half-Quarters; which, being added to the 6 Inches, makes 7 Inches and 6 Half-Quarters.

.75 of a Foot is 9 Inches: For, ſubtracting 5 from the 7, there remains 2, to which joining the 5, it makes 25: Now for the 5 ſubtracted reckon 6 Inches; then from the 25 abate 1, and the remaining 24 are Half-Quarters, or 3 Inches; which, being added to the 6 Inches, makes 9 Inches.

.94 of a Foot is 11 Inches and 2 Half-Quarters: For, ſubtracting 5 from the 9, there remains 4, to which join the 4, and it is 44: Now for the 5 ſubtracted, reckon 6 Inches; then from the 44 abate 2, and the remaining 42 are Half-Quarters, or 5 Inches and 2 Half-Quarters; which, being added to the 6 Inches, makes 11 Inches and 2 Half-Quarters or 1 Quarter.

The Reader may try the Accuracy of theſe Concluſions, by finding the Value of the Decimal Fraction, by the former Part of this Article. If the Fraction had conſiſted of three Places, and the third Figure was above 5, in ſome Inſtances the Value of ſuch Fraction may be more accurately determined by taking the third Place of Fractions into Conſideration.

Article 48. To reduce the Parts of Coin, Weights, Meaſures, *&c.* into Decimals.

Rule. Set down for a Dividend the given Parts, if they are of one Denomination, or, if they are of ſeveral, reduce them to the leaſt Denomination; to this annex Cyphers, marked as Fractions; and then divide it by that Number which makes up the Integer, and is of the ſame Denomination with the Dividend, by Art. 44; and the Quotient is the Decimal ſought.

Exam. 1. Reduce 17*s.* to the Decimal of a Pound.

20) 17.00 (85
160

100
100

0

Exam. 2. Reduce 7*s.* 9*d.* to the Decimal of a Pound.

7*s.* 9*d.*
12

240) 93.0000 (.3875
720 ...

2100
1920

1800
1680

1200
1200

0

Exam. 1. This being only Shillings, I annex Cyphers, and then divide by 20, the Shillings in a Pound: The Quotient .85 is the Decimal ſought.

Exam. 2. This being Shillings and Pence, they muſt firſt be reduced to Pence; to which annex Cyphers, and then divide by 240, the Pence in a Pound: The Quotient .3875 is the Decimal ſought.

Exam. 3. Reduce 2*d.* $\frac{1}{4}$ to the Decimal of a Pound.

2*d.* $\frac{1}{4}$
4

960) 9.000000 (.009375
8640 ...

3600
2880

7200
6720

4800
4800

0

Exam. 4. Reduce 3 $\frac{3}{4}$ Inches to the Decimal of a Foot.

3 $\frac{3}{4}$
4

48) 15.0000 (.3125
144 ...

60
48

120
96

240
240

0

Exam. 3. This being Pence and Farthings, it muſt be reduced to Farthings; then, after the Cyphers are annexed, divide by 960, the Farthings in a Pound.

Exam. 4. Reduce this to 15 Quarters of an Inch, and then divide by 48, the Quarters of an Inch in a Foot.

Exam. 5.

Reduce 9*oz.* 4*dwts.* Troy, to the Decimal of a Pound.

9*oz.* 4*dwts.*
20

240) 184.000 (.766, *&c.*
1680

1600
1440

1600
1440

160

Exam. 6.

Reduce 12 Drachms to the Decimal of a Pound Avoirdupoize.

256) 12.000000 (.046875
1024

1760
1536

2240
2048

1920
1792

1280
1280

0

Theſe Examples are done in the ſame Manner: There being 240 Pennyweights in a Pound Troy, the fifth Example is divided by that Number; where, the Remainder being always the ſame, the Quotient Figure would be 6, if the Diviſion was continued: And, in the ſixth Example, the 12 Drachms are divided by 256, the Number of Drachms in a Pound Avoirdupoize.

In the Parts of a Pound Sterling the Decimal may be found without Diviſion, as follows, *viz.* in the Parts of a Pound, for every two Shillings put a 1 in the Tenth's Place next the Dot; for one Shilling put a 5 in the ſecond Place of Decimals; and for ſix Pence put a 2 in the ſecond Place and a 5 in the third Place of Decimals: As to any leſs Quantity, reduce it to Farthings, and place the Number of Farthings in the ſecond and third Places of Decimals, if they are more than 10, otherwiſe in the third Place only, obſerving to add 1 to them, if they are more than three Pence.

Exam. 1.

Reduce 16*s.* 7*d.* $\frac{3}{4}$ to a Decimal.

16*s.* is .8
6*d.* is .025
7*q.* is .007

.832

Exam. 2.

Reduce 13*s.* 4*d.* to a Decimal.

12*s.* is .6
1*s.* is .05
4*d.* is .017

.667

Exam. 3.

Reduce 9*s.* 9*d.* $\frac{3}{4}$ to a Decimal.

8*s.* is .4
1*s.* is .05
6*d.* is .025
3*d.* is .0125
3*q.* is .063

.4905

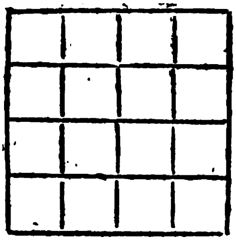
It may be obſerved, in Example 2, that, for four Pence, which is ſixteen Farthings, I ſet down 17, becauſe it is more than three Pence.

In the ſame Manner may the Parts of any Integer be reduced to a Decimal with little Trouble, by obſerving, that .5 is the Half of any Integer, .25 the Quarter, .125 the Eighth, and ſo on: But, as this Way is not quite exact in ſome Inſtances, I would recommend it to the Reader to proceed by the uſual Method of dividing the Numerator by the Denominator, in order to reduce a Vulgar Fraction to a Decimal.

To extract the Square Root of any Number.

BEFORE we inſtruct the Learner to extract the Square Root, it will be proper to explain what is meant by a Square Number, and what by the Root of ſuch Square. That Number is called a Square which is produced by multiplying any Number into itſelf: Thus, if a 6 be multiplied by itſelf, the Product is 36, which is therefore a Square Number; and 6 is called the Root or Square Root of 36, becauſe 6 times 6 is 36: In the ſame Manner, 9 is a Square, the Root of which is 3, for 3 times 3 is 9: So is 16 a Square Number, whoſe Root is 4.

If a Piece of Board, Stone, or any other Matter, be cut exactly ſquare, and each Side is four Inches, then is the Area or Content ſixteen Inches; that is, there will be ſixteen ſquare ſuperficial Inches in the Board, as may be ſeen by the following Figure.



Each Side being divided into four Parts, and Lines drawn from thoſe Diviſions, the whole Square will be divided into ſixteen little Squares: From whence it appears, that the Square Root of any Number is the Length of the Side of a Square, whoſe Area, or ſuperficial Content, is equal to the given Square Number.

If any Square Number is multiplied by its Root, the Product is called the Cube; which is the Content of a Solid, whoſe Length, Breadth, and Thickneſs are equal to the Root. Thus, if a Piece of Timber was cut exactly cubical, each Side being four Inches, the Content of the Cube would be ſixty-four Inches, ſixty-four being the Cube of four; which four Inches are the Length, Breadth, and Thickneſs, or Depth of the Piece.

Thus having explained what a Square and a Cube are, I ſhall next give the Squares and Cubes of all the nine Digits, in the folllowing Table.

A Table of the Squares and Cubes of the nine Digits.

Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	216	343	512	729

In the upper Row are placed the nine Digits, under each of which are placed their reſpective Squares and Cubes; therefore, when the Square or Cube of any Number under 10 is required, they may be found in this Table by Inſpection: Thus, the Square of 5 is 25, and of 7 is 49: The Cube of 6 is 216, and the Cube of 9 is 729; and ſo of the reſt. And, in the ſame

Manner, if the Roots of any of the Squares in the Table are required, they are found the ſame Way: But, in Caſe the Square Root of any Number not to be found in the Line of Squares in the Table is required, this muſt be done by the following Method.

Article 49. When a Number is given for the Square Root to be extracted, place a Dot over the Units Place; then, miſſing one Figure, put another Dot over the third Figure, or Place of Hundreds; and thus continue to put a Dot over every other Figure to the End; and, as many Dots as are thus placed over the Number, of ſo many Figures will the Root conſiſt.

Exam. 1. Extract the Square Root of $\dot{6}1\dot{5}0\dot{4}$.

This Number being dotted as above directed, it appears, that the Root conſiſts of three Figures or Places: To find the firſt of the three, look in the foregoing Table, in the middle Row, or Row of Squares, for the Number pertaining to the firſt Dot to the left Hand, which in this Example is 6; if it is not there, take the next leſs Square, which in this Caſe is 4, whoſe Root is 2; now draw a crooked Line to the right Hand of the Figures, and place the Root 2 as a Quotient in Diviſion; then put 4, the Square of 2, under the 6, and ſubtract it, ſetting down 2, the Remainder, underneath, as in Diviſion: Then take down the two Figures pertaining to the next Dot, *viz.* 15, and annex them to the 2 that remained, and this is called the Reſolvend; now draw a crooked Line to the left Hand of the Reſolvend, and double the Root in the Quotient for a Diviſor; then place this Diviſor to the left of the crooked Line, leaving Room ſufficient for another Figure to be put betwixt that and the crooked Line: Then the Work will ſtand thus,

$$\begin{array}{r@{\,}l}
 & \dot{6}1\dot{5}0\dot{4}\ (2 \\
 & 4 \\
\hline
4\ ) & 215
\end{array}$$

Then find how many times the Diviſor 4 can be had in 215 the Reſolvend, (but with this Reſtriction, that the Figure expreſſing the Number of times it may be had being placed to the right Hand of the 4, to make the Diviſor complete,

plete, and multiplied by the ſame Figure in the Root, may not exceed the Reſolvend) and I find it will be 4 times; ſet down 4 in the Root, and alſo in the Diviſor; then multiplying 44, this increaſed Diviſor, by the 4 laſt placed in the Root, the Product 176 muſt be placed under the Reſolvend 215, and ſubtracted from it: To the Remainder 39 bring down the next Square, that is, the laſt two Figures, thus,

$$\begin{array}{rl} & \dot{6}1\dot{5}0\dot{4}\ (24 \\ & 4 \\ \hline 44) & 215 \\ & 176 \\ \hline & 3904 \end{array}$$

Draw a crooked Line before 3904, the new Reſolvend, and double 24, the Figures in the Root, and place them as a Diviſor; then find how many times the Diviſor 48 can be had in 3904, (but with the ſame Reſtriction as before, that the Number of times it may be had, being placed to the right Hand of the 48, to make the Diviſor complete, and multiplied by the ſame Figure in the Root, may not exceed the Reſolvend) which is 8 times; ſet down 8 in the Root, and alſo in the Diviſor; then multiplying 488, the Diviſor thus increaſed, by the 8 laſt placed in the Root, the Product is 3904, which being ſubtracted, there remains 0: Therefore 248 is the true Root ſought. The whole Operation is as follows:

$$\begin{array}{rl} & \dot{6}1\dot{5}0\dot{4}\ (248\ \text{Root.} \\ & 4 \\ \hline 44) & 215 \\ & 176 \\ \hline 488) & 3904 \\ & 3904 \\ \hline & 0 \end{array}$$

$$\begin{array}{r} 248\ \text{Root.} \\ 248 \\ \hline 1984 \\ 992 \\ 496 \\ \hline 61504\ \text{Proof.} \end{array}$$

The Way to prove if the Work is right, is, to multiply the Root into itſelf: Thus 248, being multiplied by itſelf, produces 61504, the given Number.

Exam. 2. Extract the Square Root of $\dot{5}6\dot{6}4\dot{4}$ (2

$$\begin{array}{rl} & 4 \\ \hline 4) & 166 \end{array}$$

In this Example, the firſt Square is 5, and the next leſs Number in the Line of Squares in the Table is 4, whoſe Root 2 muſt be placed in the Root, and its Square 4 under the 5; then ſubtract, and there remains 1; to which bring down the next Period 66, as taught in the former Example: Double the Root 2 for a Diviſor, and place 4 its Double to the left of the crooked Line; then aſk how many times 4 the Diviſor can be had in 166, (but with the ſame Reſtriction as before, that the Number of times it may be had, being placed to the right Hand of the 4, to make the Diviſor complete, and multiplied by the ſame Figure in the Root, may not exceed the Reſolvend) and I find it will be 3 times; then, putting 3 in the Root, and alſo in the Diviſor, multiply 43, the increaſed Diviſor, by 3, and the Product is 129; which place under the Reſolvend, and ſubtract as before.

$$\begin{array}{rl} & \dot{5}6\dot{6}4\dot{4}\ (23 \\ & 4 \\ \hline 43) & 166 \\ & 129 \\ \hline 46) & 3744 \end{array}$$

Thus having taken down the laſt Period 44, and placed it to the right of 37 the Remainder, the Reſolvend is 3744; and 23, the Figures in the Root, being doubled for a Diviſor, is 46, which is placed to the left of the crooked Line: Now inquire how often 46 can be had in 3744, (under the ſame Reſtriction as before, that, the Figure being placed in the Root, and to the right Hand of the Diviſor, and multiplying the increaſed Diviſor by the Figure in the Root, it may not exceed the Reſolvend) and I find it will be 8 times; place 8 in the Root, and alſo on the right Hand of the Diviſor, betwixt that and the crooked Line: Laſtly, multiply 468, the Diviſor thus increaſed, by 8, the Figure laſt placed in the Root, and the Product is 3744; then ſubtract, and there remains 0: Therefore 238 is the true Root of the given Number, as appears by the Work.

$\dot{5}6\dot{6}4\dot{4}$ (238
4
43) 166
129
468) 3744
3744
0

238 Root.
238
1904
714
476
56644 Proof.

Exam. 3.
Extract the Square Root of
$\dot{1}8\dot{7}6\dot{9}$ (137
1
23) 87
69
267) 1869
1869
0

Exam. 4.
Extract the Square Root of
$2\dot{0}7\dot{9}3\dot{6}$ (456
16
85) 479
425
906) 5436
5436
0

These two Examples are done in the same Manner as the former; only it is to be observed, that, the fourth Example consisting of six Figures, the Dot does not fall on the last Figure to the left Hand; therefore, the first Period is 20, whose next less Square in the Table is 16, the Square of 4.

Exam. 5. Extract the Square Root
of $3\dot{4}6\dot{9}2\dot{1}$ (589
25
108) 969
864
1169) 10521
10521
0

589
589
5301
4712
2945
346921 Proof.

Exam. 5. The firſt Square is 34, and the next leſs Square in the Table is 25, whoſe Root is 5; therefore put 25 under 34, and ſubtract; alſo put 5 in the Root, and proceed as in the former Examples.

```
Exam. 6. Extract the Square Root
                                                 4057
         of 16459249 (4057                       4057
            16                                 --------
         ---------                              28399
         805) 4592                             20285
              4025                            162280
             -------                         ----------
         8107) 56749                          16459249 Proof.
               56749
              -------
                 0
```

```
Exam. 7. Extract the Square Root
                                                 3095
         of 9579025 (3095                        3095
            9                                  --------
         ---------                              15475
         609) 5790                             27855
              5481                            92850
             -------                         ----------
         6185) 30925                          9579025 Proof.
               30925
              -------
                 0
```

Exam. 6. The firſt Square being 16, and that being found exactly in the Table, when it is ſubtracted, nothing remains; ſo that the Reſolvend will be only 45, the next Period: When, therefore, the Root 4 is doubled for a Diviſor, it cannot be had in 45 (under the Reſtriction as in the other Examples); therefore, as the Diviſor cannot be had, ſet down a Cypher in the Root, and alſo to the right Hand of 8 the Diviſor: Then bring down the next Period, which is 92, and aſk how many times 80, the Double of the Root, can be had in 4592, (with the ſame Reſtriction as before) and the Anſwer is 5 times; place 5 in the Root, and alſo to the right Hand of the Diviſor; then multiply 805, the increaſed Diviſor, by 5, the Figures laſt placed in the Root, and proceed as before.

The ſeventh Example is done in the ſame Manner.

Exam. 8.

Exam. 8.	*Exam.* 9.	*Exam.* 10.
Extract the Square Root of 129.	Extract the Square Root of 540.	Extract the Square Root of 457.2459.

The Numbers in the eighth and ninth Examples not being Square Numbers (for there will be a Remainder after the Root is extracted) in order to obtain the Root near the Truth, it is neceſſary to annex Cyphers for decimal Places; obſerving, that, as many decimal Places as are required in the Root, ſo many Pairs of Cyphers muſt be annexed to the Number; therefore, let the Root be required in each Example to two Places of Decimals, then there muſt be four Cyphers annexed to each.

```
         .  . . .
        129.0000 (11.35                 11.35
        1                               11.35
       ------                          -------
    21) 29                               5675
        21                              3405
       ------                          1135
   223) 800                           1135
        669                           ---------
       -------                        128.8225
  2265) 13100                             .1775
        11325                         ---------
       -------                        129.0000 Proof.
         1775
```

```
        .  . . .
        540.0000 (23.23                 23.23
        4                               23.23
       ------                          -------
    43) 140                              6969
        129                             4646
       ------                          6969
   462) 1100                          4646
        924                           ---------
       -------                        539.6329
  4643) 17600                             .3671
        13929                         ---------
       -------                        540.0000 Proof.
         3671
```

In

· · · ·	
457.2459 (21.38	21.38
4	21.38
41) 57	17104
41	6414
	2138
423) 1624	4276
1269	457.1044
4268) 35559	.1415
34144	457.2459
1415	

In theſe Examples the Learner may ſee, by the Proofs, that the Root comes out very near the Truth, tho' the Operation is carried no farther than to two Places of Decimals. In Caſes where a greater Exactneſs is required, more Pairs of Cyphers may be added, to bring it nearer the Truth; for it muſt be obſerved, that, if a Number is not a true Square, ſo as to have no Remainder, the true Root of ſuch Number can't be attained, tho' the Operation be carried to ever ſo great a Length. The Root 21.38 of the tenth Example, being multiplied by itſelf, produces 457.1044.

Or, if they are Integers only whoſe Square Root is to be extracted, and, after all the Periods are taken down, there is a Remainder, you may then take down Pairs of Cyphers at Pleaſure, and ſo continue the Root to any Number of Decimal Fractions as may be convenient.

But, if a mixed Number be given to be extracted, and the Places of Decimals are an odd Number, as three, five, &c. there muſt be a Cypher annexed to the right Hand of the decimal Places, to make them even: The firſt Dot in the Integers muſt be over the Units Place; therefore, it is beſt to begin with dotting the Units Place, and then proceed to the reſt, whether forward or backward.

Exam. 11.	*Exam.* 12.	*Exam.* 13.
Extract the Square Root of 7.24576.	Extract the Square Root of .56644.	Extract the Square Root of 7487.441.

In each of theſe Examples, the Number of decimal Places are odd; therefore, a Cypher muſt be added to each, before the Operation is begun; after which, it is the ſame as before.

7.245760(2.691	.566440(.752	7487.4410(86.53
4	49	64
46)324	145)764	166)1087
276	725	996
529)4857	1502)3940	1725)9144
4761	3004	8625
5381)9660	936	17303)51910
5381		51909
4279		1

The Proof of theſe laſt Examples I have omitted; but the Learner may multiply them for his own Improvement, remembering to add the Remainder.

Art. 50. *To extract the Cube Root.*

THERE are ſeveral Methods of extracting the Cube Root: The Method by *Converging Series* is a neat and elegant Manner of Approximation: But the following Rule appears to me the beſt and moſt convenient, for the preſent Purpoſe, I have yet ſeen; and for which I am obliged to Mr. *Thomas Triplett*, Teacher of the Mathematics, at *Kingſton*; who likewiſe favoured me with a new Method of *Croſs-Multiplication*, which will be explained in its proper Place.

Rule. Place a Dot over the Units Place; then, miſſing two Figures, put another Dot over the Place of Thouſands; and, miſſing two Figures more, put a Dot over the Place of Millions; continue to dot every third Figure, and, as many Dots as are thus placed, of ſo many Figures will the Root conſiſt.

Then ſeek in the Table, Page 89, for a Cube Number, in the bottom Line, the next leſs to the firſt Cube or Period of the given Number on the left Hand; ſet the ſaid Cube Number under the firſt Period of the given Number, and its Root in the Place of the Root, to the right Hand of the given Number; ſubtracting the Cube from the firſt Period, and to the Remainder bring down the next Period, which call the *Reſolvend*, and draw a Line under it.

Now ſquare the Figure in the Root, and multiply it by 300, putting the Product under the Reſolvend; then multiply the Figure in the Root by 30, which place under the laſt Product; and the Sum of theſe two Products call a *Diviſor*, under which draw a Line.

Find how many times this Diviſor can be had in the Reſolvend, and this is the ſecond Figure in the Root, which place in the Root.

Multiply this laſt Figure in the Root by itſelf, and place the Product under the Diviſor; then multiply that Part of the Diviſor, which ariſes from multiplying the former Figure in the Root by 30, by the laſt Figure in the Root, and put the Product under the laſt Product; under theſe place the other Part of the Diviſor, and add them together; and multiply their Sum by the laſt Figure in the Root, calling the Product the *Subtrahend*, and ſubtract it from the Reſolvend.

If the Root is only to conſiſt of two Places, the Operation is finiſhed, as in the following Examples.

Exam. 1. To extract the Cube of Root 15625.

15625 (25 Root
8
———
7625 Reſolvend.
———
1200 the Product of the Square of 2 by 300.
60 the Product of 2 by 30.
———
1260 Diviſor.
———
25 the Product of 5 by 5.
300 the Product of 60 by 5.
1200 the other Part of the Diviſor.
———
1525
5 the laſt Figure in the Root.
———
7625 Subtrahend.
———
0 remains.

Proof.
25 Root.
25
———
125
50
———
625 Square.
25
———
3125
1250
———
15625 Cube.

The firſt Period in the given Number is 15; and, by th Table of Cubes, Page 89, the Cube leſs than 15 is 8, which place under the 15, and 2, the Cube Root of 8, in the Root. The Reader, by comparing the Directions with the Operation, will find it very eaſy. To try whether 25 is the exact Cube Root of 15625, multiply 25 by 25, and this Product again by 25; this laſt Product is 15625, therefore the true Cube Root of 15625 is 25.

Exam. 2. Extract the Cube Root of 438976.

438976 (76
343
95976 Reſolvend.
14700 Product of the Square of 7 by 300.
210 Product of 7 by 30.
14910 Diviſor.
36 Product of 6 by 6.
1260 Product of 210 by 6.
14700 the other Part of the Diviſor.
15996
6 the Figure laſt placed in the Root.
95976 Subtrahend.
0 remains.

Exam. 3. Extract the Cube Root of 205379.

205379 (59
125
80379 Reſolvend.
7500
150
7650 Diviſor.
81 Square of 9.
1350 Product of 150 by 9.
7500
8931
9
80379 Subtrahend.
0 remains.

Exam. 4. Extract the Cube Root of 18399744.

18399744 (264
8

10399 Reſolvend.

1200 Product of the Square of 2 by 300.
60 Product of 2 by 30.

1260 Diviſor.

36 Square of 6.
360 Product of 60 by 6.
1200 the other Part of the Diviſor.

1596
6 the Figure laſt placed in the Root.

9576 Subtrahend.

823744 Reſolvend.

202800 Product of the Square of 26 by 300.
780 Product of 26 by 30.

203580 Diviſor.

16 Square of 4.
3120 Product of 780 by 4.
202800 the other Part of the Diviſor.

205936
4 the Figure laſt placed in the Root.

823744 Subtrahend.

0 remains.

Exam. 5. Extract the Cube Root of 54 to three Places of Decimals.

54.000000000 (3.779
27

27000 Reſolvend.

2700 Product of the Square of 3 by 300.
90 Product of 3 by 30.

2790 Diviſor.

49 Square of 7.
630 Product of 90 by 7.
2700 the other Part of the Diviſor.

3379
7 the Figure laſt placed in the Root.

23653 Subtrahend.

3347000 Reſolvend.

410700 Square of 37 by 300.
1110 Product of 37 by 30.

411810 Diviſor.

49 Square of 7.
7770 Product of 1110 by 7.
410700 the other Part of the Diviſor.

418519
7 the Figure laſt placed in the Root.

2929633 Subtrahend.

417367000 Reſolvend.

42638700 Square of 377 by 300.
11310 Product of 377 by 30.

42650010 Diviſor.

81 Square of 9.
101790 Product of 11310 by 9.
42638700 the other Part of the Diviſor.

42740571
9 the Figure laſt placed in the Root.

384665139 Subtrahend.

32701861 Remainder.

By the two laſt Examples the Learner may ſee, that, when a new Reſolvend is made, all the Figures then in the Root muſt be ſquared, and their Square multiplied by 300, and the Figures in the Root multiplied by 30 to make a Diviſor: That, when it is known how many times the Diviſor can be had in the Reſolvend, that Figure muſt be placed in the Root, to the right Hand of thoſe already there, as in the Quotient in Diviſion: That then the Figure laſt placed in the Root muſt be ſquared, and put down under the Diviſor; and the former Part of the Root multiplied by 30 is to be multiplied by this laſt placed Figure in the Root, and put under the Square of the laſt Figure in the Root; under theſe put the other Part of the Diviſor, which being added together, their Sum muſt be multiplied by this laſt placed Figure in the Root, for the Subtrahend to be ſubtracted from the Reſolvend.

If the Subtrahend, when all the Periods are taken down, is the ſame with the Reſolvend, the Work is finiſhed, and the Root is the true Root ſought; as in the fourth Example, where 264 is the true Cube Root of the given Cube: But, if any Thing remains when all the Periods have been taken down, to that Remainder annex three Cyphers, and purſue the ſame Method as before; by which the Root may be continued to as many Places of Decimals as may be thought proper; always obſerving to annex three Cyphers to each Remainder, to make a new Reſolvend.

If a Decimal Fraction, or mixed Number, is given for the Cube Root to be extracted, and the decimal Places are neither three, ſix, nor nine, Cyphers muſt be annexed, to make the Decimals have one of thoſe Numbers of Places; ſo that the Dot may fall on the Units Place of the integral Part of the mixed Number, and that the firſt Period, in an entire Decimal, may conſiſt of three Places.

Exam. 6. Extract the Cube Root of 92775.111183.

92775.111183 (45.27
64

28775 Reſolvend.

4800 Square of 4 by 300.
120 Product of 4 by 30.

4920 Diviſor.

25 Square of 5.
600 Product of 120 by 5.
4800 the other Part of the Diviſor.

5425
5 the Figure laſt placed in the Root.

27125 Subtrahend.

1650111 Reſolvend.

607500 Square of 45 by 300.
1350 Product of 45 by 30.

608850 Diviſor.

4 Square of 2.
2700 Product of 1350 by 2.
607500 the other Part of the Diviſor.

610204
2 the Figure laſt placed in the Root.

1220408 Subtrahend.

429703183 Reſolvend.

61291200 Square of 452 by 300.
13560 Product of 452 by 30.

61304760 Diviſor.

49 Square of 7.
94920 Product of 13560 by 7.
61291200 the other Part of the Diviſor.

61386169
7 the Figure laſt placed in the Root.

429703183 Subtrahend.

0 remains.

Exam. 7. Extract the Cube Root of 80.763.

80.763 (4.32
64

16763 Reſolvend.

4800 Square of 4 by 300.
120 Product of 4 by 30.

4920 Diviſor.

9 Square of 3.
360 Product of 120 by 3.
4800 the other Part of the Diviſor.

5169
3 the Figure laſt placed in the Root.

15507 Subtrahend.

1256000 Reſolvend.

554700 Square of 43 by 300.
1290 Product of 43 by 30.

555990 Diviſor.

4 Square of 2.
2580 Product of 1290 by 2.
554700 the other Part of the Diviſor.

557284
2 the Figure laſt placed in the Root.

1114568 Subtrahend.

141432 Remainder.

I ſhall juſt obſerve to the Reader, that, when the Root conſiſts of Integers and Fractions, the Fractions are not diſtinguiſhed in forming the Diviſor, but uſed as Integers.

In the ſeventh Example, there being but three decimal Places given, the Root would conſiſt of but one Decimal, without annexing three Cyphers to the Remainder, to make a new Reſolvend: And, if to the laſt Remainder three Cyphers are annexed, the Root may then be obtained to three decimal Places.

Exam. 8. Extract the Cube Root of 146.379.

146.379 (5.27
125

21379 Reſolvend.

7500 Square of 5 by 300.
150 Product of 5 by 30.

7650 Diviſor.

4 Square of 2.
300 Product of 150 by 2,
7500 the other Part of the Diviſor.

7804
2 the Figure laſt placed in the Root.

15608 Subtrahend.

5771000 Reſolvend.

811200 Square of 52 by 300.
1560 Product of 52 by 30.

812760 Diviſor.

49 Square of 7.
10920 Product of 1560 by 7.
811200 the other Part of the Diviſor.

822169
7 the Figure laſt placed in the Root.

5755183 Subtrahend.

15817 Remainder.

Exam. 9. Extract the Cube Root of .162771336.

.162771336 (.546
125

37771 Reſolvend.

7500 Square of 5 by 300.
150 Product of 5 by 30.

7650 Diviſor.

16 Square of 4.
600 Product of 150 by 4.
7500 the other Part of the Diviſor.

8116
4 the Figure laſt placed in the Root.

32464 Subtrahend.

5307336 Reſolvend.

874800 Square of 54 by 300.
1620 Product of 54 by 30.

876420 Diviſor.

36 Square of 6.
9720 Product of 1620 by 6.
874800 the other Part of the Diviſor.

884556
6 the Figure laſt placed in the Root.

5307336 Subtrahend.

0 remains.

I ſhall here add two Examples in Decimals only.

Exam. 10. Extract the Cube Root of .474552.

$.47\dot{4}55\dot{2}$ (.78
343

131552 Reſolvend.

14700 Square of 7 by 300.
210 Product of 7 by 30.

14910 Diviſor.

64 Square of 8.
1680 Product of 210 by 8.
14700 the other Part of the Diviſor.

16444
8 the Figure laſt placed in the Root.

131552 Subtrahend.

0 remains.

Exam. 11. Extract the Cube Root of .3758.

$.37\dot{5}80\dot{0}$ (.72
343

32800 Reſolvend.

14700
210

14910 Diviſor.

4
420
14700

15124
2

30248 Subtrahend.

2552 Remainder.

In the laſt Example, the given Number is .3758; which conſiſting of four Places, two Cyphers muſt be annexed, to make ſix Places, as it is a Decimal.

If the Learner has a Mind, he may prove the Examples, by multiplying the Roots by themſelves, and that Product by the Root, as was done in the firſt Example: Where there is a Remainder, as in the fifth, ſeventh, eighth, and eleventh Examples, it muſt be added to the laſt Product: Thus, in Example 11,

.72 Root.
.72

144
504

.5184 Square.
.72 Root.

10368
36288

.373248 Cube.
2552 Remainder.

.375800 or .3758, the given Number.

I ſhall only add one Example more, where a Cypher ariſes in the Root.

Exam. 12. To extract the Cube Root of 29.508381.

29.508381 (3.09
27

2508 Reſolvend.

2700
90

2790 Diviſor.

2508381 Reſolvend.

270000
900

270900 Diviſor.

81
8100
270000

278181
9

2503629 Subtrahend.

4752 Remainder.

The Diviſor 2790 cannot be had in 2508 the Reſolvend; therefore place a Cypher in the Root, and take down 2508; to which joining 381, the next Period, it gives 2508381, a new Reſolvend; and then proceed in the Work as before.

Art. 51. *To multiply Feet, Inches, and Parts, by Feet, Inches, and Parts.*

THE Multiplication of Feet, Inches, and Parts, commonly called *Croſs-Multiplication*, being very uſeful, I ſhall endeavour to make it as eaſy to the Learner as I can, by a new Method, the beſt I have yet ſeen; for which I am obliged to Mr. *Thomas Triplett*, of *Kingſton*, who was ſo kind as to communicate it to me.

When Feet are multiplied by Feet, the Product is Feet; if Feet are multiplied by Inches, or Inches by Feet, the Product is Inches; but, if Inches are multiplied by Inches, the Product will be *Parts*, of which twelve make an Inch: From whence it ariſes, that Learners are often at a Loſs where to ſet down their Product: To prevent which Difficulty, Mr. *Triplett* invented the following Method: Divide the Multiplicand by 12, and, that being multiplied by the Inches, each Product is ſet under the Denomination from whence it is produced.

Exam. 1. Multiply 9 F. 6 I. by 4 F. 3 Inch.

F.	I.	P.
	9	6
9	6	
4	3	
2	4	6
38	0	
40	4	6

Exam. 2. Multiply 12 F. 7 I. by 6 F. 9 Inch.

F.	I.	P.
1	0	7
12	7	
6	9	
9	5	3
75	6	
84	11	3

Exam. 1. The Multiplicand and Multiplier being ſet down, draw a Line over the Multiplicand; then divide the Multiplicand by 12, thus, the twelfth Part of 9 Feet is 9 Inches; ſet 9 over the Inches in the former Multiplicand: Alſo the twelfth Part of 6 Inches is 6 Parts; which muſt, therefore, be placed one Place to the right Hand, after making Dots betwixt, as is uſual in ſuch Caſes, and as is done above.

Having thus made two Multiplicands, (the new one for the Inches of the Multiplier, and the other for the Feet) multiply the 3 Inches into the 6 Parts, and it makes 18, which, being divided by 12, gives 1, and 6 remains; ſet down the 6 Parts under the 6 Parts in the upper Multiplicand, and carry the 1; then, 3 times 9 is 27, and 1 that was carried is 28, which, being divided by 12, gives 2, and 4 remains; ſet down the 4, in the Place of Inches, under the 9, and the 2 in the Place of Feet: Then multiply the 9 Feet and 6 Inches by the 4 Feet in the Multiplier, ſaying, 4 times 6 is 24, which, being Inches, muſt be divided by 12; and 12 may be had twice in 24, and nothing remains; therefore, ſet down 0 in the Place of Inches, and carry 2; laſtly, 4 times 9 is 36, and 2 that was carried is 38, which, being Feet by Feet, muſt be ſet down entire, in the Place of Feet.

Under theſe two Products draw a Line, and add them together; by which it appears, that the Product of 9 Feet and 6 Inches, by 4 Feet and 3 Inches, is 40 Feet, 4 Inches, and 6 Parts, or Half an Inch.

Exam. 2. The 12 Feet in the Multiplicand, being divided by 12, gives 1; which ſet down in the Place of Feet, over the former, having firſt drawn a Line; and, as there is nothing remaining, put a Cypher in the Place of Inches: Then the [illegible]

Inches, being divided by 12, gives 7 Parts, which muſt be placed to the right Hand.

Then multiply the 9 Inches in the Multiplier into the 1 Foot and 7 Parts, the upper Multiplicand, thus, 9 times 7 is 63, which, being divided by 12, gives 5, and 3 remains; ſet down the 3 in the Place of Parts, under the 7, from which it was produced, and carry the 5 to the next; which being a Cypher, 9 times nothing is nothing, but the 5 that was carried is 5; which ſet down in the Place of Inches; then, 9 times 1 is 9, which put down in the Place of Feet: Next multiply the 12 Feet and 7 Inches, the lower Multiplicand, by the 6 Feet in the Multiplier, thus, 6 Feet by 7 Inches is 42 Inches, or 3 Feet and 6 Inches; put down the 6 Inches under the 5 Inches, and carry the 3; then, 6 times 12 is 72, and the 3 that was carried is 75, which put under the 9 Feet; now add theſe two Products, and their Sum, 84 Feet, 11 Inches, and 3 Parts, is the true Anſwer.

Before I proceed to any more Examples, it may be proper to obſerve, that, an Inch being the twelfth Part of a Foot, and what we call a *Part* the twelfth Part of an Inch, there can be no Number ſo large as 12, except in the Feet: Therefore, in dividing the Multiplicand by 12, nothing more is required, the Numbers being under 12, than to remove them a Place lower; for the twelfth Part of any Number of Feet, under 12, makes the ſame Number of Inches; and the twelfth Part of the Inches, they always being under 12, are ſo many *Parts*. The Multiplicand being thus reduced to one Twelfth of what it was, every Product will ſtand under the Denomination from whence it is produced, when multiplied by the Inches in the Multiplier: Otherwiſe, multiplying by Inches reduces each Kind a Place lower, which is here already done.

Exam. 3. Multiply 15 F. 5 I.
by 9 F. 10 Inch.

F.	I.	P.
1	3	5
15	5	
9	10	
12	10	2
138	9	
151	7	2

Exam. 4. Multiply 17 F. 8 I.
by 10 F. 4 Inch.

F.	I.	P.
1	5	8
17	8	
10	4	
5	10	8
176	8	
182	6	8

Exam. 3. Draw a Line over the Multiplicand, which, being divided by 12, gives 1 Foot, 3 Inches, and 5 Parts; for 12 may be had once in 15, and 3 over; put down the 1 over the Feet, and the 3 over the Place of Inches, and the 5 Inches in the old Multiplicand put in the Place of Parts in the new Multiplicand: Multiply the 1 Foot, 3 Inches, and 5 Parts, by 10, thus, 10 times 5 makes 50, which, being divided by 12, gives 4, and 2 remains; ſet down the 2 under the 5, and carry 4; then, 10 times 3 is 30, and 4 that was carried is 34, which, being divided by 12, gives 2, and 10 remains; ſet down the 10 Inches, and carry 2 to the Feet; ſaying, 10 times 1 is 10, and 2 that was carried makes 12, which, being Feet, muſt be ſet down entire: Then multiply the 15 Feet and 5 Inches by the 9 Feet, thus, 9 Feet by 5 Inches is 45 Inches, or 3 Feet and 9 Inches; put down the 9 Inches under the 10 Inches, and carry the 3; then, 9 Feet by 5 Feet is 45 Feet, and 3 that was carried is 48 Feet; put down the 8 Feet under the 2 Feet, and carry the 4; laſtly, 9 times 1 is 9, and 4 that was carried is 13, which put down as in the Example: Now draw a Line under theſe two Products, and add them; the 2 Parts put down; but the 10 Inches, being added to the 9 Inches, makes 19 Inches, or 1 Foot and 7 Inches; put down the 7 Inches, and carry the 1 Foot to the Feet, which are added in the uſual Manner.

Exam. 4. Draw a Line over the Multiplicand, and the 17 Inches in the Multiplicand, being divided by 12, gives 1 Foot and 5 Inches; place the 1 over the Feet, and the 5 Inches place over the Inches, and the 8 Inches in the old Multiplicand put in the Place of Parts in the new Multiplicand, for 8 Inches, being divided by 12, gives 8 Parts: Multiply this new Multiplicand by 4 Inches, thus, 4 Inches by 8 Parts is 32 Parts, or 2 Inches and 8 Parts; ſet down the 8 Parts under the Place of Parts in the upper Multiplicand, and carry the 2 Inches; then, 4 Inches by 5 Inches is 20 Inches, and 2 Inches that was carried is 22 Inches, or 1 Foot and 10 Inches; ſet down the 10 Inches in the Place of Inches, and carry the 1 Foot; then, 4 Feet by 1 Foot is 4 Feet, and 1 that was carried is 5, which ſet down as in the Example: Now multiply the old Multiplicand, 17 Feet and 8 Inches, by the 10 Feet, thus, 10 Feet by 8 Inches is 80 Inches, or 6 Feet and 8 Inches; ſet down the 8 Inches under the 10 Inches, and carry the 6 Feet; then multiply the 17 Feet by 10, to which carry the 6 Feet, in the uſual Manner, and it makes 176 Feet, which ſet down as in the Example; draw a Line under theſe two Products, then add them as at the third Example, and we have 182 Feet, 6 Inches, and 8 Parts, the Product required.

Exam. 5.	*Exam.* 6.	*Exam.* 7.
Multiply 56 F. 3 I.	Multiply 112 F. 5 I.	Multiply 24 F. 10 I.
by 17 F. 7 I.	by 23 F. 9 I.	by 15 F. 6 I.
F. I. P.	F. I. P.	F. I. P.
4 : 8 : 3	9 : 4 : 5	2 : 0 : 10
56 : 3	112 : 5	24 : 10
17 : 7	23 : 9	15 : 6
32 : 9 : 9	84 : 3 : 9	12 : 5
396 : 3	345 : 7	132 : 6
56	224	24
989 : 0 : 9	2669 : 10 : 9	384 : 11

Exam. 5. The Multiplicand, 56 Feet and 3 Inches, being didivided by 12, as in the former Examples, gives 4 Feet, 8 Inches, and 3 Parts, for the new Multiplicand; then multiplying this new Multiplicand by the 7 Inches, as taught in the former Examples, it gives 32 Feet, 9 Inches, and 9 Parts: The 17 Feet, being multiplied into the 3 Inches, makes 51 Inches, which, being divided by 12, gives 4 Feet and 3 Inches; ſet down the 3 Inches, and multiply the 56 Feet, firſt by the 7, remembering to carry the 4 when the 7 is multiplied into the 6; this produces 396 Feet and 3 Inches: Then multiply the 56 Feet by 1, the remaining Part of the Multiplier, and ſet the Product 56 under the 396, as in common Multiplication: The Products, being added, make 989 Feet and 9 Parts.

In the ſixth Example, the 5 Inches in the Multiplicand, being multiplied by the 23 Feet in the Multiplier, gives 115 Inches, or 9 Feet and 7 Inches; put down the 7 Inches, and carry the 9 Feet to the Product of the 112 multiplied by 3: And, in the ſeventh Example, when the 10 Inches are to be multiplied by the 15 Feet, the Product is 150 Inches, or 12 Feet and 6 Inches: The reſt of the Work is the ſame as before.

But, when Feet, Inches, and Parts, are to be multiplied by Feet, Inches, and Parts, there will be produced lower Diviſions,

viſions, called *Seconds* and *Thirds*; a Second being the twelfth Part of what we here call a *Part*, and a Third the twelfth Part of a Second. In theſe Operations, it is neceſſary to make two ſupplemental Multiplicands, one for the Inches, and the other for the Parts.

Ex. 8. Multiply 9 F. 4 I. 7 P. by 5 F. 6 I. 4 P.

F.	I.	P.	S.	T.
		9	4	7
	9	4	7	
9	4	7		
5	6	4		
	3	1	6	4
4	8	3	6	
46	10	11		
51	10	4	0	4

Ex. 9. Multiply 12 F. 9 I. 5 P. by 8 F. 4 I. 10 P.

F.	I.	P.	S.	T.
	1	0	9	5
1	0	9	5	
12	9	5		
8	4	10		
	10	7	10	2
4	3	1	8	
102	3	4		
107	5	1	6	2

Exam. 8. Having placed the given Numbers under each other, draw a Line over the Multiplicand, and divide it by 12, which is done by placing each Figure one Place lower in Denomination, they being all under 12; this is the Multiplicand for the Inches: Now divide this firſt ſupplemental Multiplicand by 12, thus, the 9 Inches, being divided by 12, gives 9 Parts, which place over the 4 Parts: The 4 Parts in the firſt ſupplemental Multiplicand, being divided by 12, gives 4 Seconds, which put over the Place of Seconds in the firſt ſupplemental Multiplicand: The 7 Seconds in the firſt ſupplemental Multiplicand, being divided by 12, gives 7 Thirds in the ſecond ſupplemental Multiplicand, which place as in the Example.

Multiply the upper ſupplemental Multiplicand by the 4 Parts in the Multiplier, thus, 4 Parts by 7 Thirds is 28 Thirds, or 2 Seconds and 4 Thirds; put down the 4 Thirds under the Thirds in the upper ſupplemental Multiplicand, and carry the 2 Seconds: Then, 4 Parts by 4 Seconds is 16 Seconds, and 2 that was carried is 18 Seconds, or 1 Part and 6 Seconds; put down the 6 Seconds in the Place of Seconds, and carry the 1 Part: And 4 Parts by 9 Parts is 36 Parts, and 1 that was carried is 37 Parts, or 3 Inches and 1 Part, which put down as in the Work.

Multiply the lower ſupplemental Multiplicand, 9 Inches, 4 Parts, and 7 Seconds, by the 6 Inches in the Multiplier, thus, 6 Inches by 7 Seconds is 42 Seconds, or 3 Parts and 6 Seconds; put the 6 Seconds under the Seconds in the laſt Product, and carry the 3 Parts: Then, 6 Inches by 4 Parts is 24 Parts, and the 3 that was carried is 27 Parts, or 2 Inches and 3 Parts; put the 3 Parts under the Parts of the former Product, and carry the 2 Inches: And 6 Inches by 9 Inches is 54 Inches, and 2 that was carried is 56 Inches, or 4 Feet and 8 Inches, which place as in the Work.

Multiply the given Multiplicand, 9 Feet, 4 Inches, and 7 Parts, by the 5 Feet in the Multiplier, and put the Product, 46 Feet, 10 Inches, and 11 Parts, as before taught.

Now add theſe three Products together, and their Sum is the Product ſought.

The ninth Example is done in the ſame Manner; only, the given 12 Feet, when divided, make 1 Foot in the firſt Multiplicand; and this Multiplicand, being divided by 12, gives 1 Inch, 9 Seconds, and 5 Thirds, for the ſecond ſupplemental Multiplicand.

Exam. 10.

Multiply 20 F. 6 I. 3 P. by 10 F. 8 I. 5 P.

F.	I.	P.	S.	T.
	1	8	6	3
1	8	6	3	
20	6	3		
10	8	5		
	8	6	7	3
13	8	2		
205	2	6		
219	7	2	7	3

Exam. 11.

Multiply 18 F. 7 I. 2 P. by 9 F. 5 I. 4 P.

F.	I.	P.	S.	T.
	1	6	7	2
1	6	7	2	
18	7	2		
9	5	4		
	6	2	4	8
7	8	11	10	
167	4	6		
175	7	8	2	8

Exam. 10. The Multiplicand and Multiplier conſiſting of Feet, Inches, and Parts, we muſt find two ſupplemental Multiplicands, as in the two laſt Examples; then drawing a Line over the Multiplicand, and dividing the 20 Feet in the Mu tiplicand by 12, the Quotient is 1 Foot, and 8 over; put t 1 over the Place of Feet; and, dividing the 8 Feet over b 12, it gives 8 Inches, which put over the Place of Inches Then, the 6 Inches in the Multiplicand, being divided by 12 gives 6 Parts, which put over the Place of Parts: And, laſtly the 3 Parts in the Multiplicand, being divided by 12, give 3 Seconds, which place as in the Example.

To form the ſecond ſupplemental Multiplicand, divide the 1 Foot in the firſt ſupplemental Multiplicand by 12, and it gives 1 Inch, which put over the Place of Inches: The 8 Inches in the firſt ſupplemental Multiplicand, being divided by 12, gives 8 Parts, which put over the Place of Parts The 6 Parts in the firſt ſupplemental Multiplicand, bein divided by 12, gives 6 Seconds in the ſecond Multiplicand And the 3 Seconds in the firſt ſupplemental Multiplicand, bein divided by 12, gives 3 Thirds in the ſecond ſupplement Multiplicand.

The upper ſupplemental Multiplicand, 1 Inch, 8 Part 6 Seconds, and 3 Thirds, being multiplied by the 5 Parts the Multiplier, gives 8 Inches, 6 Parts, 7 Seconds, and Thirds, by the Directions at the eighth Example.

The under ſupplemental Multiplicand, 1 Foot, 8 Inches 6 Parts, and 3 Seconds, being multiplied by the 8 Inches i the Multiplier, gives 13 Feet, 8 Inches, and 2 Parts, by the Directions at the eighth Example.

And the Multiplicand, 20 Feet, 6 Inches, and 3 Parts, being multiplied by the 10 Feet in the Multiplier, gives 205 Feet, 2 Inches, and 6 Parts, by the Directions at the eighth Example.

Now draw a Line, and add theſe three Products together, and their Sum is the Product required.

Exam. 11. The Multiplicand, 18 Feet, 7 Inches, and 2 Parts, being divided by 12, gives 1 Foot, 6 Inches, 7 Parts, and 2 Seconds, for the firſt ſupplemental Multiplicand; which, being divided by 12, gives 1 Inch, 6 Parts, 7 Seconds, and 2 Thirds, for the ſecond ſupplemental Multiplicand.

The upper ſupplemental Multiplicand, 1 Inch, 6 Parts, 7 Seconds, and 2 Thirds, being multiplied by the 4 Parts in the Multiplier, gives 6 Inches, 2 Parts, 4 Seconds, and 8 Thirds, by the Directions at the eighth Example.

The under ſupplemental Multiplicand, 1 Foot, 6 Inches, 7 Parts, and 2 Seconds, being multiplied by the 5 Inches in e Multiplier, gives 7 Feet, 8 Inches, 11 Parts, and 10 nds, by the Directions at the eighth Example.

nd the Multiplicand, 18 Feet, 7 Inches, and 2 Parts, g multiplied by the 9 Feet in the Multiplier, gives 167 , 4 Seconds, and 6 Parts, by the Directions at the eighth nple.

ſtly, adding theſe together, we have the Product ſought.

he two following Examples are done in the ſame Manner.

Exam. 12.

tiply 12 F. 2 I. 9 P. by
10 F. 7 I. 9 P.

F. I. P. S. T.
1 : 0 : 2 : 9
1 : 0 : 2 : 9

12 : 2 : 9
10 : 7 : 9

9 : 2 : 0 : 9
7 : 1 : 7 : 3
22 : 3 : 6

130 : 2 : 3 : 3 : 9

Exam. 13.

Multiply 15 F. 6 I. 1 P. by
7 F. 5 I. 2 P.

F. I. P. S. T.
1 : 3 : 6 : 1
1 : 3 : 6 : 1

15 : 6 : 1
7 : 5 : 2

2 : 7 : 0 : 2
6 : 5 : 6 : 5
108 : 6 : 7

115 : 2 : 8 : 5 : 2

But, when the Feet in the Multiplier are more than 12, as in the fifth, ſixth, and ſeventh Examples, it may be more convenient to do it by taking the aliquot Parts of a Foot. If the Number of Inches or Parts in the Multiplicand happen not to be aliquot Parts of 12, they may be divided into ſuch as are: The aliquot Parts of 12 are, 1, which is a twelfth Part; 2, a Sixth; 3, a Fourth; 4, a Third; and 6, the Half. The others may be made into two Parts, thus, 5 may be parted into 4, which is a Third, and the remaining 1 is the Fourth of that: So, 7 may be parted into 4, a Third, and 3, the Fourth of 12; or elſe into 6, the Half, and 1, the Sixth Part of that Half: And 8 may be parted into 4, the Third, to be taken twice; and 9 into 3, the Fourth, taken three times. Of this I ſhall ſubjoin ſome Examples.

litical
logical
xtinct
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ymmetry
[illegible]
histry
perfect
acility
ntiquity
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~~istinct~~
~~[illegible]~~
Generic
eciprocally
omentum

Exam. 14.

Exam. 14.	*Exam.* 15.	*Exam.* 16.
Multiply 49 F. 6 I. by 28 F. 4 I.	Multiply 242 F. 4 I. by 59 F. 4 I.	Multiply 84 F. 3 I. by 15 F. 9 I.
F. I. P.	F. I. P.	F. I. P.
4 : 1 : 6	20 : 2 : 4	7 : 0 : 3
49 : 6	242 : 4	84 : 3
28 : 4	59 : 4	15 : 9
16 : 6	80 : 9 : 4	63 : 2 : 3
14	19 : 8	3 : 9
392	2178	420
98	1210	84
1402 : 6	14378 : 5 : 4	1326 : 11 : 3

Exam. 14. Having multiplied the 4 Feet, 1 Inch, and 6 Parts, by the 4 Inches, and put down 16 Feet and 6 Inches, the Product, as before; then, inſtead of multiplying the 28 Feet by the 6 Inches, take the Half of 28 for Feet, becauſe 6 Inches are the Half of a Foot, which gives 14 Feet; place this under the 16 Feet, Units under Units: Then multiply the 28 Feet into the 49 Feet, that is, multiply the 49, firſt by the 8, and place the Units of this Product under the Units of the 14, the Tens under the Tens, and ſo on; and multiply the 49 by the remaining 2 of the Multiplier, putting this Product under the laſt Product, as in common Arithmetic: Now draw a Line, then add theſe into one Sum, and it will be the Product ſought.

Exam. 15. Having found the upper Multiplicand, 20 Feet, 2 Inches, and 4 Parts, as before taught, and multiplied it by the 4 Inches, put down the Product, 80 Feet, 9 Inches, and 4 Parts, as in the former Examples: But, the 4 Inches in the Multiplicand being one Third of a Foot, take the Third of 59 Feet, which is 19, and 2 Feet remains; and, the 2 Feet being 24 Inches, take the Third of 24, which is 8, and place under the Inches as uſual: Then multiply the 59 Feet into the 242, as taught in the laſt Example: Now draw a Line, and add as before, and you have the Product ſought.

The other Example is done in the ſame Manner.

Exam. 17.

Multiply 28 F. 6 I. 10 P. by 16 F. 4 I. 9 P.

F.	I.	P.	S.	T.
	2	4	6	10
2	4	6	10	
28	6	10		
16	4	9		
1	9	5	1	6
9	6	3	4	
8				
0	8			
0	5	4		
168				
28				
468	5	0	5	6

Exam. 18.

Multiply 42 F. 8 I. 4 P. by 18 F. 9 I. 6 P.

F.	I.	P.	S.	T.
	3	6	8	4
3	6	8	4	
42	8	4		
18	9	6		
1	9	4	2	
32	0	3		
9				
3				
0	6			
336				
42				
802	3	7	2	

Exam. 17. After forming the two ſupplemental Multiplicands, and multiplying the 9 Parts into the upper Multiplicand, and the 4 Inches into that next above the Line, ſet down each Product under the particular Denomination from whence it was produced, (dividing by 12, where neceſſary) as in the Work.

But now, inſtead of multiplying the 16 Feet by the 6 Inches and 10 Parts in the Multiplicand, (the 6 Inches being half a Foot) divide the 16 Feet in the Multiplier by 2, and put 8 Feet, the Quotient, in the Place of Feet, as in the Work.

Now proceed to the 10 Parts in the Multiplicand; and, the 6 Parts being half an Inch, divide the 16 Feet by 2, and the Quotient 8 will be Inches, which put in the Place of Inches.

There being 4 Parts left out of the 10 Parts, and 4 being one Third of 12, divide the 16 Feet by 3, and the Quotient 5 will be Inches, which put in the Place of Inches; and, the 1 Inch over being multiplied by 12, to bring it into Parts, divide 12 by 3, and the Quotient is 4 Parts, which put in the Place of Parts, as in the Work.

Then multiply the 28 by 16, and place the Products as in the Work, like common Multiplication.

Now add theſe together, and it will be the Product ſought.

If the Reader ſhould be at any Uncertainty to fix the Denomination of the Quotient, he may remember,

That Feet divided by Inches produce Feet in the Quotient.

Feet divided by Parts produce Inches in the Quotient.

Feet divided by Seconds produce Parts in the Quotient.

Feet divided by Thirds produce Seconds in the Quotient.

And, univerſally, the Quotient is the next higher Denomination to that in the Multiplicand whoſe aliquot Part divided the Feet in the Multiplier.

Exam. 18. Dividing the Multiplicand 42 Feet, 8 Inches, and 4 Parts, by 12, the Quotient is 3 Feet, 6 Inches, 8 Parts, and 4 Seconds, for the lower ſupplemental Multiplicand; which, being divided by 12, gives 3 Inches, 6 Parts, 8 Seconds, and 4 Thirds, for the upper ſupplemental Multiplicand.

Multiplying the 3 Inches, 6 Parts, 8 Seconds, and 4 Thirds, the upper ſupplemental Multiplicand, by the 6 Parts in the Multiplier, the Product is 1 Foot, 9 Inches, 4 Parts, and 2 Seconds, as in the Work.

Multiplying the 3 Feet, 6 Inches, 8 Parts, and 4 Seconds, the lower ſupplemental Multiplicand, by the 9 Inches in the Multiplier, the Product is 32 Feet and 3 Inches, as in the Work.

But now, inſtead of multiplying the Inches and Parts in the Multiplicand, by the 18 Feet in the Multiplier, out of the 8 Inches in the Multiplicand take 6 Inches, for half a Foot; then divide the 18 Feet in the Multiplier by 2, and put down 9 Feet, the Quotient, in the Place of Feet.

Now, the 2 Inches of the Multiplicand which are left being one Third of the 6 Inches before taken, divide the 9 Feet laſt found by 3, and the Quotient is 3 Feet, which place under the 9 Feet.

There being 4 Parts left, they are one Sixth of the 2 Inches laſt found; therefore, divide the 3 Feet by 6, and the Quotient is 6 Inches, which put in the Place of Inches.

Laſtly, multiply the 42 by 18, as in common Multiplication.

Now add theſe together, and the Sum will be the Product ſought.

When one of the given Numbers conſiſts only of Feet and Inches, tho' the other has Parts, that which is without Parts being made the Multiplier, there will then be required but one ſupplemental Multiplicand, as in the nineteenth Example: And, if one of the given Numbers conſiſts of Feet only, then the Multiplicand need not be divided by 12, as in the twentieth Example.

Exam. 19.

Multiply 17 F. 10 I. 6 P. by 25 F. 9 I.

F.	I.	P.	S.
1	5	10	6
17	10	6	
25	9		
13	4	10	6
1	0	6	
12	6		
8	4		
75			
34			
450	3	4	6

Exam. 20.

Multiply 76 F. 8 I. 7 P. by 16 F.

F.	I.	P.
76	8	7
16		
	8	
	1	4
10	8	
456		
76		
1227	5	4

Exam. 19. Inſtead of multiplying the 25 Feet into the 6 Parts, take the Half, which is 12 Inches, or 1 Foot, and the 1 that remains makes 6 Parts; then, dividing the 10 Inches into the two aliquot Parts, 6, the Half of a Foot, and 4, the Third, take firſt the Half of 25, which is 12 Feet and 6 Inches, and then the Third, which is 8 Feet and 4 Inches.

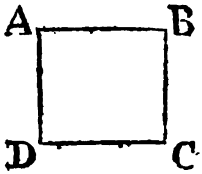
Exam. 20. The 7 Parts being divided into 6, the Half, and 1, the Sixth of that Half, take the Half of 16, which is 8 Inches, and the Sixth of that is 1 Inch and 4 Parts; then, dividing the 8 Inches in the Multiplicand into 4 and 4, each being the Third of a Foot, take the third Part of 16, which is 5 Feet and 4 Inches, and that, being doubled, gives 10 Feet and 8 Inches.

The Meaſuring of Superficies.

Art. 52. TO meaſure a Square.

Rule. Multiply the Side of the Square into itſelf, and the Product is the Area or ſuperficial Content.

Let A B C D repreſent a Square, whoſe Side is 12 Feet. Multiply 12, the Side, by itſelf, thus,



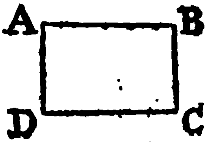
12
12
―――
144 Area,

The Product 144 is the Area or ſuperficial Content of the Square A B C D; that is, a Square, whoſe Side is 12 Feet, contains 144 Feet: This will appear, if each of the Sides A B, B C, C D, and D A, are divided into 12 equal Parts, and Lines drawn from thoſe Diviſions croſs the Square; for then will the Square be divided into 144 little Squares, whoſe Sides will be equal to the Diviſions of the Lines.

Art. 53. To meaſure a Parallelogram.

Rule. Multiply the Length by the Breadth, and the Product is the Area or ſuperficial Content.

Let A B C D repreſent a Parallelogram, whoſe Length is 12 Feet, and Breadth 9 Feet. Multiply 12 by 9, and the Product is the Content.



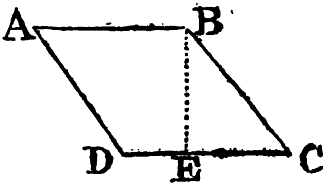
12
9
―――
108 Area.

Therefore the ſuperficial Content is 108 Feet; as may be ſeen by dividing the Lines A B and C D into 12 equal Parts, and drawing Lines from one to the other; then dividing the Lines A D and B C into 9 Parts each, and drawing Lines from one to the other, cutting the former; by which the Parallelogram will be divided into 108 little Squares, each of whoſe Sides will be equal to the Diviſions of the Lines: So that, of whatever Dimenſions the Diviſions of the Lines are, the Parallelogram will contain 108 Squares of the ſame Dimenſions.

Article 54. To meaſure a Rhombus.

Definition. A Rhombus is a Figure with four equal Sides, in the Form of a Quarry of Glaſs, or a Diamond on Cards, having two Angles greater, and two leſs, than the Angles of a Square: The former are called *obtuſe* Angles, and the latter *acute* or ſharp Angles.

Rule. Multiply the Side by the Length of a Perpendicular let fall from one of the obtuſe Angles to the Side oppoſite ſuch Angle.



Let A B C D repreſent a Rhombus, each of whoſe Sides is 16 Feet: A Perpendicular let fall from the obtuſe Angle at B, on the Side D C, will interſect it in the Point E, ſo will B E be 13 Feet; and this being multiplied into the given Side, the Product is the Area of the Rhombus.

16

13

48

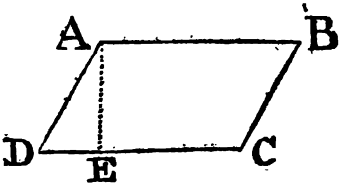
16

208 Area.

Article 55. To find the Area of a Rhomboides.

Definition. A Rhomboides is a Figure whoſe oppoſite Sides and oppoſite Angles are equal.

Rule. Multiply one of the longeſt Sides by the Perpendicular let fall from one of the obtuſe Angles on one of the longeſt Sides.



The Rhomboides ABCD; the longeſt Sides AB or CD being 20.9 Feet, and the Perper AE 11.2 Feet.

20.9

11.2

418

209

209

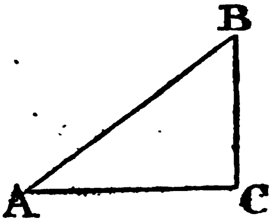
234.08 Anſwer 234.08 Feet.

Article 56. To meaſure a Triangle.

Definition. A Triangle is a Figure having three Sides and three Angles. There are three Sorts of Triangles, denominated from the Angles: For, if one of the Angles of a Triangle is an exact Square, (which, by Mathematicians, is called a *right Angle*) ſuch Triangle is called a *right-angled* Triangle: If there is no right Angle, it is called an *oblique-angled* Triangle, of which there are two Kinds; if all the three Angles are acute, that is, each leſs than a right Angle, it is called an *acute-angled* Triangle; but, if one of the Angles is greater than a right Angle, it is called an *obtuſe-angled* Triangle.

Rule. If it be a right-angled Triangle, multiply the Baſe by half the Perpendicular, or half the Baſe by the Perpendicular, and the Product is the Area: But, if it be an oblique-angled Triangle, (whether obtuſe or acute) multiply half the Baſe by the Length of the Perpendicular let fall on the Baſe from the Angle oppoſite to it, or half the perpendicular Height by the Baſe, and the Product is the Area. The longeſt Side of a Triangle is uſually called the Baſe, except in a right-angled Triangle, where the longeſt of the two Legs which include the right Angle is called the Baſe.

The right-angled Triangle A B C, right-angled at C; the Baſe A C is 21 Feet, and the Perpendicular B C 15.5 Feet.

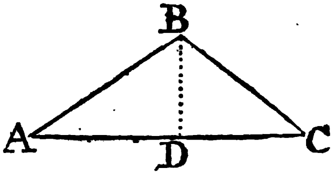


15.5 Perpendicular.
10.5 Half the Baſe.

775
1550

162.75 Area.

The oblique-angled Triangle A B C being given, let fall a Perpendicular from the Angle at B on the Baſe A C, and that Perpendicular is the Height of the Triangle. The Baſe A C being 28.6, and the Perpendicular B D 10, to find the Area?



14.3 Half the Baſe.
10 Height of the Triangle.

143.0 Area.

And here the Learner may obſerve, that this Way of meaſuring an oblique-angled Triangle is the ſame with the meaſuring a right-angled Triangle. In this Place, therefore, it may be proper to inſtruct the Reader in one of the Properties of a right-angled Triangle: Which is this, that the Square of the longeſt Side of a right-angled Triangle, uſually called the Hypothenuſe, is equal to the Sum of the Squares of the two other Sides, uſually called the Legs. This is the forty-ſeventh Propoſition of the firſt Book of *Euclid*: For the finding of which *Pythagoras* is ſaid to have ſacrificed a Hecatomb to the Muſes, it being a Thing of ſuch conſiderable Uſe; for, by this Means, any two Sides of a right-angled Triangle being given, the other may be found by common Arithmetic. Thus, in the right-angled Triangle ABC, the Baſe AC and Perpendicular BC being given, the Hypothenuſe AB may be found by extracting the Square Root of the Sum of the Squares of the Baſe and Perpendicular.

21 Baſe.	15.5 Perpendicular.	441
21	15.5	240.25
21	775	681.25 (26.1 Hypothenuſe.
42	775	4
	155	
441		46) 281
	240.25	276
		521) 525
		521
		4

And, if the Hypothenuſe and one of the Legs are given, the other may be found by ſubtracting the Square of the given Leg from the Square of the Hypothenuſe, for the Square Root of the Remainder is the other Leg.

In theſe Operations, the true Length of the Side ſought cannot always be found, becauſe the Sum of the Squares of the Legs, or the Difference of the Squares of the Hypothenuſe and given Side, rarely happen to be Square Numbers: In which Caſe, the Extraction may be continued as far as is neceſſary, in order to come as near the Truth as may be judged proper. But there are ſome Square Numbers whoſe Sums do make a Square: Of which Sort are the Numbers 3 and 4, whoſe Squares being added together make 25, which is the Square of 5: Therefore, if the Baſe of a Triangle is 4, and the Perpendicular 3, the Hypothenuſe will be 5: And, if any of theſe Numbers are multiplied by any other Number, thoſe Products will be the Sides of right-angled Triangles; as 6, 8, 10, and 15, 20, 25. Thus Artificers, when they ſet off the

the Corner of a Building, uſually meaſure 6 Feet on one Side and 8 Feet on the other, then laying croſs a ten-feet Pole, it makes the Corner a true right Angle.

Article 57. There is another Method of finding the Area of Triangles, the three Sides being given.

Rule. Add the three Sides together, then take the Half of that Sum, and out of it ſubtract each Side ſeverally: Multiply the Half of the Sum and theſe Remainders continually; then extract the Square Root of this Product, which will be the Area of the Triangle.

In the former oblique Triangle, the Baſe A C is given 28.6, the Side A B is 18.32, and the Side B C is 16.6, to find the Area?

28.6
18.32
16.6

63.52 Sum.

31.76 Half the Sum,

31.76
28.6

3.16

31.76
18.32

13.44

31.76
16.6

15.16

31.76
3.16

19056
3176
9528

100.3616
13.44

4014464
4014464
3010848
1003616

1348.8599|04
15.16

80931594
13488599
67442995
13488599

20448.716084

20448.716084 (142.999
1

24) 104
96

282) 848
564

2849) 28471
25641

28589) 283060
257301

285989) 2575984
2573901

2083

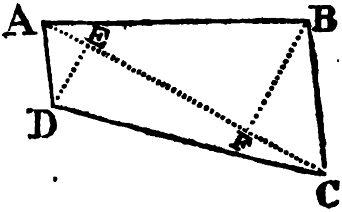
Thus is 142.999 the Area of the Triangle, which is within .001 Part of what was before found.

Article 58. To meaſure a Trapezium.

Definition. A Trapezium is an irregular Figure, of four unequal Sides, and unequal Angles.

Rule. Draw a Diagonal from one of the Angles to the oppoſite Angle, and then will the Trapezium be divided into two Triangles, of which the Diagonal is the common Baſe: Then drawing Perpendiculars from the other oppoſite Angles to the Diagonal, add thoſe Perpendiculars together; and multiply the Half of that Sum into the Diagonal, or the Half of the Diagonal into the Sum of the Perpendiculars, and that Product is the Area of the Trapezium.

In the Trapezium ABCD, the Diagonal AC is 498, the Perpendicular DE 10.8, and the Perpendicular BF 18.8: The Sum of the Perpendiculars is 29.6, whoſe Half is 14.8, which, being multiplied into 498, gives the Area.

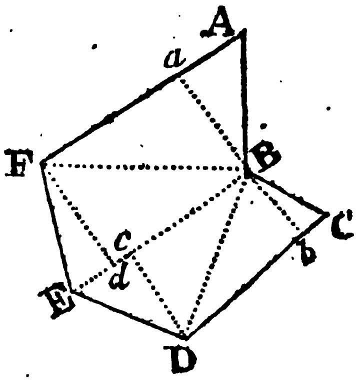


$$\begin{array}{r} 498 \\ 14.8 \\ \hline 3984 \\ 1992 \\ 498 \\ \hline 7370.4 \end{array}$$ Area of the Trapezium.

Artice 59. To meaſure any irregular Figure.

Rule. Divide the Figure into Triangles, by drawing Diagonals from one Angle to another; then meaſure all the Triangles, by either of the Rules already taught at Art. 56 or 57, and add the ſeveral Areas of all the Triangles together, which Sum will be the Area of the given Figure.

The irregular Figure ABCDEF being given, divide it into Triangles by the Diagonals FB, EB, and DB: Then may the Triangles be meaſured by letting fall Perpendiculars on their reſpective Baſes, as B*a*, B*b*, D*c*, F*d*, and multipying thoſe Perpendiculars by half their reſpective Baſes, according to the Rule of Art. 56; or by meaſuring the Sides of each Triangle,



Triangle, and multiplying the Half-Sum of the Sides of each into the Differences betwixt the Half-Sum and each Side continually; then extract the Square Root of the Product, according to the Rule of Art. 57. Thus having obtained the Areas of all the Triangles, add them together, and this will be the Area of the given Figure.

In the Triangle A F B the Baſe F A is 100, and the Perpendicular B*a* 49; in the Triangle F B E the Baſe B E is 92, and the Perpendicular F *d* 52; in the Triangle E B D the Baſe B E is the ſame as before, and the Perpendicular D *c* 44; and in the Triangle D C B the Baſe D C is 80, and the Perpendicular B *b* 38; by which the Area of each may be found, by Art. 56, as follows.

50 Half A F.
49 Perpendicular B *a*.
450
200
2450 Area of A F B.

46 Half B E.
52 Perpendicular F *d*.
92
230
2392 Area of F B E.

46 Half B E.
44 Perpendicular D *c*.
184
184
2024 Area of E B D.

38 Pependicular B *b*.
40 Half D C.
1520 Area of D C B.

2450
2392
2024
1520
9186 Area of the Figure A B C D E F.

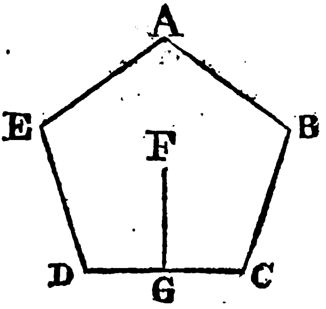
In dividing any irregular Figure into Triangles, the Triangles will be leſs by two than the Number of the Sides of the Figure, and the Diagonals leſs by three than the Number of Sides. Thus, in the preſent Caſe, the Figure has ſix Sides, and is divided into four Triangles, FAB, FBE, EBD, and DBC, by three Diagonals, BF, BE, and BD.

Article 60. To meaſure any regular Polygon.

Definition. A regular Polygon is a Figure whoſe Sides and Angles are all equal; they are uſually denominated from the Number of their Sides: Thus, a Figure of five equal Sides is called a regular *Pentagon*; of ſix Sides, an *Hexagon*, &c.

Rule. Multiply the Length of one of the Sides by the Number of Sides; then multiply this Product by the Half of a Perpendicular let fall from the Center of the Figure to the Middle of one of the Sides, and the Product is the Area of the Polygon.

In the Pentagon ABCDE each Side is 95, and the Perpendicular FG 65.36, to find the Area?



95
5

475 Sum of the Sides.
32.68 Half the Perp. FG.

3800
2850
950
1425

15523.00 Area of the Pentagon.

But, for the more ready meaſuring regular Polygons, the following Table, containing Multipliers for all regular Figures from the Triangle to the Dodecagon, or Figure of twelve Sides, will be of Uſe to the Learner. The Rule for making the Table will be ſhewn at the End of the Trigonometry.

Number of Sides.	Names.	Multipliers.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.589076
7	Heptagon	3.633959
8	Octagon	4.828427
9	Enneagon	6.181827
10	Decagon	7.694209
11	Endecagon	8.514250
12	Dodecagon	9.330125

Now, if a Side of any of theſe Figures be given, ſquare the Side, and multiply it by the Multiplier in the Table, and the Product is the Area of the Figure.

Exam. 1. A Pentagon whoſe Side is 95 Feet.

95
95

475
855

9025
1.72 Multiplier.

18050
63175
9025

15523.00 Area as before.

Exam. 2. An Octagon whoſe Side is 67 Feet.

67
67

469
402

4489
4.828 Multiplier.

35912
8978
35912
17956

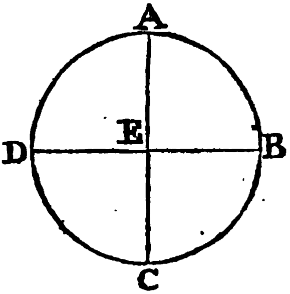
21672.892 Area of the (Octagon.

Exam. 1. The Square of the Side is 9025, which being multiplied by 1.72, the Multiplier in the Table, omitting the other Figures, as the third Place of Decimals is a Cypher, the Product is 15523, for the Area of the Pentagon, as before.

The other Example is done in the ſame Manner, only there I have taken three Places of Decimals in the Multiplier, the third Place being 8, otherwiſe two Places are ſufficient in moſt Caſes.

To Meaſure a Circle and its Parts.

What a Circle is, and how deſcribed, is ſo well known that it can't be neceſſary to define it, any farther than to acquaint the Reader with the Names of the ſeveral Parts to be meaſured or referred to in meaſuring.



In the annexed Circle A B C D, the Arch-Line A B C D is called the *Periphery*, the Length of which is called the *Circumference*: Any Line, as D B or A C, paſſing thro' the Center E, cuts the Circle into two equal Parts, called *Semicircles* or Half-Circles; and ſuch Lines are called *Diameters* of the Circle: If two Diameters are drawn thro' a Circle, at right Angles to each other, then is the Circle divided into four equal Parts, called *Quadrants*: Half the Diameter, as E B, is called the *Radius* or Semidiameter.

Article 61. The Diameter of a Circle being given, to find the Circumference.

Rule. This may be done by either of the following Proportions; in whole Numbers, as 7 is to 22, or in Decimals, as 1 is to 3.14159, ſo is the Diameter of a Circle to the Circumference.

Exam. A Circle whoſe Diameter is 17, to find the Circumference?

As 7 is to 22, ſo is 17.
17
154
22
7) 374 (53.4 Circum-(ference.
35
24
21
30
28
2

As 1 is to 3.14159, ſo is 17.
17
2199113
314159
53.40704 Circum-(ference.

The Anſwers are nearly the ſame in both; but, for greater Expedition, the Multiplier in the Decimals may be reduced one Figure, and 3.1416 may be uſed without any ſenſible Difference; by which the Diameter of a Circle being multiplied, the Product is the Circumference: The firſt Term in the Proportion being 1, the Work of Diviſion is rendered uſeleſs.

Article 62. The Circumference of a Circle being given, to find the Diameter.

Rule. As 22 is to 7, or as 1 is .31831, ſo is the Circumference of a Circle to the Diameter.

Exam. The Circumference of a Circle being 473, to find the Diameter?

As 22 is to 7, ſo is 473.

473
7

22) 3311 (150.5
22

111
110

110
110

0

As 1 is to .31831, ſo is 473.

.31831
473

95493
222817
127324

150.56063

Article 63. To find the Area of a Circle.

Rule. Multiply Half the Diameter by Half the Circumference, and the Product is the Area.

If the Diameter is given, find the Circumference by Art. 61; if the Circumference is given, find the Diameter by Art. 62.

Exam. A Circle whoſe Diameter is 17, and Circumference 53.4, to find the Area?

26.7 Half the Circumference.
8.5 Half the Diameter.

1335
2136

226.95 Area of the given Circle.

Article 64. The Diameter being given, to find the Area of a Circle without finding the Circumference.

Rule. Square the Diameter, then multiply the Square by .78539, and the Product is the Area of the Circle whoſe Diameter was given.

Exam. The Diameter of a Circle being 17, to find the Area?

```
 17
 17
----
119
17
----
289
```

```
 .78539
    289
--------
 706858
628312
157078
--------
226.97771 Area of the Circle.
```

For greater Expedition, the Multiplier may be .7854, which will be ſufficiently exact.

Article 65. The Circumference of a Circle being given, to find the Area without finding the Diameter.

Rule. Square the Circumference, then multiply that Square by .07958, and the Product is the Area of the Circle.

Exam. The Circumference of a Circle being 53.4, to find the Area?

```
  53.4
  53.4
-------
  2136
 1602
2670
-------
2851.56
```

```
    2851.56
     .07953
-----------
    2281248
   1425780
  2566404
 1996092
-----------
226.9271448 Area of the Circle.
```

The Learner may perceive, that, in the three different Ways of finding the Area, the Differences are but ſmall, tho' there was an Omiſſion of ſome of the Decimals to render the Work leſs tedious.

Article 66. The Dimenſions of any of the Parts of a Circle being given, to find the Side of a Square equal to the Circle.

Rule. If the Area of the Circle is given, extract the Square Root of the Area, which will be the Side of a Square equal to the Circle: If the Diameter or Circumference be given, find the Area by the Rules in Art. 64 or 65, and extract the Square Root of the Area found, which Root will be the Side of a Square equal to the given Circle. And this is a *general Rule* to find the Side of a Square equal to any *ſuperficial Figure*, regular or irregular: For the Square Root of the Area of any Figure whatever is the Side of a Square equal to the given Figure. But, with Regard to Circles, if the Diameter is given, multiply it by .886, and the Product will be the Side of a Square equal to the Circle whoſe Diameter was ſo given: And, if the Circumference is given, multiply it by .282, for the Side of a Square equal to the Circle whoſe Circumference was ſo multiplied.

Exam. 1.

Let the Diameter of a Circle be 17, to find the Side of a Square equal to the Circle?

.886
17
———
7202
886
———
15.062 Side of a Square.

Exam. 2.

The Circumference being 53.4, to find the Side of a Square?

53.4
.282
———
1068
4272
1068
———
15.0588 Side of a Square.

The Learner may try the Truth of theſe Operations, by finding whether the Area of a Circle whoſe Diameter is 17 is the ſame with the Area of the Square whoſe Side is 15.062. The ſecond Example may be tried in the ſame Manner.

Article 67. The Area of a Circle being given, to find the Diameter.

Rule. Multiply the given Area by 1.2732, and the Product is the Square of the Diameter; then extracting the Square Root of the Product, you have the Diameter.

Exam. The Area of a Circle being 226.9, to find the Diameter.

```
                            .  . . .
     226.9                 288.8437 (16.99 Diameter.]
     1.273                 1
   ---------              ------
     6807              26) 188
    15883                  156
    4538                  ------
   2269               329) 3284
  ----------                2961
  288.8437                 ------
                     3389) 32337
                            30501
                           ------
                             1836 Remainder.
```

Article 68. The Area of a Circle being given, to find the Circumference.

Rule. Multiply the given Area by 12.566, and extract the Square Root of the Product, which Root will be the Circumference of the Circle whoſe Area was ſo multiplied.

Exam. The Area of a Circle being 226.9, to find the Circumference?

```
                          . .  . .  .
     12.566             2851.2254 (53.39 Circumference.
      226.9             25
   ---------           -----
    113094         103) 351
    75396               309
   25132               ------
  25132           1063) 4222
 -----------             3189
 2851.2254              ------
                10669) 103354
                        96021
                       ------
                         7333 Remainder.
```

In theſe two Examples, the Numbers come out ſomewhat leſs than the Truth, but near enough for common Uſe.

Article 69. The Side of a Square being given, to find the Diameter of a Circle equal to the Square whoſe Side is given.

Rule. Multiply the given Side by 1.128, and the Product will be the Diameter of a Circle whoſe Area is equal to the Area of the given Square.

Exam. The Side of a Square being 15.04, to find the Diameter of a Circle equal to that Square?

$$\begin{array}{r} 15.04 \\ 1.128 \\ \hline 12032 \\ 3008 \\ 1504 \\ 1504 \\ \hline 16.96512 \end{array}$$

Anſwer.

Article 70. The Side of a Square being given, to find the Circumference of a Circle equal to the given Square.

Rule. Multiply the given Side by 3.545, and the Product will be the Circumference of a Circle equal to the given Square.

Exam. The Side of a Square being 15.04, to find the Circumference of a Circle equal to that Square?

$$\begin{array}{r} 15.04 \\ 3.545 \\ \hline 7520 \\ 6016 \\ 7520 \\ 4512 \\ \hline 53.31680 \end{array}$$

Anſwer.

The Truth of the Operations in the two laſt Articles the Reader may try as at Art. 66.

Article 71. To find the Area of a Semicircle, the Diameter being given.

Rule. Find the Area of the Circle, by Art. 64, and take the Half of it.

In the ſame Manner may the Area of a Quadrant, or a Quarter of a Circle, be found, by taking a fourth Part of the Area of the whole Circle.

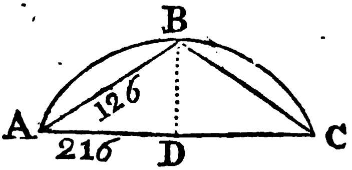
But, with Regard to meaſuring a Sector, or a Segment of a Circle, it will be neceſſary firſt to ſhew the Way to find the

Length of the Arch-Line of a Sector, and the Diameter of the Circle to a given Segment.

Article 72. A Sector or Segment of a Circle being given, to find the Length of the Arch-Line.

Rule. Divide the Segment into two equal Parts; then meaſure the Chord of the Half-Arch, from the Double of which ſubtract the Chord of the whole Segment; and one Third of that Difference, being added to the Double of the Chord of the Half-Arch, is the Length of the Arch-Line.

Exam. In the Segment A B C D the whole Chord A D C is 216, and the Chord A B or B C 126, to find the Arch-Line A B C?



126 Chord AB or BC.
2

252 Double.
216 A D C ſubtracted.

36 Difference.

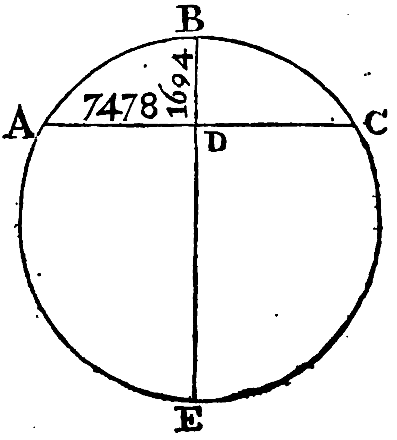
252 Double of AB.
12 $\frac{1}{3}$ of the Difference add.

264 Length of the Arch ABC.

Article 73. The Chord and verſed Sign of a Segment being given, to find the Diameter of a Circle.

Rule. Multiply Half the Chord by itſelf, and divide the Product by the verſed Sine; then add the Quotient to the verſed Sine, and the Sum is the Diameter of the Circle.

Exam. In the Segment A B C D the Chord A C is 7478, and the verſed Sine B D 1694, to find the Diameter?



3739 Half the
3739 (Chord AD.

33651
11217
26173
11217

13980121

1694)

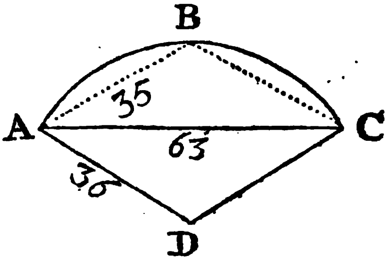
```
1694) 13980121 (8252 D E.
      13552       1694 B D add.
      -----       ----
       4281       9946 Diameter B D E.
       3388
       ----
        8932
        8470
        ----
         4621
         3388
         ----
          233
```

Article 74. To meaſure a Sector.

Definition. A Sector is a Part of a Circle, contained between an Arch-Line and two Radii or Semidiameters of the Circle.

Rule. Find the Length of Half the Arch, by Art. 72; then multiply this by the Radius or Semidiameter, and the Product is the Area.

Exam. 1. In the Sector A B C D, given the Radius A D or D C 36 Feet, the Chord A C 63 Feet, and the Chord A B or B C 35 Feet, to find the Area of the Sector?

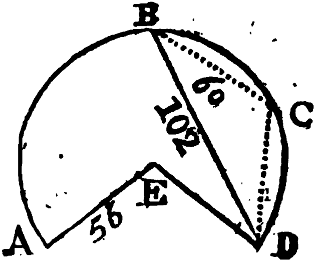


```
35 Chord A B.
 2
--
70
63 ſubtract.
--
3) 7
   -----
   2.333
  70     add.
  ------
  72.333 Length of the Arch A B C, by Art. 72.
```

```
   36.166 Half the Arch.
   36.    Radius.
  -------
  216996
 108498
 --------
 1301.976 Area of the Sector.
```

Exam.

Ex. 2. In the Sector ABCD, greater than a Semicircle, given the Radius AE or ED 56, the Chord BD of Half the Arch ABD 102, and the Chord BC of Half the Arch BCD 60, to find the Area of the Sector?



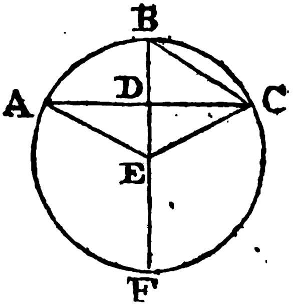
60 BC.
2
120
102 ſubtract.
3) 18
6
120 add.
126 Length of the Arch BCD, by Art. 72.

126 Half the Arch.
56 Radius.
756
630
7056 Area of the Sector.

Article 75. To find the Area of a Segment of a Circle.

Definition. A Segment of a Circle is any Part of a Circle cut off by a right Line drawn croſs the Circle, which does not paſs through the Center; for, if a right Line paſſes through the Center, it is a Diameter, and divides the Circle into two equal Parts called Semicircles; but a Segment is always either greater or leſs than a Semicircle.

Exam. 1. To find the Area of the Segment ABC, whoſe Chord AC is 86, the Chord of Half the Arch BC 52, and the verſed Sine BD 29.24?



Rule. By Art. 72, find the Length of the Arch-Line ABC, and, by Art. 73, the Diameter BF; then multiply Half the Chord of the Arch ABC by Half the Diameter, and the Product is the Area of the Sector ABCE: Then find the Area of the Triangle AEC, whoſe Baſe AC is 86, and perpendicular Height DE 17, found by ſubtracting the verſed Sine BD from Half the Diameter; and the Area of the Triangle AEC, being ſubtracted from the Area of the Sector ABCE, leaves the Area of the Segment ABC.

52 BC.
2

104
86 AC ſubtract.

3) 18

6
104 add.

110 Arch-Line ABC.

46.237 Radius.
55 Half the Arch.

231185
231185

2543.035 Area of the (Sector.

43 Half the Baſe AD.
17 Perpendicular DE.

301
43

731 Area of the Triangle.

43 Half ADC.
43

129
172

29.24)1849.00 (63.235 DEF.
17544 29.24 BD add.

9460 92.475 Diameter BF.
8772

6880
5848

10320
8772

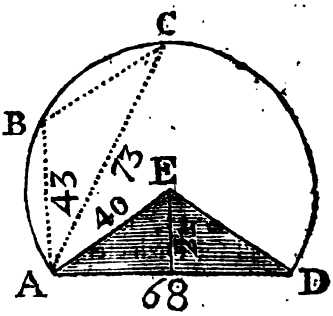
15480
14620

860

2543.
731

1812 Area of the Segment.

Exam. 2. In the Segment ABCD, greater than a Semicircle, given the Chord of the whole Segment AD 68, the Chord AC of Half the Arch ACD 73, the Chord AB or BC of one Fourth of the Arch ACD 43, and the Radius AE or ED 40, to find the Area of the Segment ABCD?



Firſt find the Area of the Sector A B C D E, by Art. 72, at the ſecond Example; then find the Area of the Triangle A E D, by Art. 56; and, adding the Area of the Triangle to the Area of the Sector, that Sum is the Area of the Segment.

43 Chord A B.
2
86
73 Chord A C ſubtract.
3)13
4.333
86 Double of A B add.
90.333 Arch-Line A B C.
40
3613.320 Area of the Sector.

34 Half the Baſe A D.
21 Perpendicular E 68.
34
68
714 Area of the Triangle (A E D.
3613.32 Area of the Sector add.
4327.32 Area of the Segment.

The Arch-Line A B C being but Half the Arch of the Sector, multiply it by 40, the Radius, and the Product 3613.32 is the Area of the Sector; to which adding 714, the Area of the Triangle, their Sum 4327.32 is the Area of the Segment.

Article 76. To find the Area of an Ellipſis.

Definition. An Ellipſis, commonly called an Oval, is a Curve which returns into itſelf like a Circle, but has two different Diameters, one longer than the other.

Rule. Multiply the two Diameters of the Ellipſis together; then multiplying that Product by .78539, or .7854, this laſt Product is the Area of the Ellipſis.

In the Ellipſis A B C D, the tranſverſe or longeſt Diameter B D is 44, and the conjugate or ſhorteſt Diameter A C 36, to find the Area?

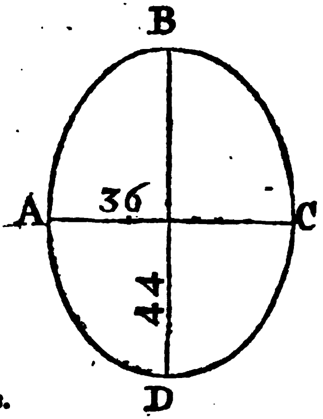
44
36

264
132

1584
.7854

6336
7920
12672
11088

1244.0736 Area of the Ellipſis.



Article 77. To meaſure a Parabola.

Definition. A Parabola is a Figure made by cutting a Cone [*See the Definition of a Cone, Art.* 81.] by a Plane parallel to one of the Sides: But the Figure will give the Learner a better Idea than any Definition.

Rule. Multiply the Baſe by the Height, and that Product by 2; then divide this laſt Product by 3, and the Quotient is the Area.

Exam. The Baſe AC is 120, and the Height BD 84, to find the Area?

120
84

480
960

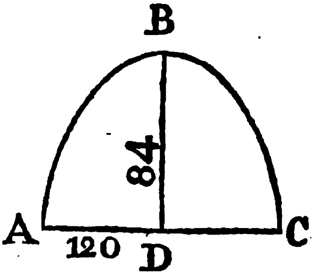
10080
2

3) 20160 (6720 Area of the Parabola ABCD.
18

21
21

6
6

0



The

The Meaſuring of Solids.

THE ſolid Foot is compoſed of 1728 Inches: For a Solid that is 1 Foot, or 12 Inches, every Way, the ſolid Content is 1728 Inches, as appears by the following Article.

Article 78. To meaſure a Cube.

Definition. A Cube is a Solid of ſix equal Sides, each of which is an exact Square, in the Form of a Die.

Rule. Multiply the Side by itſelf, and that Product by the ſame Side, and this laſt Product is the ſolid Content of the Cube.

Exam. The Side of a Cube being 15 Inches, or 1 Foot and 3 Inches, to find the Content?

```
     1.25              15
     1.25              15
    ------            ----
      625              75
     250              15
    125               -----
    -------           225
    1.5625             15
      1.25            -----
    -------           1125
     78125            225
    31250             -----
   15625          1728) 3375 (1.95 ſolid Content.
   ---------            1728
   1.953125           -------
                       16470
                       15552
                      -------
                        9180
                        8640
                       ------
                         540 Remainder.
```

I have done this two different Ways, that the Learner may ſee they come out the ſame. The Content in Inches is 3375, which being divided by 1728, the Inches in a ſolid Foot, and the Diviſion continued by annexing more Cyphers, it will come out exactly 1.953125, as in the Decimal Operation.

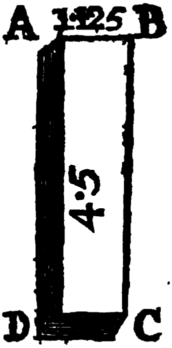
Article 79. To meaſure a Parallelopipedon.

Definition. A Parallelopipedon is a Solid of three Dimenſions, Length, Breadth, and Thickneſs; as a Piece of Timber exactly ſquared, whoſe Length is more than the Breadth or Thickneſs. The Ends are called Baſes, which are equal.

Rule. Find the Area of the Baſe, then multiplying that by the Length, it gives the ſolid Content.

Exam. 1. The Side AB is 1.125, and the Length CD 4.5, to find the ſolid Content?

$$\begin{array}{rl}
1.125 & \\
1.125 & \\
\hline
5625 & \\
2250 & \\
1125 & \\
1125 & \\
\hline
1.265625 & \text{Area of the Baſe.} \\
4.5 & \\
\hline
6328125 & \\
5062500 & \\
\hline
5.6948125 & \text{ſolid Content.}
\end{array}$$



Exam. 2. A Veſſel being 2.5 Feet each Side within, and 3 Feet deep, to find the Content?

$$\begin{array}{rl}
2.5 & \\
2.5 & \\
\hline
125 & \\
50 & \\
\hline
6.25 & \\
3 & \\
\hline
18.75 & \text{the Content.}
\end{array}$$

If a Piece of Timber, or any other Thing, is of an equal Bigneſs through its whole Length, tho' there is a Difference between the Breadth and Thickneſs, if the Breadth and Thickneſs are multiplied together, and that Product multiplied by the Length, this laſt Product is the ſolid Content. The ſame is the Caſe with Regard to a Veſſel of three different Dimenſions, as a Ciſtern. *Exam.*

Exam. A Piece of Timber being 1 Foot and 3 Inches, or 15 Inches, broad, 9 Inches thick, and 8 Feet and 6 Inches, or 102 Inches, long, to find the Content?

```
     1.25                       15 Inches.
      .75                        9 Inches.
     ----                      -----
      625                      135
     875                       102 Inches.
    -----                      -----
    .9375                      270
      8.5                     1350
   ------                     -----
    46875             1728) 13770 (7.96875
   75000                    12096
   -------                  -----
  7.96875 Content.           16740
                             15552
                             -----
                              11880
                              10368
                              -----
                               15120
                               13824
                               -----
                                12960
                                12096
                                -----
                                  8640
                                  8640
                                  ----
                                     0
```

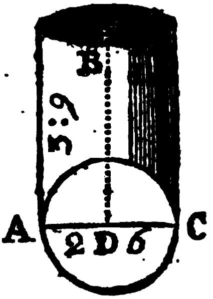
In this Operation the Inches are changed into the Decimal Fractions of a Foot, by Article 48.

Article 80. To meaſure a Cylinder.

Definition. A Cylinder is a round Body, whoſe Baſes are Circles, like a round Column, or a Rolling-Stone of a Garden.

Rule. The Diameter of the Baſe being given, find the Area, by Art. 64; then multiplying the Area of the Baſe by the Length, that Product is the Content of the Cylinder.

Exam. The Diameter of the Baſe A C being 2 Feet and 6 Inches, and the Length B D 3 Feet and 9 Inches, to find the Content?



If the Inches are changed into the Decimal Fractions of a Foot, as in the laſt Article, then,

2.5 Feet, Diameter of the Baſe.
2.5
———
125
50
———
6.25

.7854
6.25
———
39270
15708
47124
———
4.9086510 Area of the Baſe.
3.75 Length.
———
24543255
34360557
14725953
———
18.4074375 Content.

The Reader may, for his Practice, reduce the Dimenſions to Inches, and find the ſolid Content in Inches, which being divided by 1728, the Quotient will be the ſolid Content in Feet, as in the laſt Article.

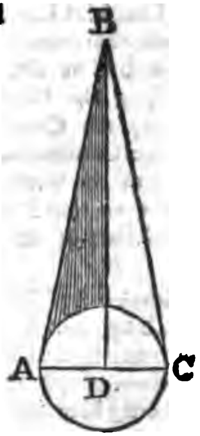
This is a general Rule for finding the Content of any ſtraight ſolid Body, of equal Bigneſs from End to End, of whatever Form the Baſes are: For, if the Area of the Baſe is multiplied by the Length, the Product is the ſolid Content. In ſuperficial Meaſure, we have given Rules for computing the Area of all Kinds of Figures; and, in Art. 60, a Table of Multipliers for all regular Figures: It will, therefore, be ſuperfluous to trouble the Learner with Examples of triangular or multangular Priſms.

Article 81. To meaſure a Cone.

Definition. A Cone is a Solid, which, from a circular Baſe, leſſens regularly 'till it ends in a Point at the Top, in the Form of a round Spire.

Rule. Having the Diameter of the Baſe, find the Area, by Art. 64; then multiplying the Area of the Baſe by one Third of the perpendicular Height, that Product is the Content of the Cone.

Exam. The Diameter A C being 20, and the Height B D 60, to find the Content?



20
20
400

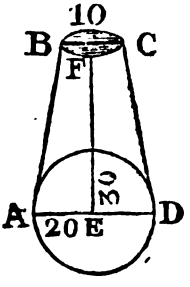
·7854
400
314.16|00 Area of the Baſe.
20 ⅓ of the Height.
6283.20 Content.

Article 82. To meaſure the Fruſtum of a Cone.

Definition. The Fruſtum of a Cone is the Part that remains when the Top is cut off by a Plane parallel to the Baſe.

Rule. Multiply the Diameters of the two Baſes together, and to this Product add one Third of the Square of the Difference of the Diameters; then multiplying this Sum by ·7854, it will be a mean Area between the two Baſes, which, being multiplied by the Length of the Fruſtum, gives the ſolid Content.

Ex. The former Cone being cut off in the Middle, the greater Diameter A D is 20, the leſſer Diameter B C 10, and the Height E F 30, to find the Content of the Fruſtum?



20
10
200
33·333
233·333
·7854
933332
1166665
1866664
1633331
183·2597382 mean Area.
30
5497·7921460 Content of the Fruſtum.

20
10
10
10
3) 100 Square of the Difference.
33·333 ⅓ of the Square.

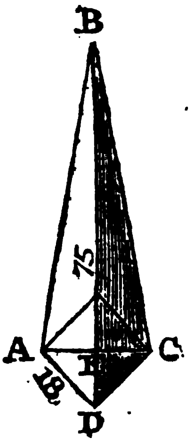
The ſolid Content of the Fruſtum is 5497.79; and, if the Cone that was cut off is meaſured, (whoſe Diameter at the Baſe B C is 10, and perpendicular Height 30, found by ſubtracting the Height of the Fruſtum from the Height of the Cone) the Content will be found 785.4, which being added to the Content of the Fruſtum is 6283.19, which is the Content of the whole Cone within leſs than one hundredth Part; and even that Difference is owing to the Defect in taking the third Part of the Square of the Difference of the Baſes.

Article 83. To meaſure a Pyramid.

Definition. A Pyramid is a Solid, which, from a triangular or other figured Baſe, leſſens regularly 'till it ends in a Point.

Rule. Find the Area of the Baſe, whether a Triangle, Square, Heptagon, or the like, by the Rules laid down in Superficial Meaſure; then multiplying the Area of the Baſe by one Third of the Height, that Product is the ſolid Content of the Pyramid.

Exam. 1. A quadrangular Pyramid, the Height BE being 75, and each Side of the Baſe 18, to find the Content?



18
18
144
18
324 Area of the Baſe,
25 ⅓ of the Height.
1620
648
8100 Content.

Exam. 2. A triangular Pyramid, the Height BE being 96, and each Side of the Baſe 25; and the Baſe of this Figure being a Triangle, let the perpendicular Height D E be 21.65; to find the Content?

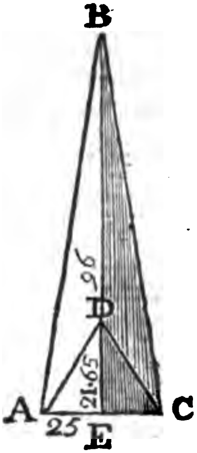
21.65 Perpendicular D E.
12.5 Half A C.

10825
4330
2165

270.625
32 $\frac{1}{3}$ of the Height.

541250
811875

8660.000 Content.

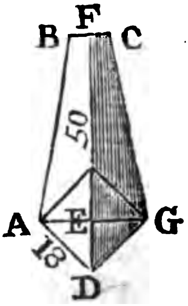


Article 84. To meaſure the Fruſtum of a Pyramid.

Definition, The Fruſtum of a Pyramid is what remains after the Top is cut off by a Plane parallel to the Baſe.

Rule. Multiply the Side of the greater Baſe by the Side of the leſſer, and to the Product add one Third of the Square of the Difference of the Sides: If it is a quadrangular Baſe, this is the mean Area between the Baſes; but, if the Baſe is any other regular Figure, take the Multiplier in the Table, Art. 60, by which multiplying this Sum, the Product is the mean Area between the Baſes: Laſtly, multiplying this by the Height, it gives the Content of the Fruſtum.

Exam. 1. The Side of the greater Baſe A D 18, as before, the Side of the leſſer B C 6, and the Height E F 50, to find the Content?



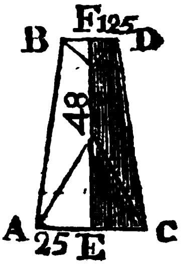
18 18 156
6 6 50

108 12 Difference. 7800 Content.
48 12

156 144 Square of the Difference.

48 $\frac{1}{3}$ of the Square.

Exam. 2. The Side of the greater Baſe A C 25, as before, the Side of the leſſer Baſe BD 12.5, and the Height EF 48, to find the Content?



12.5
25
―――
625
250
―――
312.5
52.08
―――
364.58

25
12.5
―――
12.5 Difference of
12.5 (the Sides.
―――
625
250
125
―――
3) 156.25 Square of the Difference.
―――
52.08 $\frac{1}{3}$ of the Square.

364.58
.433
―――
109374
109374
145832
―――
157.86|314 mean Area.
48 Height.
―――
126290
63145
―――
7577.40 Content.

When one Third of the Square of the Difference is added to the Rectangle of the Sides of the Baſes, multiply by .433, the tabular Multiplier for Triangles, Art. 60, which gives 157.86, the other Decimals being cut off to ſhorten the Work; only, in multiplying, remember to add what is to be carried, that is, 2 when you multiply by 8, for 8 times 3 is 24, and 1 when you multiply by 4, for 4 times 3 is 12.

If the Pyramids that were cut off are meaſured, by Art. 83, and their Content added to the Content of their reſpective

Fruſtums, they will be the ſame as the Content found in that Article, abating what may be loſt in retrenching the Decimals. The Baſes of theſe Pyramids are the ſame with the leſſer Baſes of the Fruſtums: The Height of the triangular Pyramid that was cut off is 48, and the Height of the quadrangular 25, found as at Art. 82: The Area of the firſt will be found 1082.5, and of the other 300, which the Learner may try for his Exerciſe and Improvement.

Article 85. To meaſure a Globe.

Definition. A Globe is a round ſolid Body, in the Middle of which is a Point, from which all Lines drawn to the Surface are equal.

Rule. Cube the Diameter, then multiply that Cube by .5236, and the Product is the ſolid Content of the Globe.

Exam. The Diameter of a Globe is 3 2, to find the Content?

```
        3.2
        3.2
       ----
         64
         96
      -----
      10.24
        3.2
      -----
       2048
      3072
     ------
     32.768
      .5236
    -------
     196608
     98304
    65536
   163840
  ---------
 17.1573248 Content.
```

If the Circumference of the Globe is given, cube the Circumference, and multiply the Cube by .016887 for the Content.

Article 86. To meaſure the Surface of a Globe.

Rule. Multiply the Circumference and Diameter together, and the Product is the ſuperficial Content, or Area, of the Surface of the Globe.

Exam. The Diameter of a Globe being 3.2, to find its Area?

3.1416
3.2
62832
94248
10.05312 Circumference, (by Art. 61.

10.053
3.2
20106
30159
32.1696 Area of the (Surface.

Article 87. To meaſure the Solidity of a Fruſtum or Segment of a Globe.

Definition. The Fruſtum of a Globe is any Part cut off by a Plane.

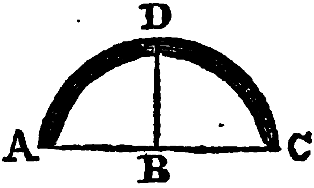
Rule. To three times the Square of the Semidiameter of the Baſe add the Square of the Height; then multiplying that Sum by the Height, and the Product by .5236, it gives the ſolid Content.

Exam. The Height BD being 12 Inches, and the Diameter of the Baſe AC 26 Inches, to find the Content?

13 Semidiameter.
13
39
13
169
3
507
144
651
12
7812

12 Height.
12
144

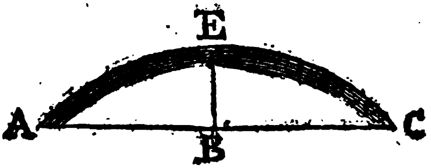
7812
.5236
46872
23436
15624
39060
4090.3632 ſolid Content.



Article 88. To meaſure the Surface of a Fruſtum or Segment of a Globe.

Rule. Find the Diameter of the Globe, by Art. 73, and the Superficies of the whole Globe, by Art. 86; then ſay, as the Diameter of the Globe is to the Height of the Fruſtum, ſo is the Superficies of the Globe to the Superficies of the curve Part of the Fruſtum; then find the Area of the Baſe, by Art. 64, and add theſe two together, which will be the whole Surface of the Fruſtum.

Exam. The Diameter A C being 72, and the Height B E 13, to find the Superficies?



36 Half the Diameter
36
216
108
13) 1296 (99 Reſidue of the Diameter.
117 13
126 112 Diameter of the Globe.
117
9

As 112 is to 13, ſo is 39408
13
118224
39408
112) 512304 (4574
448
643
560
830
784
464
448
16

3.1416
112
62832
31416
31416
351.8592 Circumference.
112
703718
351859
351859
39408.208 Area of the Surface of the Globe.

5184 Square of the Diameter of the Baſe.

.7854

20736

25920

41472

36288

4071.5136 Area of the Baſe of the Fruſtum.

4574 Area of the curve Surface add.

8645.5 Area of the whole Segment.

The Operation explained: Squaring the Half of the Diameter of the Baſe, and dividing by the Height, or verſed Sine B E, the Quotient is the Reſidue of the Diameter of the Globe; to which adding the verſed Sine 13, it gives 112 for the whole Diameter: Then, by Art. 61, finding the Circumference, and multiplying that by the Diameter, the Product 39408 (omitting the fractional Part) is the Area of the Surface of the whole Globe: Then, by the Rule of Three, finding the Area of the Segment's curve Surface 4574, we next compute the Area of the Baſe, by Art. 63; and, adding theſe Sums together, we have 8645.5 for the Area of the Surface of the whole Segment or Fruſtum.

As to the meaſuring the Surfaces of the other Solids, I eſteem it uſeleſs, ſince the Rules laid down in ſuperficial Meaſure are ſufficient to find the Superficies of any of the foregoing Bodies: Unleſs it is neceſſary to inform the Learner, that the curve Surface of a Cone is a Sector, whoſe Arch-Line is equal to the Cone's Baſe, and the Radii equal to the Side of the Cone; ſo that, if Half the Circumference of the Cone's Baſe is multiplied by the Length of its Side, the Product is the Area of the curve Surface; to which if the Area of the Baſe is added, the Sum is the Area of the whole Surface of the Cone.

Article 89. To meaſure a Spheroid.

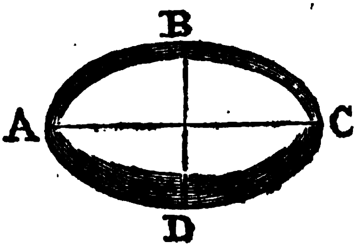
Definition. A Spheroid is a ſolid Body like an Egg, only both its Ends are the ſame.

Rule. Multiply the Square of the Diameter of the greateſt Circle, that is, the Diameter of the Middle, by the Length; then multiplying that Product by .5236, this Product is the ſolid Content.

Exam. The Diameter AC being 24, and the Length BD 30, to find the Content?

24
24
96
48
576
30
17280

17280
.5236
103680
51840
34560
86400
9047.8080 ſolid Content.



Art. 90. To find the Side of a Cube equal to any ſolid Body.

Rule. The ſolid Content of any Body being given, extract the Cube Root of the given Solidity, which will be the Side of a Cube equal to the Body whoſe ſolid Content was ſo given.

Exam. 1. Find the Side of a Cube equal to a Globe whoſe Content is 16974.593.

16974.593 (25.7 Side of (the Cube.
8
8974
1200
60
1260
25
300
1200
1525
5
7625
1349593
187500
750
188250
49
5250
187500
192799
7
1349593

Exam. 2. Find the Side of a Cube equal to a Spheroid whoſe Content is 9047.808.

9047.808 (20.8 Side of the (Cube.
8
1047
1200
60
1260
1047808
120000
600
120600
64
4800
120000
124864
8
998912
48896 Remainder.

If the Dimenſions of any ſolid Body were given, and it was required to find the Side of a Cube that is equal to that ſolid Body, find the Content of the given Body, by the Rules already laid down, and extract the Cube Root of that Number, which will be the Side of the Cube required.

Article 91. To meaſure the Solidity of any irregular Body, whoſe Dimenſions cannot be taken.

Get any regular Veſſel, either ſquare or round, and into it put the Body to be meaſured; then pour Water into the Veſſel ſo as to cover the Body, and meaſure the dry Part from the Top of the Veſſel to the Surface of the Water; then take out the Body, and meaſure again from the Top of the Veſſel to the Water, and ſubtract the firſt Meaſure from the ſecond, which is the Fall of the Water: Then, if the Veſſel is ſquare, meaſure one Side and multiply it by itſelf, and that Product by the Fall of the Water, which laſt Product is the Content of the Body: But, if it is a long Square, as a Ciſtern, multiply the Length by the Breadth, and that Product by the Fall of the Water: Or, laſtly, if it is a round Veſſel, ſquare the Diameter, and multiply the Square by .7854, and that Product by the Fall of the Water, which gives the ſolid Content of the Body.

Exam. 1. A Body being put in a Veſſel 18 Inches ſquare, the Water ſunk 6 Inches. What was the Content of the Body?

$$\begin{array}{r l} 1.5 & \text{Foot.} \\ 1.5 & \\ \hline 75 & \\ 15 & \\ \hline 2.25 & \text{(Inches.} \\ .5 & \text{Decimal of 6} \\ \hline 1.125 & \text{Content.} \end{array}$$

Exam. 2. A Body being put in a Ciſtern 2 Feet wide, and 4 Feet long, the Water ſunk 9 Inches. What was the Content of the Body?

$$\begin{array}{r l} 4 & \\ 2 & \\ \hline 8 & \text{(Inches.} \\ .75 & \text{Decimal of 9} \\ \hline 40 & \\ 56 & \\ \hline 6.00 & \text{Content.} \end{array}$$

Exam. 1. Eighteen Inches is 1.5 Foot; which being ſquared is 2.25 Feet, and that being multiplied by .5, the Decimal of 6 Inches, gives 1.125 Foot; which is 1 Foot and an eighth Part of a Foot.

Of Board and Timber Meaſure.

The Meaſuring of Boards is ſuperficial Meaſure, the Thickneſs of a Board not being taken into the Account in meaſuring; ſo that a Board is a Parallelogram, the Meaſuring of which is taught Art. 53. In regard to how much in Length is required to make a Foot to any aſſigned Breadth, there is a Table on the Joint-Rules, which every Carpenter is acquainted with; and when any Breadth is given, to find how much in Length makes a Foot, divide 144, the Square Inches in a Foot, by the given Breadth, and the Quotient is the Number of Inches required to make a Foot.

Exam. 1. A Board being 18 Inches broad, how many Inches in Length go to a Foot?

18) 144 (8
 144

Anſwer 8 Inches.

Exam. 2. A Board being 6 Inches broad, how many Inches make a Foot?

6) 144 (24
 24

Anſ. 24 Inches, or 2 Feet.

Of Timber there are three different Sorts: If it is ſquare, and of the ſame Bigneſs all the Way, it is a Parallelopipedon, the Meaſuring of which is taught Art. 79; if it is bigger at one End than the other, it is a Fruſtum of a ſquare Pyramid, the Meaſuring of which is taught Art. 84; and, if it be round Timber, it is the Fruſtum of a Cone, the Meaſuring of which is ſhewn Art. 82.

But there is a cuſtomary Way of meaſuring both round and ſquare Timber, which, tho' it be erroneous, is uſed by all Dealers: It will therefore be uſeleſs to repeat what has been ſo often proved, of the Errors of thoſe Rules. In round Timber, they uſually girt the Piece in the Middle with a Cord, which they divide into four Parts; this fourth Part of the Circumference they call the Side of a Square equal to the Circumference; this fourth Part ſquar'd, and the Square multiply'd by the Length, they call the ſolid Content: In which there are two Errors; for the Bigneſs in the Middle is not a Mean between the greater and leſſer Baſes, neither is the fourth Part of the String equal to the Side of a Square, of equal Area with the Circumference: And as to the Timber, after it is ſquar'd, if the Sides are unequal, they add the two Sides together, and take the Half of that for a mean Side; which is another Error, tho' not of ſo much Conſequence as thoſe relating to round Timber: But, by Means of theſe ſucceſſive Errors, a Piece of Timber, when ſquar'd, contains more Feet than when it was round. It will therefore be proper to work an Example each Way, that the Difference may be ſeen.

Exam. 1. A Piece of round Timber being 15 Feet long, and Girt in the Middle 42 Inches, how much Timber is contain'd in it?

4) 42.0 (10.5 one Fourth of the Girt.

.875 Decimal of 10 $\frac{1}{2}$ (Inches. Girt 42 Inches, or 3.5 Feet.

.875
.875
4375
6125
7000
.765625
15
3728125
765625
11.384375 Content.

3.5
3.5
175
105
1225
.07958
9800
6125
11025
8575
.9748550
15
5274275
974855
15.022825 Content.

According to the common Way of Meaſuring, this Piece contains 11 Feet and about a Quarter and Half-Quarter; but by ſquaring the Circumference, and multiplying the Square by .07958, as taught at Art. 65, and multiplying that Product by the Length, which is the true Method, we have 15 Feet, which is a conſiderable Difference; and this Difference would be increaſed, if a mean Baſe were to be taken, which the Circumference in the Middle is not.

Exam. 2. A Piece of ſquared Timber being 9 Inches wide, 10.5 Inches deep, and 16 Feet long, how much Timber is contain'd in it?

.875 Decimal of 10 ½ Inches.
.75 Decimal of 9 Inches.

2) .1625 (.8125 mean Side.

.8125
16

48750
8125

12.9900 Content.

.875 Decimal of 10 ½ Inches.
.75 Decimal of 9 Inches.

4375
6125

.65625

.655250(.809 mean Side.
64

1609) 15250
14481

769 Remainder.

.809
16

4854
809

12.944 Content.

In this Example the Difference is but little, as I obſerv'd before, being not one Tenth of a Foot.

To find how much in Length is requir'd to make a Foot of Timber to any given Square, there is on the two-foot Joint-Rules a Table of Timber-Meaſure, on the contrary End to that of Board-Meaſure; the Uſe of which is known to every Carpenter: And to find it to any given Square, not in the Table, divide 1728, the cubic Inches in a Foot, by the given Inches ſquare of the Piece, and the Quotient is the Length requir'd to make a Foot.

Exam. 1. A Piece of ſquare Timber, whoſe Side is 8.5 Inches, how much in Length will make a Foot?

8.5
8.5

425
680

72.25)1728.00(23.9
14450

28300
21675

66250
65025

1225 Remainder.

Anſwer 1 Foot, 11.9 Inches.

Exam. 2. A Piece of Timber whoſe Side is 16.5 Inches, how much in Length will make a Foot?

16.5
16.5

825
990
165

272.25) 1728.00 (6.38
163350

94500
71675

228250
217800

10450 Rem.

Anſwer 6.38 Inches.

Of the Five Regular Bodies.

There are five Solids contain'd under equal, regular Sides, which, by way of Diſtinction, are called *The five regular Bodies.* Theſe are the *Tetraedron*, the *Hexaedron* or *Cube*, the *Octaedron*, the *Dodecaedron* and the *Eicoſiedron*. The Meaſuring of the Cube was ſhewn at Art. 78. I ſhall now ſhew how to Meaſure the other four.

Of the Tetraedron.

The *Tetraedron* is a Solid contain'd under four equal and equilateral Triangles; that is, it is a triangular Pyramid of four equal Faces, the Side of whoſe Baſe is equal to the ſlant Height of the Pyramid, from the Angles to the Vertex.

The firſt Thing to be done is, to find the perpendicular Height of the Triangles; which is done thus; from the Square of the given Side ſubtract the Square of Half the Side, and extract the Square Root of the Remainder, and this will be the perpendicular Height of the Triangle.

The Side of a Tetraedron being 12, to find the ſolid and ſuperficial Content.

```
Side  12        Half the Side  6        108.0000 ( 10.39
      12                       6        1
     ----                     ---      ------
     144                      36       203) 800
      36                                    609
     ----                                  -----
     108                              2069) 19100
                                             18621
                                             -----
                                               479
```

If four more Cyphers were annex'd, the Root would be 10.3923 for the perpendicular Height of the Triangle; therefore, by Art. 56, Half the Side multiplied into the perpendicular Height gives the Area of the Baſe.

```
10.3923
      6 Half the Side.
-------
62.3538 Area of the Baſe.
```

To find the ſolid Content: At Art. 83, we have ſhewn the Way to find the Solidity of a Pyramid, by multiplying the Area of the Baſe by one Third of the Height, or, which is the ſame, one Third of the Area by the Height of the Pyramid: Now, to find the perpendicular Height of the Tetraedron, ſubtract the Square of Half the given Side from the Square of the Triangle's Height before found, and divide the Remainder by the Double of the Triangle's Height, and the Quotient will be the Diſtance of the Center of the Baſe from the Sides; and this Quotient, ſubtracted from the Height of the Triangle, gives the Diſtance of the Center from the Angles.

```
          108 The Square of the Triangle's Height.
           36 The Square of Half the Side.
          ---
20.7846) 72.0000 ( 3.464 Diſtance of the Center from
         623538                          (the Sides,
         ------
          964620
          831384
          -------
          1332360        From     10.3923
          1247076        Subtract  3.464
          -------                 -------
            852840                 6.9283 Diſt. from the
            831384                             (Angles.
            ------
             21456
```

If the Square of the Diſtance of the Center of the Baſe from the Sides be ſubtracted from the Square of the Triangle's Height, the Square Root of the Remainder is the perpendicular Height of the Pyramid.

```
      3.464
      3.464
     ------
     13856
    20784
   13856
  10392
  ---------
  11.999296
```

```
From 108 Square of the Triangle's Height.
Subt.  12 Square of the Diſtance of the Center from the Sides.
      ---
      96.0000 (9.79
      81
     ----
187) 1500
     1309
     ----
1949) 19100
      17541
      -----
       1559
```

As we took 12 for the Square of the Diſtance of the Center, inſtead of 11.999296, we may call the Height of the Tetradron 9.8, it being near the Truth: This multiply'd into 20.7846, the Third of the Area of the Baſe, gives the Solidity of the Tetraedron.

```
      20.7846
          9.8
     --------
     1662768
   1870614
   ---------
   203.68908 Solidity.
```

For the Superficies, the Area of the Baſe multiply'd by 4, as the Faces are all equal, gives the Surface of the Tetraedron.

If the Square of the Diſtance of the Center from the Angles had been ſubtracted from the Square of 12, the given

Side, the Square Root of the Remainder would be the Height of the Pyramid, as before.

Square of 12 is 144
Square of 6.9283 is 48

96 Remainders; whoſe Root being extracted gives the perpendicular Height of the Pyramid the ſame as before.

Of the Octaedron.

The *Octaedron* is a Solid contained under eight equal and equilateral Triangles, which may be conceiv'd to conſiſt of two quadrangular Pyramids, of equal Baſes, joined together: Therefore, to meaſure the Solidity of this Body, it will be ſufficient to conſider it as two ſquare Pyramids, the Sides of whoſe Baſes are equal to the given Side of the Triangles, under which it is contained.

The Side of an Octaedron being 12, to find the ſolid and ſuperficial Content.

The Sides of the Baſes being 12, their Area will be 144, by Art. 52. If the Square of Half the given Side be ſubtracted from the Square of the perpendicular Height of the Triangle, the Square Root of the Remainder is the Height of each Pyramid.

The Side of the Triangle being the ſame here as in the Tetraedron, the Square of the Perpendicular of the Triangle will be 108, as before; from which ſubtracting 36, the Square of Half the given Side, the Remainder is 72; whoſe ſquare Root multiply'd by 48, the third Part of the Area of the Baſe, gives the ſolid Content of one Pyramid; which doubled gives the Solidity of the Octaedron.

72.0000 (8.485 Height of each (Pyramid.
64

164 (800
656

1688) 14400
13504

16965) 89600
84825

4775

8.485
48

67880
33940

407.28
2

814.56 Solidity.

The Area of the Baſe being the ſame here as in the Tetraedron, the Superficies will be the Double of the Superficies of that, as here are Eight Triangles of the ſame Dimenſions.

Of the Dodecaedron.

The *Dodecaedron* is a Solid contain'd under twelve equilateral Pentagons, and may be conceiv'd to conſiſt of twelve pentagonal Pyramids, of equal Baſes and Altitude, whoſe Vertexes meet in the Center of the Dodecaedron: Therefore, if we find the ſolid Content of one of thoſe Pyramids, that multiplied by 12 gives the Solidity of the Dodecaedron.

The Side of the Dodecaedron being 12, to find the ſolid and ſuperficial Content.

To find the Solidity of a Pyramid, we muſt firſt find the perpendicular Height: Now, the Side of a Dodecaedron being given, the ſlant Height of each Pyramid of which it conſiſts, meaſur'd from the Angles to the Vertex, is to the given Side of the Pentagon, as 1 is to .7136441; and therefore this will be to that, as .7136 to 1, nearly.

As .7136 is to 1, ſo is 12.
12

.7136) 12.0000 (16.816 ſlant Height,
7136

48640
42816

58240
57088

11520
7136

43840
42816

1024

Next find the Diſtance of the Center of the pentagonal Baſes from the Angles: In all regular Pentagons, as 117556 is to 100000, ſo is the Side of the Pentagon to the Diſtance of the Center from the Angles.

As 117556 is to 100000, ſo is 12.

```
                        12
                     -------
117556) 1200000 ( 10.2079
        117556
        -------
          244400
          235112
          -------
            928800
            822892
            -------
            1059080
            1058004
            -------
               1076
```

If the Square of the Diſtance from the Center be ſubtracted from the Square of the ſlant Height, the Square Root of the Remainder is the perpendicular Height of the Pyramid.

```
      16.816                          10.2079
      16.816                          10.2079
     --------                        ---------
      100896                           918711
     16816                            714553
    134528                          2041580
   100896                          1020790
  16816                           ------------
  ----------                      104.20122241
  282.777856
  104.201222
  ----------
  178.576634 ( 13.36  perpendicular Height.
  1
  ---
23) 78
    69
   ----
263) 957
     789
    -----
2666) 16866
      15996
     ------
        870
```

The Square of 12, the Side of the Pentagon, multiplied into the tabular Number for Pentagons, in the Table of Art. 60, will give the Area of the Baſe.

12	1.720477
12	144
144	6881908
	6881908
	1720477

247.748688 Area of the Pentagon.
4.45 one Third of the Height.

1238743440
990994752
990994752

1102.4816616 Solidity of one Pyramid.
12

13229.7799392 Solidity of the Dodecaedron.

If the laſt Extraction had been farther continued, the perpendicular Height would be 13.3632, whoſe third Part would be 4.4544; in which Caſe the Solidity would be 13241, and ſomewhat more.

For the Superficies, the Area of the Pentagon multiplied by 12 gives the Surface of the whole Dodecaedron.

247.748688
12

2972.984256 Superficies.

Of the Eicoſiedron.

The *Eicoſiedron* is a Solid contained under twenty equal and equilateral Triangles, and may be conceived to conſiſt of twenty equal triangular Pyramids, whoſe Vertexes all meet in the Center: Hence, if we find the ſolid Content of one of thoſe Pyramids, that being multiplied by 20 gives the Content of the Eicoſiedron.

The Side of an Eicoſiedron being 12, to find the ſolid and ſuperficial Content: We muſt firſt find the Diſtance of the Center of the Baſes from the Angles, and then the ſlant Height the Pyramids; from which the perpendicular Height may be found, by ſubtracting the Square of the Diſtance of the Center from the Square of the ſlant Height, and extracting the Square Root of the Remainder.

The Diſtance of the Center of the Triangle from the Angles we have found in the Tetraedron (the Triangles being here of the ſame Dimenſions) to be 6.9283; and for the ſlant Height of the triangular Pyramids of the Eicoſiedron, this is the Proportion, as 1.05146 is to one, ſo is the given Side of the Eicoſiedron to the ſlant Height of the Pyramids, of which it conſiſts.

As 1.05146 is to 1, ſo is 12.

```
                 12
                -----
1.05146) 12.00000 ( 11.4127 ſlant Height.
         105146
         ------
          148540
          105146
          ------
           433940
           420584
           ------
            133560
            105146
            ------
             284140
             210292
             ------
              738480
              736022
              ------
                2458
```

The Square of the Diſtance of the Center from the Angles being ſubtracted from the Square of the ſlant Height, the Square Root of the Remainder is the Height of the Pyramid.

```
Square of 11.427 is 130.2497
Square of 6.9283 is  48
                    --------
                    82.2497 ( 9.069 perpendicular Height.
                    81
                    -----
             1806) 12497
                   10836
                   ------
            18129) 166100
                   163161
                   ------
                     2939
```

The Area of the Baſe is the ſame here as in the Tetraedron, which multiplied by 3.023, the Third of the Height, gives the ſolid Content of one Pyramid; and this multiplied by 20, the Number of Pyramids, gives the Solidity of the *Eicoſiedron*.

62.352
3.023

187056
124704
1870560

188.49|0096
20

3769.86 Solidity.

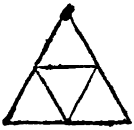
Then the Area of the Baſe, multiplied by 20, gives the ſuperficial Content.

62.352
20

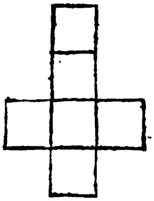
1247.040 Superficies.

I have omitted the Figures of theſe Bodies, as ſome of them would give but a confuſed Idea to ſuch as are entirely unacquainted with the Bodies: But the following Figures, cut out in Paſteboard, and the Lines cut half through, will fold up into the ſeveral Bodies.

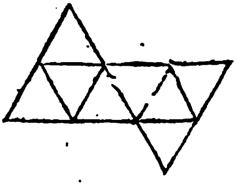
Tetraedron.



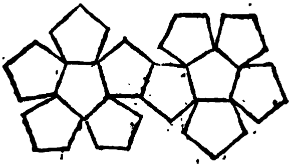
Hexaedron, or Cube.



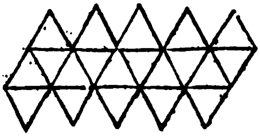
Octaedron.



Dodecaedron.



Eicoſiedron.



A TABLE *of the ſolid and ſuperficial Content of each of the Five Bodies, the Sides being* 1, *or Unity.*

Names of the Bodies.	Solidity.	Superficies.
Tetraedron.	0.11785	1.73205
Octaedron.	0.4714	3.464
Hexaedron.	1.0000	6.000
Eicoſiedron.	2.18169	8.66025
Dodecaedron.	7.663119	20.6457

All like ſolid Bodies being in Proportion to one another as the Cubes of their like Sides, and their Superficies as the Squares of their like Sides; the ſolid Content of any of theſe Bodies may be found, by multiplying the Cubes of their Sides by the Numbers in the ſecond Column under *Solidity*, and their Superficies, by multiplying the Squares of their Sides into the Numbers in the third Column under *Superficies.*

The Deſcription and Uſe of the Sliding-Rule.

Soon after the Invention of Logarithms, Mr. *Edmund Gunter* contrived the laying them on a Ruler or Scale, which from him was called *Gunter's Scale*; on which, with the Help of a Pair of Compaſſes, any Queſtion relating to Proportion may be ſpeedily wrought: Since that Mr. *Oughtred* contrived to make two Scales ſlide one by the other, which performs the Work without Compaſſes, and is called the *Sliding-Rule*: Of theſe there are ſeveral Sorts; but, as that uſually called *Coggeſhall's* is the moſt common and beſt known, I ſhall give the Deſcription and Uſe of that.

This is uſually put on one of the Sides of a Carpenter's Joint-Rule, by a Piece made to ſlide in a Groove: This ſliding Piece, as alſo the Line above it, is divided and numbered 1, 2, 3, *&c.* to 9, and again from 1 in the Middle, to 10 at the End: Theſe two are called the Lines of Numbers; each of thoſe Diviſions betwixt the Figures is divided into ten Parts; and thoſe Diviſions, from 1 at the Beginning and Middle to the 2, are divided into five Parts, and, from each 2, to each 3, into Halves; the Diviſions being afterwards too ſmall to be divided. The Sliding-Piece has two of thoſe Lines on it; one on the upper Edge, to anſwer that on the Rule; another on the under Edge, to anſwer the Girt-Line on the Rule, below the Slider: Theſe four Lines are marked A, B, C, D.

The Girt-Line is a broken Line of Numbers, contrived for meaſuring Timber at one Operation: It is numbered from 4 at the Beginning, to 5, 6, 7, 8, 9, 10, 20, 30, 40, at the End; each Diviſion, from the Beginning to 8, divided into ten, and thoſe again halved; but, from 8 to 9, and from 9 to 10, are only divided into ten Parts; then, from 10 to the End, each of the numbered Diviſions is divided, firſt into ten Parts, and each of thoſe Tenths into four ſmaller Diviſions.

To find a given Number on the Line of Numbers.

In the firſt Part of the Line, to the left Hand, if the Numbers 1, 2, 3, *&c.* be accounted as Units, then the larger Diviſions are, each, tenth Parts of an Unit, and the ſmaller Diviſions are Parts of thoſe Tenths: The 1 in the Middle of the Line ſtands for 10, and the other Figures 2, 3, *&c.* to the End, ſtand for 20, 30, 40, *&c.* to the 10 at the End, which ſtands for 100. In this Caſe, therefore, all Numbers under 10 are to be look'd for in the firſt Part of the Line, and Numbers from 10 to 100 in the laſt Half: Thus, if it be required to find 12 on the Line, the 1 in the Middle ſtanding for 10, count from thence two of the larger Diviſions, and you'll find this mark'd 12: If 37 were requir'd, from 3 on the laſt Part of the Line count ſeven of the Diviſions; and ſo of any other Number under 100. If it were requir'd to find 2.48, from 2 in the firſt Part of the Line, count four of the larger Diviſions, this is 2.4; then, as each of theſe Diviſions are here divided into Half, ſomewhat more than half Way betwixt this ſmaller Diviſion and the Fifth of the larger, is the Place of of .8 of the larger Diviſions, which is .08 of the number'd Diviſions, and is the Place of 2.48, the given Number. The ſame Way may other decimal Parts be found, either in the firſt or laſt Part of the Line.

Of the Uſe of the Line of Numbers.

To multiply one Number by another, ſet 1 on the Line B to the Number given for the Multiplier in the Line A; then will the Number given for the Multiplicand in the Line B ſtand againſt the Product in the Line A. Example: To multiply 8 by 7, ſet 1 in the Line B againſt 7 in the Line A; then will 8 in the Line B ſtand againſt 56, the Product, in the Line A; and while, the Rule is in this Poſition, you have the Products of all the Numbers on the Line B, from 1, to ſo far as the Line A reaches, multiplied by 7, and the Products in the Line A: Thus 2, 3, 4, 5, &c. on the Line B, ſtand againſt 14, 21, 28, 35, &c. in the Line A. The ſame is the Caſe of any mixed Number found on the Line B; thus 1.6 ſtands againſt 11.2 in the Line A.

To divide one Number by another, ſet the Diviſor on the Line B againſt 1 in the Line A; then againſt the Dividend in the Line B, you have the Quotient in the Line A. Example: To divide 96 by 6, ſet 1 in the Line A againſt 6 on the Line B; then will 96 on the Line B ſtand againſt 16, the Quotient, in the Line A; and, while the Rule is in this Poſition, you have the Quotient on the Line A of all the Numbers on the Line B, divided by 6.

For the Meaſuring of Timber, the Girt-Line mark'd D is particularly deſign'd; for the Uſe of which take the following Examples.

Example 1. A Piece of Timber 10 ½ Inches ſquare, 44 Feet long, to find the Content.

Set the 12 on the Line D to 44 on the Line C; then will 10 ½ on the Line D ſtand againſt 33.68 Feet, the Content; which is 33 Feet, 8 Inches.

Example 2. A Piece of Timber 22 Inches ſquare, 34 Feet long, to find the Content in Feet.

Set 12 on the Girt-Line D to 34, the Length of the Piece of Timber in Feet, on the Line C; then will 22 on the Line D, ſtand againſt the Content in Feet on the Line C: But here it is to be obſerved, that 22 on the Line D, will reach beyond the Line C, the Content being more than 100 Feet: Therefore, in this Caſe, the 12 in the Girt-Line D muſt be ſet to 34 in the former Part of the Line C; accounting the Figures as Tens, the ſame as in the latter Part; then the 1 in the Middle is 100, the 2, 3, 4, &c. following, 200, 300, 400, &c. to the 10 at the End, which is now 1000. Hence the larger Diviſions, which before were Units, are in this Caſe Tens, and the ſmaller Diviſions increaſe their Value in the ſame Proportion.

Having ſet the 12 in the Line D againſt 34 in the firſt Part of the Line C, then will 22, the Inches ſquare in the Line D, ſtand againſt 114, and ſomewhat more; that is, it will ſtand a little beyond the ſecond ſmall Diviſion, betwixt the firſt and ſecond of the large Diviſions, anſwering to 114.277 Feet, or 114 Feet, 3 Inches.

By the Method, uſed in this Example, of making the Numbers in the firſt Part of the Line ſtand for Tens (or Hundreds, if neceſſary) the Uſe of the Sliding-Rule may be extended to larger Numbers; always remembering, that all the Numbers on the laſt Part of the Scale will be of the ſame Denomination with the 1 in the Middle; and, though in large Numbers we cannot come to the Degree of Exactneſs neceſſary in ſome Things, yet it may be of great Uſe to correct a Miſtake.

I ſhall now add ſome Queſtions, to exerciſe the Learner, in ſome of the foregoing Rules.

Queſtion 1. Seven Men bought a Grinding-Stone of 5 Feet (or 60 Inches) Diameter: How much of the Stone's Diameter muſt each grind, to have an equal Share of the Stone, if one firſt grind his Share, and then another his, till the Stone is ground away?

In order to ſolve this Queſtion, ſquare the Diameter, and divide the Square by 7, the Number of Perſons; ſubtract the Quotient from the Square of the Diameter, and extract the Square Root of the Remainder, which is the Length of the Diameter, after the firſt Man has ground his Share; this Work repeated, by ſubtracting the ſame Quotient, or ſeventh Part, from the Remainder, for every one, to the laſt, extract the Square Root of the Remainders, and ſubtract thoſe Roots from the Diameter, one after another, and the ſeveral Remainders are Anſwers.

60
60
7) 3600(514.2857
35
10
7
30
28
20
14
60
56
40
35
50
49
1

3600
514.2857
3085.7143
514.2857
2571.4286
514.2857
2057.1429
514.2857
1542.8572
514.2857
1028.5715
514.2858

3085.7143)55.549
15
105(585
525
1105(6071
5525
11104(54643
44416
111089(1022700
999801
22899

The Square Root of
3085.7143 is 55.549.
2571.4286 is 50.709.
2057.1429 is 45.355.
1542.8572 is 39.279.
1028.5715 is 32.071.
514.2857 is 22.678.

60
55.549
4.451 1ſt Share.
55.549
50.709
4.84 2d Share.
50.709
45.355
5.354 3d Share.
45.355
39.279
0.076 4th Share.
39.279
32.071
7.208 5th Share.
32.071
22.678
9.393 6th Share.

Having found the Roots of the ſix Remainders, ſubtract the Root of the firſt Remainder from the Diameter, and the Difference is what the firſt Man is to grind down; then ſubtracting the Root of the ſecond Remainder from the Root of the firſt, the Difference is what the ſecond Man is to grind down; ſubtracting the Root of the third Remainder from the Root of the ſecond Remainder, the Difference is what the third Man is to grind down; proceeding in this Manner, as in the Work, find what the fourth, fifth, and ſixth Man is to grind down, and the Square Root of the Quotient, which ariſes from dividing the Square of the Diameter by 7, is the Share of the ſeventh Man. The Shares added together are to make the Diameter of the Stone.

4.451
4.84
5.354
6.076
7.208
9.393
22.678

60.000

I have omitted the Operations of extracting the Square Root of all the Numbers but the firſt, partly becauſe it would take up too much Room, and partly for the Learner to exerciſe himſelf. The laſt is 22.6778; for which Reaſon I put 22.678, as that is not ſo much more, as a 7 in the third Place would have been leſs, than the Truth.

Queſt. 2. By the Law of *England*, a Statute Buſhel for meaſuring Corn is to be made 8 Inches high, and 18 Inches and an Half Diameter, to contain 2176 cubic Inches, tho' the Content of the Dimenſions is but 2150.425 Inches: I demand what the Diameter of the Buſhel muſt be, the Height being 8 Inches, and what the Height, the Diameter being 18.5 Inches, to contain 2176 cubic Inches?

For the firſt, divide the given Content 2176 by 8, the given Height, and the Quotient is the Area of a Circle, whoſe Diameter, found by Art. 67, will be the Diameter of the Buſhel: But, when the Diameter is 18.5, to find the Height, by Art. 61, find the Area of the Circle, then divide 2176 by the Area ſo found, and the Quotient is the Height ſought.

8) 2176

8) 2176
272 Quotient.

1.273
272
2546
8911
2546
346.2560 (18.6 Diameter.
1
28) 246
224
366) 2225
2196
29

18.5
18.5
925
1480
185
342.25
.7854
136900
171125
273800
239575
268.803150

268.8) 2176.0 (8.0956 Height.
21504
25600
24192
15080
13440
16400
16128
272

Queſtion 3. If a Man hire an Acre of Graſs for his Horſe, how long muſt the Chord be tied to his Head, ſo that he might not graſe more than an Acre?

A Piece of Ground forty Poles or Perches in Length, and four Poles or Perches in Breadth, is an Acre. A Pole is 16.5 Feet. Bring the forty Poles and four Poles into Feet, multiply theſe by one another, the Product is the Area of an Acre of Land in ſquare Feet.

16.5

16.5
40
660.0

16.5
4
66.0

660
66
3960
3960
43560 ſquare Feet in an (Acre.

By Art. 67, find the Diameter of a Circle, whoſe Area is 43560 Feet, and Half the Diameter is the Length of the Tether.

43560
1.273
130680
304920
87120
43560
55451.880 (235.48
4
43) 154
129
465) 2551
2325
4704) 22688
18816
47088) 387200
376704
10496 Remainder.

117.74 Feet, the Anſwer.

Queſtion 4. A ſquare Pyramid of Marble, each Side of its Baſe ſix Inches, its Height four times the Side of the Baſe, twenty-four Inches, weighs twenty-nine Pounds: What will be the Weight of a Pyramid of the ſame Marble, whoſe Baſe is thirty Inches, and Height an hundred and twenty Inches?

The Height of each of theſe Pyramids being four times their reſpective Baſes, the Contents will be as the Cube of any one of their Dimenſions; and, as the Matter is the ſame, the

Weight will be as their Content: Therefore, as the Cube of 6, which is 216, is to the Weight of its Pyramid, ſo is the Cube of 30, which is 27000, to the Weight of its Pyramid.

As 216 to 29, ſo is 27000

```
                  27000
                     29
                 ------
                 243000
                 54000
                 ------
           216) 783000 (3625 Pounds, the Anſwer.
                648
                ----
                 1350
                 1296
                 ----
                   540
                   432
                   ---
                   1080
                   1080
```

If the *Heights* of the Pyramids had been cubed, the Anſwer had been the ſame: But, if their Heights had not been the ſame Number of Times as their Baſes, then their Weights would have been as their ſolid Contents, the Matter being ſuppoſed the ſame.

Queſt. 5. What will be the Diameter of a Globe, whoſe ſolid and ſuperficial Content are the ſame?

In Art. 85, the Rule for finding the ſolid Content of a Globe is, to multiply the Cube of the Diameter by .5236; and, in Art. 86, the Rule for finding the ſuperficial Content is, to multiply the Diameter by the Circumference: But, the Circumference being found by multiplying the Diameter by 3.1416, and this Product being again multiplied by the Diameter, to find the ſuperficial Content, hence the ſuperficial Content of the Globe is found by multiplying 3.1416 into the Square of the Diameter.

Now, as the Cube of the Diameter of the Globe, multiplied by .5236, is to be equal to the Square of the ſame Diameter, multiplied by 3.1416, hence it follows, that the Square of the Diameter of the Globe here propoſed muſt be to its Cube, as .5236 is to 3.1416: But the Proportion of .5236 to 3.1416 may be found by dividing the latter by the former, and the Quotient will be the Diameter of the Globe ſought.

.5236) 3.1416 (6 The Diameter of a Globe, whoſe ſuperficial
31416 and ſolid Content will be equal, as will appear by the Work.

3.1416	6	.5236
6	6	216
18.8496 Circumference.	36	31416
6	6	5236
		10472
113.0976 ſuperficial Content.	216 Cube.	113.0976 ſolid Content.

Of Meaſuring the Work of the ſeveral Artificers belonging to Building; as Carpenters, Bricklayers, Joyners, Plaiſterers, Painters, Glaziers, Maſons, and Paviours.

Of CARPENTERS' *Work.*

THE Carppenters' Work is uſually meaſured by the Square of 100 Feet; that is, ten Feet in Length, and ten in Breadth, make a Square, either of Flooring, Partitioning, or Roofing; for meaſuring which, take this general *Rule*. Multiply the Length by the Breadth, and divide the Feet by 100; the Quotient is the Number of Squares: The Remainder, or Figures cut off, are Feet.

Exam. 1. A Floor being 47 Feet and 3 Inches long, 34 Feet and 9 Inches wide, how many Squares of Flooring does it contain?

F. I. P. (47 F. 3 I. by 12. See Art. 51.
3 11 3 the Multiplicand for 9 Inches, found by dividing
47 3
34 9
35 5 3 the Product of 3 F. 11 I. 3 P. by 9 Inches.
188 } the Product of 47 by 34.
141
8 6 the $\frac{1}{4}$ of 34 Feet, taken for the 3 Inches.
16|41 11 3 Anſwer 16 Squares, 41 Feet.

The Reader may change the Inches into the Decimals of a Foot, by Art. 48, and then perform the Operation decimally; which I leave for his Practice.

The 1641 Feet require no other Diviſion, to reduce them to Squares, than cutting off two Figures to the right Hand, which is dividing by 100: The Inches and Parts are not regarded in theſe Meaſurements.

Exam. 2. Partitions betwixt divers Rooms being 10 Feet, 8 Inches high, 104 Feet, 6 Inches in Length; how many Squares are there contain'd in it?

8	8	6	the Multiplicand for 8 Inches, found by dividing (104 F. 6 I. by 12. See Art. 51.
104	6		
10	8		
69	8	0	the Product of 8 F. 8 I. 6 P. by 8 I.
1040			the Product of 104 F. by 10 F.
5			the Half of 10 Feet, taken for the 6 Inches.
11|14	8	0	Anſwer 11 Squares, 14 Feet; the Truth of which the Reader may try by Decimals, for his own Practice.

Exam. 3. A Houſe meaſured to the Outſide of the Walls 56 Feet, 9 Inches in Length, and 16 Feet, 10 Inches and a Half wide: How many Squares does the Roof contain?

Here it is to be obſerved, that the moſt common Proportion for the Length of the Rafters to the Width of the Houſe, is as 3 to 4; this being uſually called the true Pitch: In this Caſe, the uſual Method is, to multiply the Dimenſions of the Flatts, and add the Half of the Product to it; this Flatt and Half are equal to the whole Roof.

F.	I.	P.	S.	
	4	8	9	Multiplicand for 6 Parts, or ½ In. } See Art. 51.
4	8	9		Multiplicand for 10 Inches. } See Art. 51.
56	9			
16	10	6		
2	4	4	6	the Product of 4 I. 8 P. 9 S. by 6 Parts.
47	3	6		the Product of 4 F. 8 I. 9 P. by 10 Inches.
8				To multiply 16 F. by 9 I. for 6 I. take the ½ of 16 F. which is 8, and for the remaining 3 I. take the ½ of 8 F. which is 4.
4				
336				the Product of 56 by 16.
56				
957	7	10	6	
478	9	11	3	Half.
14|36	7	9	9	Anſwer 14 Squares, 36 Feet.

If the Reader has any Mind to do this Operation decimally, he will find the Decimal of 10 Inches and a Half to be .875; for a Foot reduced to Half-Inches is 24, and 10 ½ Inches reduced to Half-Inches are 21; ſo that 10 ½ Inches are $\frac{21}{24}$, or $\frac{7}{8}$ of a Foot, which, by Art. 48, might be reduced to .875, the Decimal of 10 ½ Inches.

But, when Roofs are made of a different Pitch, as is often the Caſe, the trueſt Way of meaſuring is by a Line thrown over, which takes the Dimenſions of both Sides of the Roof; this multiplied by the Length of the Building gives the true Content of the Roof: And, with Regard to Buildings that are hipt at one or both Ends, the Roof is of the ſame Dimenſions as when ſtraight rooft all the Way; ſo that this makes no Difference: But, in meaſuring Floors and Partitions, the Doors, Windows, Stairs, &c. muſt be meaſured and deducted.

Of BRICKLAYERS' *Work.*

Brick-Work, as Walls and Chimnies, are meaſured by the Rod: A Rod is 16.5 Feet, the Square of which is 272.25 Feet, or 272 Feet and a Quarter, which is a Rod of Brick-Work: But Regard muſt be had to the Thickneſs of the Wall, for one Brick and a Half is the Standard; ſo that, if a Wall be thicker or thinner than one Brick and a Half, it is uſually reduced to that, before it is computed in Rods. The Way to do this is, to multiply the Number of Feet in a Piece of Brick-Work by the Number of Half-Bricks in the Thickneſs of the Wall; this Product, divided by 3, gives the Content of the Wall in Feet, reduced to the Standard.

Exam. 1. A Wall being 64 Feet long, 18 Feet high, and four Bricks thick, how many Rods of Brick-Work are contained in it, reduced to the Standard?

```
            64
            18
           ----
           512
           64
          ----
          1152
             8 Half-Bricks thick.
        -------
       3) 9216
      ---------
  272) 3072 (11 Rods.
       272
      -----
        352
        272
       -----
         80 Feet.
```

In this Example, after reducing it to a Brick and a Half, I divide by 272, rejecting the fractional Part, as is uſually done in theſe Caſes, the Difference being inconſiderable: If the Feet remaining require it, they are uſually divided by 68, the Feet in a Quarter of a Rod; and here the Anſwer may be 11 Rod one Quarter, the odd Feet may be allowed on Account of rejecting the Decimal in the Deviſor.

In ſome Places Brick-Work is meaſured by the Rod of 18 Feet ſquare; in which Caſe the ſquare Rod is 324 Feet; In others they make but 63 ſquare Feet, called a Rood, *viz.* 21 Feet long, and 3 Feet high; and in this laſt Way of meaſuring, no Reduction is made of the Thickneſs, but that is allowed for in the Price: Therefore it muſt be inquired what Meaſure is to be uſed.

Exam. 2. A Wall being 72 Feet long, 34 Feet high, and 3 Bricks and a Half thick, how many Rods of Brick-Work, at 18 Feet the Rod, are contained in it?

```
   72        2448   (thick.       324) 5712 (17 Rods.
   34           7 Half-Bricks          324
  ----      -------                   ----
  288    3) 17136                     2472
 216        -------                   2268
 ----        5712                     ----       (ters.
 2448                              81) 204 ( 2 Quar-
                                       162
                                       ----
                                        42 Feet.
```

A Quarter of this Rod is 81 Feet; therefore, dividing the remaining Feet by 81, I find it 17 Rods and a Half and 42 Feet.

The next Article in Bricklayers' Work is Tiling; which, being meaſured by the Square of 100 Feet, is much the ſame with meaſuring the Roofs in Carpenters' Work; only there we meaſure but to the Outſide of the Wall, but Bricklayers muſt be allowed for the Eaves; and therefore the beſt Way is to throw a Line over the Ridge, and meaſure from Eaves to Eaves on both Sides: They are alſo often allowed Double for Hipps and Vallies.

Exam. 3. A Roof cover'd with Tiles 48.75 Feet from Eaves to Eaves, and 54.5 Feet in Length, how many Squares of Tiling are contained in it?

F. I. P.

F.	I.	P.
4	0	9
48	9	
54	6	
24	4	6
27		
13	6	
192		
240		
26|56	10	6

48.75
54.5

24375
19500
24375

26|56.875

Anſwer 26 Squares and a Half.

Inſtead of multiplying 54 Feet into 9 Inches, I divide the 9 Inches into 6, the Half of a Foot, and 3, the Half of that; then taking the Half of 54, which is 27, I place it under the Title Feet, and the Half of that for the 3 Inches, being 13 Feet and 6 Inches, I place them under their proper Titles, and proceed to multiply the 48 Feet by 54 Feet, as in common Multiplication.

Chimnies are meaſured as a Solid, by multiplying the Height by the Width, and that Product by the Depth, in each Story, only deducting the Vacancy of the Hearth; after which it may be reduc'd to Standard, in the ſame Manner as was ſhewn in meaſuring Walls, by multiplying the Number of Feet by the Number of Half-Bricks in the Depth, from the Front to the Back, and dividing the Product by 3, as was there taught: But, in regard the Depth of Chimnies is meaſured by Foot-Meaſure, and a Foot is $\frac{8}{9}$ of a Brick and a Half, it will be a more ready Way to multiply the Number of Feet by 8, and divide the Product by 9, and the Quotient will be the Quantity of Brick-Work reduced to Standard; which then muſt be divided by 272, to bring it into ſquare Rods.

A Chimney being 10 Feet high, 6 Feet and 3 Inches wide, and 2 Feet deep, in the firſt Floor; 8 Feet high, 6 Feet and 2 Inches wide, and 2 Feet deep, in the ſecond Floor; 7 Feet high, 5 Feet and 9 Inches wide, and 2 Feet deep, in the third Floor; the Shaft 8 Feet high, 5 Feet wide, and 2 Feet deep; to find the Quantity of Brick-Work? the Vacancy in the Ground-Floor being 4 Feet and 9 Inches wide, 4 Feet high, and 1 Foot and 10 $\frac{1}{2}$ Inches deep; in the ſecond Floor, the Hearth 4 Feet and 3 Inches wide, 3 Feet and 6 Inches high, and 1 Foot and 10 $\frac{1}{2}$ Inches deep; in the third Floor, 2 Feet and 6 Inches high, 1 Foot and 3 Inches wide, and 1 Foot and 10 $\frac{1}{2}$ Inches deep.

F. I.

F.	I.	F.	I.	F.	I.	F.		F.	I.
6	3	6	2	5	9	8		125	
10		8		7		5		98	8
								80	6
62	6	49	4	40	3	40		80	
2		2		2		2			
125		98	8	80	6	80		384	2

Having found the Dimenſions of the ſeveral Parts, and added them together, we are next to compute the Quantity of the Vacancies to be deducted: In doing which, it will be beſt to put thoſe Quantities firſt which have the ſmalleſt Diviſions, as this will ſave the Trouble of making Multiplicands for thoſe ſmaller Diviſions; removing the Work after the firſt Multiplication, in order to make a Multiplicand for the Inches.

F.	I.	P.	
1	10	6	
4			
7	6		
	7	6	
7	6		
4	9		
5	7	6	
30			
35	7	6	
27	10	8	3S.
5	10	3	9
69	4	6	

F.	I.	P.	S.	
	1	10	6	
1	10	6		
4	3			
	5	7	6	
7	6			
7	11	7	6	
	7	11	7	6T.
7	11	7	6	
3	6			
3	11	9	9	
23	10	10	6	
27	10	8	3	

F.	I.	P.	S.
	1	10	6
1	10	6	
2	6		
	11	3	
3	9		
4	8	3	
	4	8	3
4	8	3	
1	3		
1	2	0	9
4	8	3	
5	10	3	9

Dimenſions of the Vacancy in the firſt Floor are 4 Feet, 9 Inches wide, 4 Feet high and 1 Foot 10 Inches and a Half deep: After I have multiplied 1 F. 10 I, 6 P. by 4, the Product, 7 Feet and 6 Inches, I put below, to have Room to place the new Multiplicands over it; as in the Work; and the like of the other two Vacancies.

Having found the Quantity of the Vacancies to be 69 Feet, 4 Inches, 6 Parts, ſubtract it from the Quantity before found, then multiply the Remainder by 8, and divide by 9, to reduce it to the Standard Thickneſs of a Brick and a Half; and, laſtly, divide it by 272 to bring it into Rods.

	F.	I.	P.
Dimenſions of the Chimney	384	1	
Vacancies to be deducted	69	4	6
	314	9	6
	8		
	2518	4	

```
9) 2518 (279            272) 279 (1 Rod.
   18                        ---
   --                         7
    71
    63
    --
     88
     81
     --
      7
```

Of JOYNERS' Work.

Joyners' Work is done by the Yard: But the Dimenſions are taken with a String, becauſe the Cornice, Mouldings, and ſwelling Pannels cannot be meaſured by a ſtraight Inſtrument; but a String may be thruſt into the Hollows, and bent over the Swelling; all which require to be meaſured, in regard of Trouble in working them: By this Means, the Work meaſures conſiderably more in Height than the real Height of the Room. But, in Regard to meaſuring about the Room for the Length, it is done in the ſame Manner as in meaſuring the Floor: When the Dimenſions are multiplied, divide by 9, to bring it into ſquare Yards.

Exam. 1. A Room of Wainſcoting, meaſured as above, is 12 Feet, 9 Inches high, and 96 Feet, 3 Inches in Compaſs: How many Yards of Wainſcoting are contained in it?

F. I. P.

	F.	I.	P.	
	8	0	3	See Art. 51.
	96	3		
	12	9		
	72	2	3	
	3			
	1152			
9)	1227	2	3	
	136	3		

96.25
12.75
48125
67375
19250
9625
9) 1227.1875
136.3

Anſwer 136 Yards, 3 ſquare Feet.

In meaſuring Joyners' Work that is wrought on both Sides, as Doors, Window-Shutters, and ſuch-like, it is cuſtomary to allow for Work and Half: Therefore, in theſe Caſes, after the Dimenſions are multiplied, take Half the Product, and add it to the Whole; then divide that Sum by 9, to bring it into ſquare Yards.

Exam. 2. The Window-Shutters of a Room (or of ſeveral Rooms, if of equal Height) in Breadth 72 Feet, 6 Inches, in Height 4 Feet, 9 Inches; how many Yards are contain'd in them, allowing for Work and Half?

	F.	I.	P.	
	6	0	6	See Art. 51.
	72	6		
	4	9		
	54	4	6	
	2			
	288			
	344	4	6	
Half	172	2	3	
9)	516	6	9	
	57	3		

72.5
4·75
3625
5075
2900
344·375
Half 172.1875
9) 516.5625
57·3

Anſwer 57 Yards, 3 ſquare Feet.

In meaſuring Wainſcot, you muſt deduct Doors, Windows, *&c.* meaſuring the Window-Boards, Soffits, and Cheeks by themſelves.

Of PLAISTERERS' Work.

All Plaiſterers' Work is meaſured by the Yard, as the Joyners' is.

Exam. 1. A Ceiling 35 Feet long, 20 Feet, 6 Inches wide; how many ſquare Yards are contain'd in it?

```
      F.  I.                 20.5
      20  6                  35
      35                    ------
     -------                1025
      17  6                 615
     100                    ------
      60                  9)717.5
     --------              -------
   9)717  6                 79.6
    --------
      79  6          Anſwer 79 Yards, 6 ſquare Feet.
```

Exam. 2. The Height of the Walls of ſeveral Rooms is 9 Feet, 6 Inches, the Compaſs of the Rooms 246 Feet, 9 Inches: How many Yards of Plaiſtering are contained in them?

```
    F.  I.  P.                       246.75
    20  6   9 See Art.51.              9.5
   -----------                      --------
   246  9                            123375
     9  6                           222075
   -----------                      --------
   123  4   6                     9)2344.125
  2214                             ---------
     4  6                           260  4
     2  3
   -----------               Anſ. 260 Yards, 4 ſquare Feet.
9) 2344  1  6
   -----------
   260.4
```

In meaſuring Walls, Deductions muſt be made for Doors and Windows: Alſo, in rendering between Quarters, you may deduct one fifth Part for the Quarters, Braces, &c. but, in Whiteing and Colouring, one fourth or fifth Part muſt be added, where the Quarters and Braces project beyond the Wall.

Of

Of PAINTERS' *Work.*

The Painters' Work is meaſured in the ſame Manner as Joyners' Work, by a String, becauſe the Bruſh goes into the Hollows and over the Swellings: When the Dimenſions are multiplied, divide by 9, the ſquare Feet in a Yard. Windows, Bars, Caſements, *&c.* are not meaſured, but done by the Piece. It will not be neceſſary to give Examples, as the Work is in all Reſpects the ſame as in Joyners' and Plaiſterers' Work.

Of GLAZIERS' *Work.*

Glaziers' Work is meaſur'd by the Foot; and they uſually meaſure to a great Degree of Exactneſs: Therefore, in ſetting down their Dimenſions for multiplying them in Feet and Inches, the Inch is divided into twelve Parts, as the Foot is; ſo that a Quarter of an Inch is three Parts, Half an Inch ſix Parts, *&c.*

Exam. 1. A Light of Glaſs being 4 Feet and 2 ¾ Inches long, and one Foot and 7 ½ Inches broad, how many Feet of Glaſs are contained in it?

F.	I.	P.	S.	T.
		4	2	9
	4	2	9	
4	2	9		
1	7	6		
	2	1	4	6
2	5	7	3	
4	2	9		
6	10	5	7	6

See Art. 51.

.229 } Decimal of { 2¾ } Inch.
.625 }   { 7½ }

```
  4.229
  1.625
 ------
  21145
  8458
 25374
 4229
 --------
 6.872125
```

Anſwer 6 Feet, 10 ½ Inches.

Exam. 2. If 10 Lights be, each 3 Feet and 6 ½ Inches long, 1 Foot and 4 ¼ Inches broad, how many Feet of Glaſs are contained in them?

F.	I.	P.	S.	T.
		3	6	6
	3	6	6	
3	6	6		
1	4	3		
		10	7	6
1	2	4		
3	6	6		
4	9	6	7	6
10				
47	11	6	3	

.542 } Decimal of { 6 1/2 } Inch.
.354 } Decimal of { 4 1/4 } Inch.

3.542
1.354

14168
17710
10626
3542

4.795868
10

47.958680

Answer 47 Feet, 11 1/2 Inches.

When Windows are made round at the Top, the Height is to be taken in the Middle, and they are to be meaſured as if they were ſquare; no Allowance being made for what is wanting at the Corners, becauſe the Trouble, and the Waſte of the Glaſs, is more than is ſaved by it. The ſame is the Cuſtom as to round or oval Windows, which are to be meaſured in their wideſt and longeſt Parts, and computed as ſquare or oblong.

Of MASONS' *Work.*

Maſons' Work is meaſured by the Foot. Some Sorts of Work are meaſured only ſuperficially; but, in others, they meaſure the Solidity, taking in all the three Dimenſions, Length, and Thickneſs.

Exam. 1. A Pavement being 96 Feet long, 16 Feet and 9 Inches wide, how many ſuperficial Feet are contained in it?

F. I.
16 9
96

(6 Inches.
48 the 1/2 of 96, for
24 the 1/4 of 48, for
96 (3 Inches.
144

1608

16.75
96

10050
15075

1608.00

Answer 1608 Feet.

Exam. 2. How many ſolid Feet are contained in a Wall 64 Feet, 6 Inches long, 20 Feet, 6 Inches high, and 2 Feet, 3 Inches thick?

F. I. P.

F.	I.	P.
5	4	6
64	6	
20	6	
32	3	
10		
1280		
1322	3	
110	2	3
1322	3	
2	3	
330	6	9
2644	6	
2975	0	9

64.5
20.5
3225
12900
1322.25
2.25
661125
264450
264450
2975.0625

Anſwer 2975 Feet.

Having multiplied two of the Dimenſions, 64 F. 6 I. by 20 F. 6 I. the Product, 1322 F. 3 I. I place below, to have Room for making a new Multiplicand, by Art. 51. as in the Work.

In ſome Places Walling is done by the Rood of 63 Feet, ſuperficial Meaſure, as in Bricklayers' Work; the Thickneſs being allowed for in the Price.

Exam. 3. A Wall being 89 Feet and 3 Inches long, 15 Feet and 6 Inches high, how many Roods of 63 ſquare Feet, are contained in it?

	F.	I.	P.	
	7	5	3	
	89	3		
	15	6		
	44	7	6	
	3	9		
	445			
	89			
63)	1383	4	6	(21
	126			
	123			
	63			
	60			

89.25
15.5
44625
44625
8925
63) 1383.375 (21
126
123
63
60

Anſwer 21 Rood, 60 Feet.

Of PAVIOURS' Work.

Paviours' Work is done by the Yard; ſo that nothing more is neceſſary than to multiply the Length by the Breadth, and, if it were meaſured by the Foot, to divide by 9, to bring it into Yards: Therefore, Examples are not neceſſary, as it is the ſame with Joyners' and Painters' Work.

Of GEOMETRY.

GEOMETRY is uſually defined, *The Doctrine of Extenſion.* The Word ſtrictly means the Meaſuring of Land, but is applied to every Thing capable of being meaſured; that is, to all Things ſuſceptible of more and leſs. 'Tis uſually ſuppoſed that the *Ægyptians* were the firſt Inventors of Geometry: Their Lands being annually overflowed by the Inundations of the *Nile*, its Impetuoſity bore away their Land-Marks, and the Mud left behind made all Diſtinctions of their Lands impracticable, any other Way than by keeping Plans of their Figure and Quantity; ſo that the natural Circumſtance of their Country obliged them to the Study and Cultivation of Geometry.

Practical Geometry, or the Reſolution of Problems, which is our preſent Buſineſs, is divided into Plain and Spherical.

Of *Plain* GEOMETRY.

Definitions.

1. A *Point* in the Mathematics is conſidered only as a Mark, without any Regard to Dimenſions.

2. A *Line* is conſidered as Length, without Regard to Breadth or Thickneſs.

3. A *Plain*, or *Superfice*, has two Dimenſions, Length and Breadth, but is not conſidered as having Thickneſs.

4. A *Solid* has three Dimenſions, Length, Breadth, and Thickneſs, and is uſually called a *Body*.

5. A Line is either *ſtraight*, which is the neareſt Diſtance betwixt two Points, or *crooked*, called a curve Line, whoſe Ends may be drawn farther aſunder.

6. If two Lines are at equal Diſtance from one another in every Part, they are called *parallel* Lines, which, if continued infinitely, will never meet.

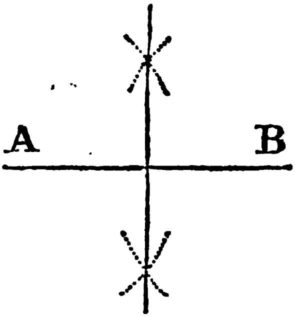
7. If two Lines incline one towards another, they will, if continued, meet in a Point; by which Meeting is formed an *Angle.*

8. If one Line fall directly upon another, ſo that the Angles on both Sides are equal, the Line ſo falling is called a *Perpendicular*, and the Angles ſo made are called *right* Angles.

9. All Angles, except they are right Angles, are called *oblique* Angles; whether they are *acute*, that is, leſs than a right Angle; or *obtuſe*, that is, greater than a right Angle.

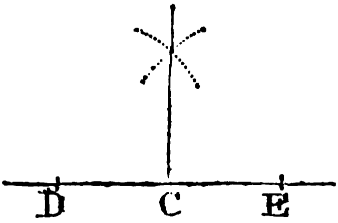
Geometrical Problems.

Problem 1. *To divide a Line*, A B, *into two equal Parts.*



Set one Foot of the Compaſſes in the Point A, and, opening them beyond the Middle of the Line, deſcribe Arches above and below the Line; with the ſame Extent of the Compaſſes, ſet one Foot in the Point B, and deſcribe two Arches croſſing the former: Draw a Line from the Interſection of the Arches above the Line, to the Interſection below the Line; this will divide the Line A B into two equal Parts.

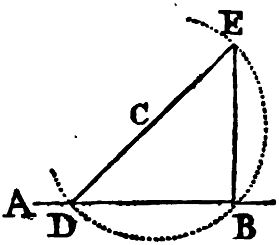
Prob. 2. *To erect a Perpendicular on the Point* C *in a given Line.*



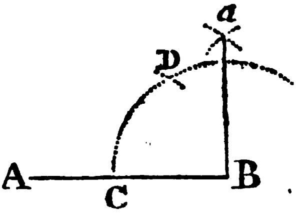
Set one Foot of the Compaſſes in the given Point C, extend the other Foot to any Diſtance at Pleaſure, as to D, and with that Extent make the Marks D and E. With the Compaſſes, one Foot in D, at any Extent above Half the Diſtance of D and E, deſcribe an Arch above the Line; and with the ſame Extent, and one Foot in E, deſcribe an Arch croſſing the former: Draw a Line from the Interſection of the Arches to the given Point C, which will be perpendicular to the given Line in the Point C.

Prob. 3. *To erect a Perpendicular upon the End of a Line, or from a Point near the End.*

There are diverſe Methods of doing this; of which I ſhall give two.



Firſt Method. If it be required to draw a Perpendicular to the Point B, near the End of the Line A B, ſet one Foot of the Compaſſes in the Point B, and, opening them at Pleaſure, ſet the other any where above the Line A B, as at C; let C be the Center, on which deſcribe an Arch cutting the Line A B in D, and paſſing through the Point B, continue it to more than a Semicircle, till it comes over the Point B; draw the Line D C E from the Interſection at D, through the Center C, till it cuts the Arch in E; laſtly, a Line drawn from E to B will be perpendicular to A B in the Point B.



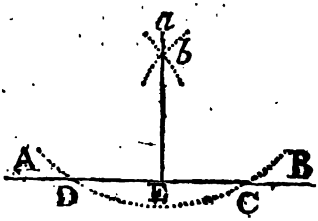
Second Manner. Set one Foot of the Compaſſes in the given Point B, open them to any convenient Diſtance, and deſcribe the Arch C D E; ſet one Foot in C, and with the ſame Extent croſs the Arch at D, and with the ſame Extent croſs the Arch again from D to E; then, with one Foot of the Compaſſes in D, and with any Extent above the Half of D E, deſcribe an Arch *a*; take the Compaſſes from D, and, keeping them to the ſame Extent, with one Foot in E, interſect the former Arch *a* in *a*; from thence draw a Line to the Point B, which will be a Perpendicular to A B.

Prob. 4. *From a given Point, a, to let fall a Perpendicular to a given Line.*

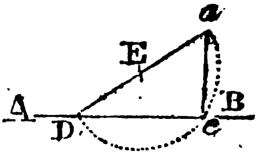
This Problem has two Caſes, according as the Point is ſituate, over the middle Part, or the End of the Line.

Caſe 1. To let fall a Perpendicular from the Point *a* over the Middle of A B.

Set

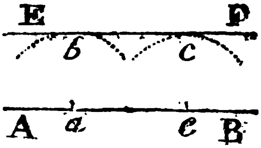


Set one Foot of the Compaſſes in the Point *a*, extend the other ſo as to reach beyond the Line A B, and draw an Arch to cut the Line A B in C and D; put one Foot of the Compaſſes in C, and, with any Extent above Half C D, draw an Arch, *b*; keeping the Compaſſes at the ſame Extent, put one Foot in D, and interſect the Arch *b* in *b*; through which Interſection, and the Point *a*, draw *a* E, the Perpendicular required.



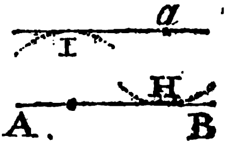
Caſe 2. To let fall a Perpendicular from a Point, *a*, being nearly over the End of the Line A B: From any Part of the given Line, as D, draw a Line to the Point *a*; divide the Line D *a* into two equal Parts in E, by Prob. 1; ſet one Foot of the Compaſſes in E, and with the Diſtance D E deſcribe an Arch from D to *a*, which will interſect the given Line A B in *e*; draw the Line *a e*, which will be perpendicular to the given Line A B.

Prob. 5. *To draw a Line parallel to a given Line*, A B.



Set one Foot of the Compaſſes in any Part of the Line, as at *a*; extend the Compaſſes at Pleaſure, unleſs a Diſtance be aſſigned, and deſcribe an Arch, *b*; with the ſame Extent, in ſome other Part of the Line A B, as at *e*, deſcribe the Arch *c*, lay a Ruler to the Extremities of the Arches, and draw the Line E F, which will be parallel to the given Line A B.

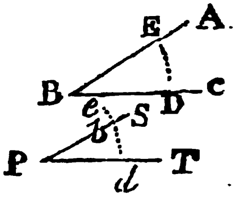
Prob. 6. *To draw a Line parallel to a given Line*, A B, *and to paſs through a given Point*, *a*.



Set one Foot of the Compaſſes in the given Point *a*; extend the other ſo as it may touch the Line A B, and not paſs it, in deſcribing the Arch H; with the ſame Extent, ſet one Foot in the Line A B, and deſcribe the Arch I; then draw a Line through the Point *a* to the Extremity of the

the Arch I, which will be parallel to the given Line A B, and paſs through the Point *a*.

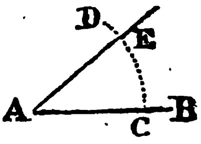
Prob. 7. *To make an Angle equal to a given Angle*, A B C.



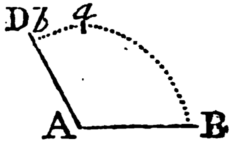
Draw the Line P T at Pleaſure; ſet one Foot of the Compaſſes in B, and with any Diſtance, B D, draw the Arch D E; keep the Compaſſes at the ſame Extent, and, placing one Foot in P, draw the Arch *d e* at Pleaſure: Set one Foot of the Compaſſes in D, extend the other to E, ſet this Extent from *d* towards *e*, draw the Line P S through the Point *b* in the Arch *d e*; and ſo will S P T be an Angle equal to A B C.

Prob. 8. *To make an Angle to any Number of Degrees.*

This Problem has two Caſes, as the Angle required is leſs or more than 90 Degrees.



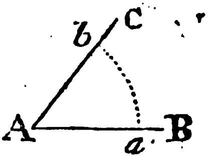
Caſe 1. To make an acute Angle of 30° a given Line AB: Take 60 Degrees from a Line of Chords in the Compaſſes, [*The Deſcription and Uſe of the Line of Chords will be given after the Definitions in Trigonometry.*] Set one Foot of the Compaſſes in the Point A, and deſcribe an Arch, C D, at Pleaſure; then ſet one Foot of the Compaſſes in the Braſs Center in the Beginning of the Line of Chords, and bring the other to 30 on the Line; with this Extent, ſet one Foot in C, with the other interſect the Arch C D in E, and through E draw the Line A E; ſo will E A B be an Angle of 30 Degrees.



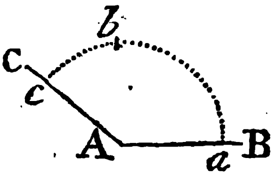
Caſe 2. To draw an obtuſe Angle of 120° on a given Line, A B: Take 60 Degrees in the Compaſſes from the Line of Chords, ſet one Foot in A, and with the other deſcribe a large Arch C D; take 90 Degrees from the Line of Chords, and ſet one Foot in C, and interſect the Arch in *a*; then take 30 Degrees from the ſame Line of Chords; and ſet them from *a* to *b*; a Line drawn from A to *b* will make an Angle, D B A, of 120 Degrees.

Prob. 9. *To meaſure an Angle.*

This Problem has two Caſes, the Angle being either leſs o more than 90°.

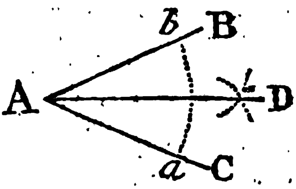


Caſe 1. To meaſure an acute Angle, A B C: Take 60 Degrees from the Line of Chords, ſet one Foot in A, and with the other deſcribe the Arch *a b* from one Leg to the other, then ſet one Foot of the Compaſſes in *a*, and extend the other to *b*; with this Extent, ſet one Foot on the Braſs Center at the Beginning of the Line, the other will reach to 54 on the Line of Chords, which is the Number of Degrees contained in the Angle B A C.



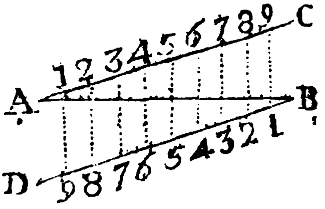
Caſe 2. To meaſure an obtuſe Angle, A B C: Take 60 Degrees in the Compaſſes from the Line of Chords, ſet one Foot in the angular Point A, and deſcribe the Arch *a c* from one Leg to the other, as before; then take 90 Degrees from the Line of Chords, and ſet it from *a* to *b*; then take the Extent from *b* to *c* in the Compaſſes, as in the laſt Problem, and ſet one Foot in the Beginning of the Line of Chords, the other will reach to 41 Degrees, which, added to 90, is 131 Degrees, the Meaſure of the Angle B A C.

Prob. 10. *To divide a given Line,* B A C, *into two equal Parts.*



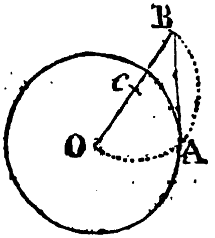
Set one Foot of the Compaſſes in the angular Point A, extend the Compaſſes to any convenient Diſtance, and interſect each Leg, as at *a* and *b*; with the ſame, or any other Extent above Half the Diſtance *b a*, ſet one Foot in *a*, and draw the Arch D; then, keeping the Compaſſes to the ſame Extent, ſet one Foot in *b*, and interſect the Arch D in D; a Line drawn from the angular Point A through the Interſection of the Arches will divide the Angle A into two equal Parts.

Prob. 11. *To divide a given Line,* A B, *into any Number of equal Parts.*



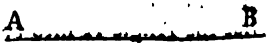
From the End of the Line, A, draw the Line A C, ſo as to make any Angle at A; from the other End of the Line, B, by Prob. 7, draw a Line B D, making an Angle at B equal to the Angle at A; which Lines A C and B D will be parallel to each other: On each of theſe Lines, with any Extent of the Compaſſes, from the Points A B, reſpectively, ſet off as many Parts equal to one another, ſave one, as the given Line is to be divided into; as ſuppoſe 10, ſet off 9 equal Parts, numbering them 1, 2, 3, &c. on each Line; draw Lines from 1, 2, 3, &c. in the Line A C, to 9, 8, 7, &c. in the Line B D; then will the Line A B be divided into ten equal Parts.

Prob. 12. *From a Point,* B, *aſſigned out of a Circle, to draw a Tangent to the Circle, or a Line that ſhall touch the Circumference, but not interſect it.*



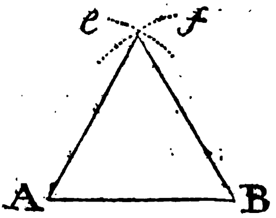
From the given Point B, draw the Line B *o* to the Center of the Circle, which is called the Secant; [*Sines, Tangents, and Secants will be defined in the Beginning of the Trigonometry*]. Divide this Line into two equal Parts in *c*, by Prob. 1; then, with one Foot of the Compaſſes in the Point *c*, with the Extent *c* B or *c o*, draw the Semicircle *o* A B, cutting the Circle in A; laſtly, draw a Line from B through the Interſection at A; this will be a Tangent to the given Circle.

Prob. 13. *To make a Triangle, each of whoſe Sides ſhall be equal to a given right Line.*



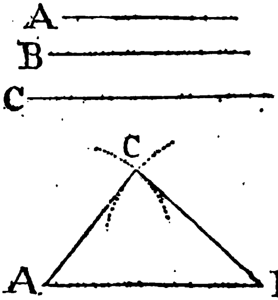
Let A B be the given Line; draw a Line at Pleaſure, larger than the given Line; then take the given Line A B in the Compaſſes, ſet one Foot in A, the

Lin.



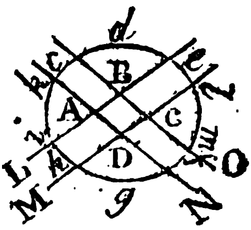
Line laſt drawn, with the other make a Mark at B, and with the ſame Extent of the Compaſſes draw the Arch *e*; keeping the Compaſſes to the ſame Extent, put one Foot in B, and draw the Arch *f*, interſecting the former Arch; from the Interſection of the Arches draw Lines to A and B, and the Triangle is completed, having each Side equal to the given Line A B.

Prob. 14. *To draw a Triangle whoſe Sides ſhall be equal to three given Lines, if any two of them be longer than the third.*



Let A, B, C, be three given Lines: Draw a Line, A B, at Pleaſure; take the Line C in the Compaſſes, ſet one Foot in A, and with the other make a Mark at B; then take the given Line B in the Compaſſes, ſet one Foot in B and deſcribe the Arch C; then take the given Line A in the Compaſſes, and ſetting one Foot in A, interſect the Arch C in C; laſtly, draw the Lines A C and B C, and the Triangle is completed, having three Sides, each equal to one of the given Lines.

Prob. 15. *To deſcribe Ovals mechanically.*



Draw two parallel Lines, as L and M, at a moderate Diſtance, by Prob. 5; then draw two others at the ſame Diſtance, croſs the former, as N and O; by the Croſſing of theſe Lines will be made a Figure, A B C D, of four Sides: Extend the Compaſſes at Pleaſure, and ſetting one Foot in D, deſcribe the Arch *c d e*; with the ſame Extent, ſet one Foot in B, and deſcribe the Arch *f g h*; then ſet one Foot of the Compaſſes in C, and contract them ſo as to reach the Point *e*, and deſcribe the Arch *l m*; with the ſame Extent, and one Foot in A, deſcribe the Arch *i k*, and the Oval is completed: In the ſame Manner, with a greater or leſs Extent of

The Deſcription and Uſe of the Logarithms.

The firſt Page of the Table is divided into ſix Columns, three of which, *viz.* the firſt, third, and fifth, contain the natural Numbers from 1 to 99 incluſive: In the ſecond, fourth, and ſixth Columns, right againſt each Number, you have its Logarithm; only obſerve, that the *Index*, or Characteriſtic of the Logarithm is omitted. The Logarithm of every Number conſiſts of two Parts, which are uſually ſet down, like Integers and Decimal Fractions, with a Dot betwixt them; that Part which ſtands to the left Hand of the Dot is called the Index, which is uſually omitted in the Tables. The Learner therefore muſt carefully remember, that the Index to all Logarithms under 10, is ——— 0.
From 10 to all under 100, is 1.
From 100 to all under 1000, is 2.
From 1000 to all under 10000, is 3.

That is, the Index of the Logarithm is always leſs by *Unity* than the Number of Places of Integers in the Number whoſe Logarithm is required. The Reader carefully remembering this neceſſary Caution, I ſhall now ſhew him how to find the Logarithm of any Number uſed in common Operations.

For which Purpoſe, it will be neceſſary to obſerve the Structure of the following Pages of the Table. Every left-hand Page is divided into ſix Columns: In the firſt, on the left Hand Pages, are contained the natural Numbers, beginning at 100 at the Top of the firſt Page, and proceeding to 999, in the natural Order: The five other Columns, on the left hand Pages, are Logarithms without their Indexes; and the ſecond, third, fourth, fifth and ſixth on the right Hand Pages, likewiſe contain Logarithms without their Indexes: Over theſe ten Columns of Logarithms are placed the Cypher and nine Digits, that is, a Cypher over the firſt, and 1, 2, 3, *&c.* to 9, over the others; the Uſe of which will be ſhewn hereafter: In the firſt Column of the right-hand Pages, the Numbers are repeated from the firſt on the left hand Pages, for the more ready finding the Logarithms: The ſeventh Column of the right hand Pages, marked at the Top D, contains the Differences of the Logarithms in each Line.

To find the Logarithm of 9, look in the firſt Page of the Table for 9, under the Title N, in the firſt Column; and right againſt it in the ſecond Column, under the Title *Logarithm*, you'll find .954242 the Logarithm of 9, without the Index, which, according to the Rule above, is 0: So that the Logarithm of 9 is 0.954242; for we always put down the Index of a Logarithm, though it be a Cypher.

It it be required to find the Logarithm of 15, in the Table, againſt 15, you have .176091; to which prefixing the proper Index, (which in this Caſe is 1, as the Number 15 conſiſts of two Places) the true Logarithm of 15 is 1.176091. Again, let it be required to find the Logarithm of 95; in the Table you have .977723, to which prefixing the Index 1, as 95 conſiſts of two Places, and is under 100 the true Logarithm of 95 is 1.977723; and the ſame of any other Number under 100.

But, for Numbers from 100 to 1000, the Rule for finding the Logarithm is, to look for the given Number in the firſt Column of one of the future left-hand Pages of the Table, under the Title *Number*, and right againſt the Number, in the next Column, over which is placed 0, is the Logarithm to the Number, without the Index, as before: Thus the Number 115, being given, will be found in the ſecond Page of the Table; and againſt it, in the ſecond Column of that Page, is .060698; to which prefixing the Index 2, (as the given Number 115 conſiſts of three Places) then 2.060698 is the Logarithm ſought: So the Logarithm of 487 being required, look in the firſt Column of the left-hand Pages for 487, it will be found in the ſixteenth Page; and right againſt it, in the ſecond Column of the ſame Page, you have .687529, which is the Logarithm, without the Index; to which prefixing 2, you have 2.687529, for the true Logarithm of 487, the Number given. If the Logarithm of 796 were required, look in the firſt Column of the left-hand Pages for the given Number, and againſt it you'll find .900913; to which prefixing the proper Index, you have 2.900913, the true Logarithm ſought. Again, the Number 890 being given, look in the firſt Column of the left-hand Pages for your Number, and againſt it, in the next Column, you have .949390; to which prefix 2, the proper Index, and 2.949390 is the Logarithm for 890, the Number given.

But when a Number conſiſts of four Places, the firſt three only to the left Hand are to be ſought in the firſt Columns of the left-hand Pages, and the laſt Place, or Place of Units, is to ſought at the Top of the Table: Thus, if the Number 4270 were given, look in the firſt Column of the left-hand Pages for 427, and look among the Figures 0, 1, 2, 3, &c. at the Top of each Column for 0; then in the Column over which 0 ſtands, and againſt 427, before found, you'll find .630428; to which prefixing the proper Index 3, we have 3.630428, the true Logarithm of 4270.

If the Logarithm of 7816 were required, look for 781 in the firſt Column of the left-hand Pages, and then look among the Numbers 0, 1, 2, 3, &c. at the Top of each Column, till you find the 6; then againſt the 781, in the Column under the Number 6, you have .892985; to which prefixing the Index 3, it gives 3.892985, the true Logarithm of 7816, the given Number. Again, if the Logarithm of 9415 be required, look for 941, as before, and look among the Numbers 0, 1, 2, 3, &c. at the Top of each Column, till you find the 5; then againſt 941, in the Column under the Number 5, you have .973820, for the Logarithm of 9415, excluſive of the Index 3, which muſt be ſupplied, as before.

If the Logarithm of a mixt Number, that is, Integers and decimal Fractions, be required, ſeek it as if the Number were all Integers: Thus, if it be required to find the Logarithm of 24.7, you muſt look for the Logarithm of 247 in the Table; only inſtead of prefixing 2 for the Index, you muſt put 1, which is the Index for 24, not regarding the fractional Part, but the Integers alone, in putting down the Index; ſo that the Logarithm of 24.7 is 1.392697, which would be the Logarithm of 247, if 2 were prefixed for the Index. If the Logarithm of 57.6 were required, look in the Table for the Logarithm of 576, and prefix 1 for the Index, being the Index of the integral Part 57. Again, if the Logarithm of 8.19 be required, look in the Table for the Logarithm of 819, as though it were an integral Number, and you'll find .913284; to which prefixing 0, the Index for 8, (being under 10) 0.913284 is the true Logarithm of 8.19, the given Number.

To find the Logarithm of a Number conſiſting of five Places.

Rule. Firſt find the Logarithm for the firſt four Figures without the Index, as before taught; then take the Difference in the laſt Column under D; multiply this Difference by the laſt Figure of the given Number, and divide the Product by 10, and the Quotient, added to the before-found Logarithm, will be the true Logarithm of the given Number, when the proper Index is prefixed to it. If it were required to find the Logarithm of 35274, look for the Logarithm of 3527, in the Manner before taught, and you will have .547406, the Logarithm of 3527, without the Index; but, for the Logarithm of 35274, take the Difference in the laſt Column marked D, where you will find 123, which is the Difference betwixt the Logarithm of 3527 and 3528; multiply this Difference by 4, divide the Product by 10, add the Quotient to the before-found Logarithm, and prefix the proper Index 4, you will have 4.547455, for the true Logarithm of 35274.

Log. of 3527 is .547406   Difference 123
Part proportional add   49   4
.547455   49|2

To find the Logarithm for 32716: The Logarithm of 3271, without the Index, is .514681, and the Difference in the laſt Column under D, is 132, which multiplied by 6, and the Product divided by 10, the Quotient, added to the Logarithm of 3271, gives the Logarithm of 32716, without the Index, as follows.:

Log. of 3271 is .514681   Difference 132
79   6
.514760   79|2

Therefore the proper Index 4, being prefixed, gives 4.514760, for the true Logarithm of 32716, the given Number.

In the ſame Manner may a Logarithm be found for a Number of ſix Places, as 149635, by firſt finding the Logarithm of the firſt four Figures 1496, and the Difference in the laſt Column; this Difference multiplied by the two laſt Figures 35, and the Product divided by 100, by cutting off the two laſt Figures, the reſt added to the Logarithm of 1496, and the proper Index 5 prefixed, you have the true Logarithm of the given Number.

Log. of 1496 is .174932   Difference is 291
Parts proportional add 102   35
Log. of 149635 is 5.175034   1455
873
101|85

To find the Number to a given Logarithm.

Look in the ſecond Column of one of the left-hand Pages of the Table for the Logarithm, without the Index; if you find it there exactly, then the Number which ſtands againſt it in the firſt Column, is the Number ſought: If it is not to be found exactly in any of theſe Columns, look for the next leſs Logarithm, and caſt your Eye ſtraight along that Line to the other Columns; if it be found in any of theſe, the Figure at the Top of the Column in which it is found muſt be joined to the other three, and theſe four Figures are the Number ſought, the Value of which will be determined by the Index.

Exam. 1. To find the Number to the Logarithm 0.796574.	*Exam.* 2. To find the Number to the Logarithm 3.532409.
Anſwer 6.26.	Anſwer 3408.

Exam. 1. I find the given Logarithm in the ſecond Column of one of the left-hand Pages, and againſt it the Number 626; but as the Index of the Logarithm is 0, I thence know, that the Number, to which it belongs, conſiſts of but one Place of Integers, or is under 10; therefore I place a Dot between the firſt Figure 6, and the other two, marking them off for Decimals, ſo that 6.26 is the Number ſought.

Exam. 2. Not finding the Logarithm given in the ſecond Column of any of the left-hand Pages, I obſerve the next leſs is .531479; from which looking along the Line, I find my Logarithm exactly in the laſt Column of Logarithms but one of the right-hand Page; and at the Top of that Column I ſee 8, which muſt be annexed to the Figures in the firſt Column of the left-hand Page, againſt the Logarithm firſt found, where we have 340; to which annexing 8, it makes 3408, for the Number to the Logarithm 3.532499 given.

Here it may be obſerved, that had the Index been either more or leſs than 3, the Number muſt have conſiſted of a greater or a leſs Number of integral Figures: Thus, if the Index had been 4, there muſt have been five integral Figures, by annexing a Cypher to thoſe found; but had the Index been 2, the 8 muſt have been marked off by a Dot for a Decimal; and in the ſame Manner the Numbers are found to the following Logarithms:

0.759668

0.759668	is	5.75.
0.991669		9.81.
1.060698		11.5.
1.294466		19.7.
1.397940		25.6.
2.392697		247.

Where the Reader may obſerve, that, when the Index is 0, then the Number ſought has only one Place of Integers; if the Index is 1, then the Number ſought has two Places of Integers; if the Index is 2, the Number ſought has three Places of Integers.

If the given Logarithm is neither to be found in the ſecond Column of the left-hand Pages, nor in either of the other nine Columns, take the next leſs, and ſubtract it from the given Logarithm; if the Difference be conſiderable, annex 9 to it, and divide this Sum by the Difference in the laſt Column under D, and the Quotient will be the fifth Figure, to be annexed to the four already found.

Exam. 1. To find the Number to the Logarithm 4.571090
The next leſs Log. (whoſe Number is 3724) .571009

Difference — — — 81

To 81 annexing 9, it is 819.

Tab. Diff. 117) 819 (7, which annexed to 3724, before
819 (found, gives 37247 Anſwer.

Exam. 2. To find the Number to the Log. 2.694210
The next leſs Log. (whoſe Number is 4945) 694166

44

To 44 annexing 9, it is 449.

Tab. Diff. 88 (449 (5, which joined to 4945, the Number
440 (before found, the Anſ. is 49455.

To find a Number conſiſting of ſix Places to a given Logarithm.

Rule. Firſt find the next leſs Logarithm in the Table, as alſo the Difference in the laſt Column under D; then take out the four Figures belonging to the Logarithm in the Table, and, in order to find the two Figures to be annexed to theſe to complete the Number, ſubtract the Logarithm found in the Table from the given Logarithm; to this Difference annex 99, and divide it by the Difference found in the laſt Column under D, and the Quotient will be the Figures to be annexed to the four before found.

But it is here to be obſerved, that in theſe Tables, the decimal Part of the Logarithms conſiſting of but ſix Places, a Number of ſix Places cannot be found ſufficiently exact, but only in the former Part of the Table, before the Logarithm of 4000, becauſe the Differences are afterwards too ſmall to calculate Numbers to ſix Places.

To find a Number to ſix Places to the Logarithm 5.454306: The next leſs Logarithm is .454236, whoſe Number is 2846, and the tabular Difference 153.

The given Log. is .454306 Tabular Diff. 153 (7199 (47
The next leſs .454235 612

The Difference is 71 1079
1071

The Quotient 47, being annexed to the four Figures found in the Table, gives 284647, for the true Number to the Logarithm 5.454306: The Index being 5, ſhews the whole to be Integers.

To find a Number to ſix Places to the Logarithm 3.241877: The next leſs Logarithm in the Table is .241795, whoſe Number is 1745, and the Difference in the laſt Column is 249.

The given Log. is .241877 Tabular Diff. 249) 8299 (33
The next leſs .241795 747

82 829
747

The Quotient 33, annexed to 1745, makes 174533, for the Number to the given Logarithm: But, as the Index is but 3, there can be no more than four Places of Integers; ſo that 1745.33 is the true Number to the given Logarithm.

The Deſcription and Uſe of the Table of Sines and Tangents.

The Table of Sines and Tangents has the 90 Degrees of the Quadrant, diſpoſed in the following Manner: At the Top of the firſt two Pages is placed o Degree, at the Top of the two next 1 Degree, and ſo proceeding on to 44 Degrees at the Top of the two laſt Pages; and then beginning with 45 Degrees at the Bottom of the two laſt Pages, and proceeding back again to 89 Degrees at the Bottom of the two firſt Pages.

The Minutes are contained in the firſt and laſt Column of each Page, marked with the Letter M, for Minutes: In the firſt Column to the left Hand of each Page, the Minutes increaſe as they deſcend; on the left-hand Page, from 0 to 30, and on the right-hand Page, from 30 to 60: But in the laſt Column of each Page, the Minutes increaſe as they aſcend; on the right-hand Page, from 0 to 30, and on the left-hand Page, from 30 to 60: Between theſe Columns, in each Page, are four other Columns, marked at the Top, *Sine*, *Co-ſine*, *Tangent*, *Co-tangent*, and at the Bottom, *Co-ſine*, *Sine*, *Co-tangent*, *Tangent*.

The Degrees, Minutes, Seconds, *&c.* are uſually wrote in this Manner, 26° . 15′ . 30″, which are to be read, 26 Degrees, 15 Minutes, 30 Seconds.

To find the Logarithmic Sine of 23° . 21′.

Find the 23° on the Top of the Table, and in the Column to the left Hand, marked M, the Numbers beginning at 0, and increaſing as they deſcend, thus, 0, 1, 2, 3, to 30, find the 21′; againſt which, in the Column over which is the Word *Sine*, you have, for the Sine of 23° . 21′, the Logarithm 9.598075.

In the ſame Manner, the Sine of	19° . 12′	is found to be	9.517020.
	34 . 16		9.750543.
	41 . 24		9.820406.

To find the Sine of 24° . 41′: Having found the 24° at the Top of the Table, as before, the Minutes, being above 30, muſt be ſought in the firſt Column of the right-hand Page, marked M, where the Numbers, beginning at 30, increaſe as they deſcend thus, 30, 31, 32, to 60; amongſt which ſeek for 41, and right againſt it, in the Column marked *Sine* at the Top, you have the Sine of 24° . 41′, 9.620763.

In the ſame Manner, the Sine of	9° . 58′	is found to be	9.238235.
	20 . 45		9.549360.
	35 . 36		9.765015.

But, when the Number of Degrees exceed 45, you muſt look for them at the Bottom of the Page, and for the Minutes in the laſt Column to the right Hand, marked M, where the Numbers increaſe as they aſcend: Thus, to find the Sine of 51° . 20′,

Find the 51° at the Bottom of the Page, and in the laſt Column to the right Hand, in the right-hand Page, find 20′; againſt which, in the Column having the Word *Sine* at the Bottom, you have 9.892536, for the Sine of 51° . 20′.

$$\text{In the ſame Manner, the Sine of}\left\{\begin{array}{l}68^\circ . 15'\\ 47 . 9\\ 56 . 24\end{array}\right\}\text{is found to be}\left\{\begin{array}{l}9.967927.\\ 9.865185.\\ 9.920604.\end{array}\right.$$

To find the Sine of 48° . 39′: Having found the 48° at the Bottom of the Table, as before, the Minutes, being more than 30, muſt be ſought in the laſt Column of the left-hand Page, where the Numbers, beginning at 30, increaſe as they aſcend, to 60 at the Top; amongſt which find the 39′, and againſt it, in the Column of the ſame Page marked *Sine* at the Bottom, you have 9.875459, for the Sine of 48° . 39′.

$$\text{In the ſame Manner, the Sine of}\left\{\begin{array}{l}51^\circ . 32'\\ 62 . 54\\ 79 . 46\end{array}\right\}\text{is found to be}\left\{\begin{array}{l}9.893745.\\ 9.949494.\\ 9.993036.\end{array}\right.$$

The Tangents may be found in the ſame Manner, to any Degree and Minute of the Quadrant; always obſerving, that, when the Degrees are found at the Top of the Page, the Minutes are to be ſought in the firſt Column to the left Hand, where the Numbers increaſe as they deſcend downward; and, when the Degrees are found at the Bottom of the Page, the Minutes muſt be ſought in the laſt Column, where the Numbers increaſe as they aſcend upwards.

The Co-ſines and Co-tangents are to be found in the ſame Manner, being no more than the Sines and Tangents of the Complement of the given Number of Degrees and Minutes to 90°, or a whole Quadrant.

Plain TRIGONOMETRY.

TRIGONOMETRY, or the Meaſuring of Triangles, is of two Kinds. The Meaſuring of right-lined Triangles, otherwiſe called plain Triangles, is called *Plain Trigonometry*: The Meaſuring of curve-lined Triangles, commonly called ſpherical Triangles, is called *Spherical Trigonometry*.

Definitions.

Definitions.

A plain Triangle is a Figure contained between three right Lines, which at their Meeting make three Angles.

If one of the three Angles of a Triangle be a right Angle, the Triangle is called a *right-angled* Triangle.

If neither of the Angles of a Triangle is a right Angle, the Triangle is called an *oblique-angled* Triangle.

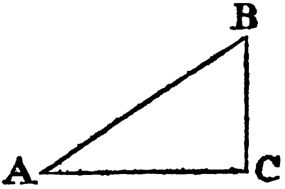
If, in an oblique-angled Triangle, one of the Angles is obtuſe, that is, greater than a right Angle, it is called an *obtuſe-angled* Triangle.

If all the three Angles are acute, that is, leſs than right Angles, then is the Triangle called an *acute angled* Triangle.

If all the three Sides of a Triangle are equal, it is called an *equilateral* Triangle.

If two of the Sides only are equal, it is called an *Iſoſceles.*

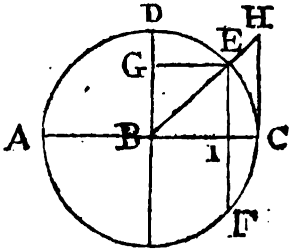
If the three Sides are all unequal, it is called *Scalenum.*



In a right-angled Triangle, as A B C, the longeſt Side, A B, is called the *Hypothenuſe*, and the other two Sides the *Legs*, of which, that on which the Triangle ſeems to ſtand, as A C, is called the *Baſe*, and the other Side, B C, the *Perpendicular.*

In an oblique-angled Triangle, the longeſt Side is uſually called the *Baſe.*

Angles are meaſured by the Arch of a Circle. The Periphery of every Circle, whether great or ſmall, is divided into 360 Degrees, each Degree into 60 Minutes, every Minute into 60 Seconds, and ſo on to Thirds, Fourths, &c.



Any Portion of the Periphery of a Circle, as E C F, is called an *Arch*; and a Line drawn from the Ends of an Arch, as E I F, is called the *Chord* of the Arch. Half the Chord of any Arch, as E I, is called the *Sine* of the Arch E C, and I C is called the *verſed Sine* of the ſame Arch E C; ſo alſo E G is the Sine of the Arch E D. A Line drawn perpendicular to the Diameter of a Circle, ſo as to touch the Circle, and not cut it, is called *a Tangent*, as [illegible]H, which is the Tangent of the Arch E C, becauſe the Line B H, drawn from the Center B through E, called the *Secant*, meets it in the Point H. Any Radius, as B D, is the Sine of 90°, uſually called the whole Sine.

The Deſcription and Uſe of a Scale of equal Parts.

The common plain Scales have on them a Line of equal Parts, marked L, divided into 10 equal Parts, marked 10, 20, 30, *&c.* to 100: At the Beginning of the Diviſions there is one other Part not numbered, which is ſubdivided into ten Parts by ſmall Diviſions; the middle one, being 5, is drawn longer than any of the reſt to diſtinguiſh it. The Uſe of this Line of Parts is, to lay any Thing down upon Paper, having the Dimenſions given: As ſuppoſe a Triangle was given, one Side being 21 Inches, another 28, and the third 35: To lay this down on Paper from the Line of Parts, you muſt draw a Line on your Paper; then, to ſet off the longeſt Side on the Line, ſet one Foot of the Compaſſes in the Diviſion marked 30 of the larger Diviſions, and extend the other backward on the Line to the 5 in the ſmall Diviſions; this Extent mark in your Line: Then, for the 28, ſet one Foot of the Compaſſes on the Diviſion marked 20, extending the other back to 8 on the ſmall Diviſions; with this Extent, ſet one Foot in the Mark made at the End of the Line, and with the other draw a Portion of an Arch; which done, ſet one Foot of the Compaſſes on the ſame Diviſion, marked 20, and contract the Compaſſes, till they reach to the firſt of the ſmall Diviſions; with this Extent, ſet one Foot in the other Mark made in your Line, and draw another Arch, croſſing the former: Laſtly, from the Interſection of theſe Arches, draw Lines to the two Marks made in the firſt Line; ſo will you have a Triangle in the ſame Proportion as was given. But, in Caſes where the Numbers are Hundreds, Tens, and odd Ones, there are on thoſe Scales uſually two diagonal Scales, one divided into Inches, the other into Half-Inches; theſe Scales are numbered with 1, 2, 3, *&c.* the Diviſions at the End are uſually numbered 2, 4, 6, 8 at the Side, there not being Room to put all the nine Digits, therefore 1, 3, 5, 7, 9, have no Numbers: There are likewiſe ten parallel Lines run the Length of the Scale, and at the End, among the ſmall Diviſions, there are diagonal Lines, croſſing from one of thoſe Parallels to another. To take off any Number from theſe Scales, ſuppoſe 768, look out the great Diviſion marked 7, and then look on the ſmaller Diviſions at the End for that marked 6;

the Compaſſes being extended from 7 in the greater Diviſions to 6 in the ſmall, is 760; but, for the 8, you muſt count eight of the Parallels, and ſet one Foot of the Compaſſes where the eighth Parallel croſſes the Line marked 7, before found, and extend the other Foot to where the Diagonal 6, before found, croſſes the ſame Parallel; that Extent is 768, the given Number.

The Deſcription and Uſe of a Line of Chords.

I have before obſerved, that all Circles are ſuppoſed to be divided into 360 Parts, called Degrees; therefore each Quadrant contains 90 Degrees, as appears by the Diameters cutting each other at right Angles: If then a Quadrant of a Circle be divided into 90 Degrees, and the Diſtance from one End of the Arch to each of theſe Degrees be transferred to a ſtraight Line, this is what we call a Line of Chords. The Radius of every Circle, being taken in the Compaſſes, will, if the Compaſſes be ſet in the Periphery and turned about, at ſix Times go round the Circumference; therefore the Radius is the Chord of 60 Degrees. On the plain Scale there are uſually two Lines of Chords to different Radii, marked at the End *Cho.* the Uſe of which is to meaſure Angles; for, if you take in your Compaſſes 60 Degrees from the Line of Chords, and, ſetting one Foot of the Compaſſes in the angular Point, draw an Arch from one Leg of the Angle to the other, and then take in the Compaſſes the Length (or rather the Chord) of the Arch, *that*, meaſured on the ſame Line of Chords, will ſhew the Degrees of the Angle, as has been already ſhewn, Prob. 8.

There are uſually on the Scale divers other Lines; as a Line of Sines, a Line of Tangents, a Line of Secants, a Line of Half-Tangents, &c. whoſe Uſe we ſhall have Occaſion to explain hereafter.

The great Uſe of Logarithms is in Trigonometrical Calculations. All the Caſes in Trigonometry being done by the Rule of Proportion, to avoid the Trouble of Multiplication and Diviſion, Mr. *Gunter*, as was obſerved above, calculated the Logarithms of the Sines and Tangents, by which all the Problems in Trigonometry are performed: So that, having alſo Tables of the Logarithms of Numbers, nothing more is neceſſary, but to add the Logarithms of the ſecond and third Numbers in the Proportion together, and from their Sum ſubtract the Logarithm of the firſt; the Remainder is the Logarithm of the fourth Number ſought; that is, it is the Logarithm of the Length of the Side, when a Side is required, or of the Sine of the Angle, when an Angle is required: And, when the firſt Number in the Proportion happens to be the Radius whoſe Logarithmic Sine is 10.000000, that is, 10. there is then no Occaſion for any other Subtraction, but only to reject 10 out of the Sum of the ſecond and third Numbers.

Of right-angled plain Trigonometry.

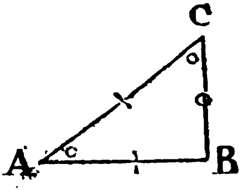
There are eight Caſes in right-angled plain Trigonometry, ſeven of which are ſolved by the following general Rule: The Method of ſolving the other will be explained when we come to it. The general Rule is this:

As the Sine of any Angle
Is to its oppoſite Side,
So is the Sine of any other Angle
To its oppoſite Side:

Or,

As the Length of any Side
Is to the Sine of the Angle oppoſite,
So is the Length of any other Side
To the Sine of the Angle oppoſite to it.

Prob. 18. *In the right-angled plain Triangle* A B C, *right-angled at* B, *given the Baſe* A B 47.5, *the Hypothenuſe* A C 59.4; *to find the Perpendicular* B C, *and the Angles at* A *and* C.



Conſtruction. Draw the Baſe-Line A B at Pleaſure; ſet one Foot of the Compaſſes on the Diviſion of the diagonal Scale at 4, and extend the other Foot to ſeven of the Diviſions at the End of the Scale; this is 47: Now, to take off the .5, take your Compaſſes from the Scale, and go down five of the Lines which are drawn parallel to the Sides of the Scale; place one Foot of the Compaſſes there, and you will obſerve the other will fall a little ſhort of the Line they before touched at the End of the Scale; extend the Compaſſes till they reach that Line; this Extent is 47.5; which place on the Baſe-Line from A to B, and at B erect B C perpendicular to A B, by Prob. 3, and continue it at Pleaſure. Now, for the Hypothenuſe, ſet one Foot of the Compaſſes on the diagonal Scale at 5, and extend the other Foot to nine of the Diviſions at the End of the Scale; this is 49; take your Compaſſes from the Scale, go down four of the Lines which are drawn parallel to the Sides of the Scale, and there place one Foot, and the other will fall a little ſhort of the Line it before touched at the End of the Scale; extend the Compaſſes 'till they touch that Line, and this Extent is 59.4: Now place one Foot in A, with the other croſs the Perpendicular B C in C, and draw the Hypothenuſe A C, which completes the Triangle A B C.

The given Parts are marked with —, and the Part required with o, whether they are Sides or Angles.

Now I obſerve, that the Hypothenuſe A C is given, and oppoſite to the right Angle at B, which, in a right-angled Triangle, is always known from the Triangle; and that the Baſe A B is given, and is oppoſite to the Angle at C, which is one of the Parts required to be found; which is done by the following Proportion according to the general Rule:

As the Hypothenuſe A C, 59.4,	——	1.773786
Is to the Radius, or Sine of the Angle at B,		10.
So is the Baſe A B, 47.5,	——	1.676694
		11.676694
		1.773786
To the Sine of the Angle at C, 53° . 6′,	—	9.902908

The ſecond and third Numbers being added together, and the firſt ſubtracted from their Sum, leaves 9.902908 for the Sine of the Angle at C; againſt which, among the Sines in the Tables of Sines and Tangents, you will find 53° . 6′, for the Angle at C; which Angle being meaſured by the Scale of Chords, by Prob. 9, if it is found to meaſure right, it proves that the Conſtruction and Calculation are both true.

The Angle at B is	90° . 00′
Subtract the Angle at C	53 . 06
Remains the Angle at A	36 . 54, which may be meaſured (in the ſame Manner as was the Angle at C.

Now I obſerve, that the Baſe A B is given, and oppoſite to the Angle C, which was found by the former Proportion; and that the Angle A is alſo found, being the Complement of the Angle C to 90°, and is oppoſite to the Perpendicular, the other Part required to be ſound; which is done by the following Proportion:

As the Sine of the Angle at C, 53°. 6', — 9.902919

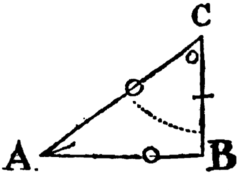
Is to the Baſe A B, 47.5, —— 1.676694
So is the Sine of the Angle at A, 36°. 54', 9.778455

11.455149
9.902919

To the Perpendicular B C, 35.66, — 1.552230

The ſecond and third Numbers being added together, and the firſt ſubtracted from their Sum, there remains 1.552230 for the Logarithm of the Perpendicular B C. Now, looking for this Logarithm in the Table of Logarithms, neglecting the Index 1, the neareſt Logarithm to it is .551400; againſt which, in the Column on the left Hand, I find 356; and, among the Figures at the Tops of the Columns, over that where my Logarithm is, I find 6, which, being placed to the right Hand of the 356, makes 3566 for the Number anſwering to the Logarithm in the Operation. But now, the Index of the Logarithm being only 1, therefore of the 3566 there muſt be only two Places Integers; whence I conclude 35.66 is the Length of the Perpendicular B C: And, to meaſure this from the diagonal Scale, place one Foot of the Compaſſes at A, and extend the other to B; which Extent will reach on the diagonal Scale from where the ſixth Parallel croſſes the Diviſion marked 3, to the fifth of the ſmall Diviſions at the End, and ſomewhat more, which ſhews it to be more than 35.6, as by the Calculation.

Prob. 19. *In the right-angled Triangle* A B C, *right-angled at* B, *given the Perpendicular* B C 35.66, *and the Angle at* A 36°. 54', *to find the Hypothenuſe* A C, *and the Baſe* A B.



Conſtruction. Draw the Baſe-Line A B at Pleaſure, on the End of which, by Prob. 3, erect the Perpendicular B C to any convenient Length: Set one Foot of the Compaſſes on the diagonal Scale at 3, and extend the other to 5 at the End of the Scale; take your Compaſſes from the Scale, and go down ſix of the parallel Lines, and extend them ſo as to touch the Point, where this Line, and the Line marked 5 at the End of the Scale, interſect; this is 35.6, which ſet off on the Perpendicular from B to C: Then, as the Angle at A is 36°.54′, the Angle at C, being the Complement of it to 90°, is 53°.6′; take 60° from the Line of Chords, and ſet one Foot of the Compaſſes in C, and draw a Portion of an Arch; then take 53°.6′ from the Line of Chords, and, ſetting one Foot of the Compaſſes where the Arch cuts the Perpendicular, with the other interſect the Arch: A Line drawn from C, thro' the Interſection, 'till it cuts the Baſe in A, completes the Triangle.

Here we may obſerve, that B C is given, and oppoſite to the given Angle at A; and that the Angle B is always known in a right-angled Triangle; and that this is oppoſite to the Hypothenuſe A C, one of the Parts required; which may be found by the following Proportion:

As the Sine of the Angle at A, 36°.54′, —	9.778455
Is to the Perpendicular B C, 35.66, —	1.552241
So is the Radius, or Sine of the Angle at B,	10.
	11.552241
	9.778455
To the Hypothenuſe A C, 59.4, —	1.773786

To meaſure the Hypothenuſe, put one Foot of the Compaſſes on the diagonal Scale at 5, and extend the other to nine of the Diviſions at the End of the Scale; take the Compaſſes from the Scale, go down four of the parallel Lines, and extend them to the Point where this Line croſſes the ninth Diagonal at the End of the Scale, and this will be the Meaſure of the Hypothenuſe, if the Work is true.

Here we are to obſerve, that the Angle A is known, and oppoſite to the Perpendicular, which was given; and that the Angle C is known, and oppoſite to the Baſe A B, the other Part required; which may be found by the following Proportion:

As the Sine of the Angle at A, 36°. 54',	9.778455
Is to the Perpendicular B C, 35.66, —	1.552241
So is the Sine of the Angle at C, 53°. 6',	9.902919
	11.455160
	9.778455
To the Baſe A B, 47.5 ———	1.676705

To meaſure the Baſe A B, ſet one Foot of the Compaſſes on the diagonal Scale at 4, and extend the other to the ſeventh Diagonal at the End; take the Compaſſes from the Scale, go down to the fifth Parallel, and place one Foot there, and extend the other to the Point where the fifth Parallel croſſes the ſeventh Diagonal at the End of the Scale; this is the Meaſure of the Baſe. And here the Reader may obſerve the following

General Rule for taking off any Number from the diagonal Scale.

If it conſiſts of two Places, put one Foot of the Compaſſes on the Scale, to that Number which ſtands firſt to the left Hand in the given Number, and extend the other Foot to the other Number at the End of the Scale; this is the Extent required.

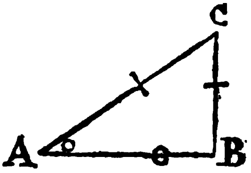
Example. To take 78 from the Scale, ſet one Foot of the Compaſſes to 7, and extend the other to the Diagonal marked 8 at the End of the Scale; this Extent is 78, the given Number.

If the Number conſiſts of three Places, ſet one Foot of the Compaſſes on the Scale, to the firſt Figure to the left Hand, and extend the other Foot to the ſecond Figure of the given Number at the End of the Scale, as before; take the Compaſſes from the Scale, and go down the parallel Lines, 'till you come to the Line that has the ſame Figure againſt it at the End with the laſt Figure to the right Hand in your given Number; here place one Foot of the Compaſſes, and extend the other to where this Parallel croſſes the Diagonal at the End of the Scale which is marked with the ſecond Figure of your given Number; this is the Extent required.

Example. To take 946 from the Scale, ſet one Foot of the Compaſſes to 9, and extend the other to the Diagonal marked 4 on the Edge at the End of the Line; take the Compaſſes from the Scale, and go down to the Parallel marked 6 at the End; fix one Foot of the Compaſſes on this Parallel, and extend the other to where the Diagonal marked 4 croſſes the Parallel; this is the true Extent for the given Number.

Here it is to be obſerved, that both the Diagonals and Parallels, for Want of Room, have only the Numbers 2, 4, 6, 8; ſo that 1, 3, 5, 7, and 9, are not marked.

Prob. 20. *In the right-angled Triangle* A B C, *right-angled at* B, *given the Hypothenuſe* A C 59.4, *and the Perpendicular* B C 35.66, *to find the Baſe* A B, *and the Angles at* A *and* C.



Conſtruction. Draw the Baſe-Line A B at Pleaſure, and on the Point B erect the Perpendicular B C, of a proper Length; then take the given Length 35.6 from the Scale, by the general Rule, Prob. 19, and ſet it from B to C: Take, by the ſame Rule, the Hypothenuſe 59.4 from the Scale, and, with one Foot of the Compaſſes in C, interſect the Baſe-Line A B in A; then draw the Line A C, and the Triangle is completed.

Here the Hypothenuſe is given, and oppoſite the right Angle; and the Perpendicular B C is alſo given, which is oppoſite the Angle A, one of the Parts required; which may be found by the following Proportion:

As the Hypothenuſe A C, 59.4, ——	1.773786
Is to the Radius, or the Sine of the Angle at B,	10.
So is the Perpendicular B C, 35.66, —	1.552241
	11.552241
	1.773786
To the Sine of the Angle at A, 36°. 54′,	9.778455

The Angle at A may be meaſured by Prob. 9.

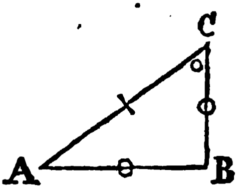
Now, the Angle at A being found, and oppoſite to the given Perpendicular B C, the Baſe A B, oppoſite to the Angle

C 53° . 6′, the Complement of A, may be found by the following Proportion, as in the laſt Problem:

As the Sine of the Angle at A, 36° . 54′, —	9.778455
Is to the Perpendicular B C, 35.66, —	1.552241
So is the Sine of the Angle at C, 53° . 6′,	9.902919
	11.455160
	9.778455
To the Baſe A B, 47.5, ———	1.676705

Which may be meaſured by the general Rule, Prob. 19.

Prob. 21. *In the right-angled Triangle* A B C, *right-angled at* B, *given the Hypothenuſe* A C 59.4, *and the Angle at* A 36° . 54′, *to find the Baſe* A B, *and the Perpendicular* B C.



Conſtruction. Draw a Baſe-Line at Pleaſure, as before, and, from the Point A, ſet off the given Angle 36° . 54′ from the Line of Chords, by Prob. 8; draw the Hypothenuſe A C to a convenient Length, and, from the diagonal Scale, by the general Rule, Prob. 19, take 59.4 in the Compaſſes, and ſet it from A to C; from the Point C let fall a Perpendicular to the Line A B, by Prob. 4, Caſe 2, and the Triangle is completed.

Here the right Angle, which is always known, is oppoſite the given Hypothenuſe; and the Angle at A is given, and oppoſite the Perpendicular, one of the Parts required; which may be found by the following Proportion:

As the Radius, or Sine of the Angle at B,	10.
Is to the Hypothenuſe A C, 59.4, ——	1.773786
So is the Sine of the Angle at A, 36° . 54′,	9.778455
To the Perpendicular B C, 35.66, ——	1.552241

Which may be meaſured by the general Rule, Prob. 19.

90° . 00′

90° . 00′
36 . 54 Angle at A.

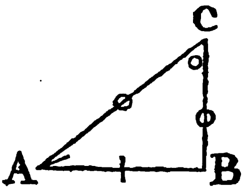
53 . 6 Angle at C.

In this Proportion, the firſt Term, being the Radius, is ſubtracted, by omitting ten in the Index of the Sum of the ſecond and third Terms. And, for finding the Baſe, having the right Angle oppoſite the Hypothenuſe, and the Angle at C 53°. 6′ oppoſite the Baſe A B, the other Part required, uſe the following Proportion:

As the Radius, or Sine of the Angle at B,	10.
Is to the Hypothenuſe A C, 59.4, —	1.773786
So is the Sine of the Angle at C, 53°. 6′, —	9.902919
To the Baſe A B, 47.5, —	1.676705

Which may be meaſured by the general Rule, Prob. 19. The Radius, being here the firſt Term, is deducted, as in the laſt Proportion.

Prob. 22. *In the right-angled Triangle* A B C, *right-angled at* B, *given the Baſe* A B 47.5, *and the Angle at* A 36°. 54′, *to find the Hypothenuſe* A C, *and the Perpendicular* B C.



Conſtruction. Draw a Baſe-Line A B at Pleaſure, and from the Scale, by the general Rule, Prob. 19, take 47.5, which ſet from A to B; then, by Prob. 8, make an Angle of 36°. 54′ at A, thro' which draw the Hypothenuſe A C to a convenient Length; laſtly, raiſe a Perpendicular on the Point B, by Prob. 3, to interſect the Hypothenuſe in the Point C, and the Triangle is completed.

90° . 00′
36 . 54 Angle at A.

53 . 6 Angle at C.

The Angle at C is oppoſite the given Baſe, and the right Angle, which is always known, is oppoſite the Hypothenuſe A C, one of the Parts required; whence, As the Sine of the Angle at C, 53° . 6′, — 9.902919

Is to the Baſe A B, 47.5, ——— 1.676694
So is the Radius, or Sine of the Angle at B, 10.

11.676694
9.902919

To the Hypothenuſe A C, 59.4, —— 1.773775

Now, we have the ſame Angle C oppoſite the given Baſe as before, and the given Angle A oppoſite the Perpendicular B C, the other Part required; therefore,

As the Sine of the Angle at C, 53° . 6′, 9.902919

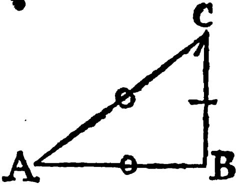
Is to the Baſe A B, 47.5, ——— 1.676694
So is the Sine of the Angle at A, 36° . 54′, 9.778455

11.455149
9.902919

To the Perpendicular B C, 35.66, —— 1.552230

The Hypothenuſe and Perpendicular may be meaſured by the general Rule, Prob. 19.

Prob. 23. *In the right-angled Triangle* A B C, *right-angled at* B, *given the Perpendicular* B C 35.66, *and the Angle at* C 53° . 6′, *to find the Hypothenuſe* A C, *and Baſe* A B.



Conſtruction. Draw a Baſe-Line at Pleaſure, and on the Point B erect a Perpendicular, by Prob. 3; from the diagonal Scale, by the general Rule, Prob. 19, ſet off 35.66 from B to C; then, by Prob. 8, make an Angle at C of 53° . 6′, thro' which draw the Hypothenuſe A C, 'till it interſects the Baſe A B in the Point A, and the Triangle is completed.

90° . 00′
53 . 6 Angle at C.

36 . 54 Angle at A.

The Angle at A is oppoſite the given Perpendicular B C, and the Angle at B is oppoſite to the Hypothenuſe A C, one of the Parts required; therefore,

As the Sine of the Angle at A, 36° . 54′,	9.778455
Is to the Perpendicular B C, 35.66, —	1.552181
So is the Radius, or Sine of the Angle at B,	10.
	11.552181
	9.778455
To the Hypothenuſe A C, 59.4, ——	1.773726

The Hypothenuſe may be meaſured by the general Rule, Prob. 19.

Now we have the right Angle at B oppoſite the Hypothenuſe found by the former Proportion, and the Angle at C 53° . 6′; whence,

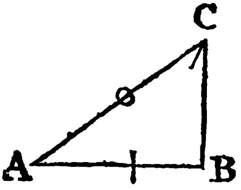
As the Radius, ——— ——	10.
Is to the Hypothenuſe, 59.4, ———	1.773786
So is the Sine of the Angle at C, 53° . 6′,	9.902919
To the Baſe A B, 47.5,. ———	1.676705

Which may be meaſured by the general Rule, Prob. 19. The firſt Term being the Radius, I omit ten in the Sum of the ſecond and third Numbers, as at Prob. 21.

Prob. 24. *In the right-angled Triangle* A B C, *right-angled at* B, *given the Baſe* A B 47.5, *and the Angle at* C 53° . 6′, *to find the Hypothenuſe* A C, *and Perpendicular* B C.

90° . 00′
53 . 6 Angle at C.

36 . 54 Angle at A.



Conſtruction. Draw the Baſe-Line A B at Pleaſure; take 47.5 from the Scale, by the general Rule, Prob. 19, and ſet from A to B; and on the Point B erect a Perpendicular, by Prob. 3, to any convenient Length; then make an Angle at A of 36° . 54′, the Complement of the given Angle C, by Prob. 8, and draw the Hypothenuſe A C 'till it intereſects the Perpendicular in C, and the Triangle is completed.

Here is given the Angle C, and the Baſe A B oppoſite to it; and the right Angle B, which is always known, is oppoſite the Hypothenuſe A C, one of the Parts required; whence,

As the Sine of the Angle at C, 53°. 6′, — 9.902919

Is to the Baſe A B, 47.5, ——— 1.676694
So is the Radius, or Sine of the Angle at B, 10.

11.976694
9.902919

To the Hypothenuſe A C, 59.4, —— 1.773775

Now we have the right Angle at B oppoſite the Hypothenuſe found by the laſt Proportion, and the Angle at A 36°. 54′ oppoſite the Perpendicular B C, the Part required; whence,

As the Radius, ——— ——— 10.

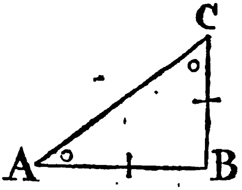
Is to the Hypothenuſe A C, 59.4, — 1.773786
So is the Sine of the Angle at A, 36°. 54′, 9.778455

To the Perpendicular B C, 35.66, —— 1.552241

Here the Radius being the firſt Term, ten is omitted in the Index of the Sum of the other two Terms, as at Prob. 21. And the Hypothenuſe and Perpendicular may be meaſured by the general Rule, Prob. 19.

Prob. 25. *In the right-angled Triangle* A B C, *right-angled at* B, *given the Baſe* A B 47.5, *and the Perpendicular* B C 35.66, *to find the Angles* A *and* C, *and the Hypothenuſe* A C.

Conſtruction. Draw a Baſe-Line A B at Pleaſure; take from the diagonal Scale 47.5, by the general Rule, Prob. 19, and ſet from A to B; at B erect a Perpendicular B C, on which, by the ſame Rule, lay 35.66 from B to C; then draw the Hypothenuſe A C, and the Triangle is completed.



In this Caſe the Learner will obſerve, that there are no two oppoſite Parts known; the Hypothenuſe A C, oppoſite the right Angle at B, being unknown, as are the two Angles A and C, oppoſite the given Sides: Hence this can't be ſolved by the general Rule, Page 212; but may be ſolved by the following Proportion:

As the Baſe A B, 47.5, ——	1.676694
Is to the Radius, —— ——	10.
So is the Perpendicular B C, 35.66, —	1.552181
	11.552181
	1.676694
To the Tangent of the Angle at A, 36°. 54′,	9.875487

Having found the Angle at A oppoſite the given Perpendicular, we have the right Angle at B oppoſite the Hypothenuſe, the other Part required; which may be found as in Prob. 23; thus,

As the Sine of the Angle at A, 36°. 54′,	9.778455
Is to the Side B C, 35.66, ——	1.552181
So is the Radius, or Sine of the Angle at B,	10.
	11.552181
	9.778455
To the Hypothenuſe A C, 59.4, ——	1.773726

The Hypothenuſe may be meaſured by the general Rule, Prob. 19. And, ſubtracting 36°. 54′, the Angle at A, from 90°, it leaves 53°. 6′ for the Angle at C, both which may be meaſured by Prob. 9.

Oblique Plain Trigonometry.

The five Caſes of oblique-angled Triangles are ſolved by the three following Rules.

When the Sides and Angles are oppoſite:

Rule 1. As in right-angled Triangles.

As the Sine of any Angle
Is to its oppoſite Side,
So is the Sine of any other Angle
To its oppoſite Side:

Or,

As any one Side
Is to the Sine of its oppoſite Angle,
So is any other Side
To the Sine of its oppoſite Angle.

Rule 2. When any two Sides with the Angle between them are given.

As the Sum of any two Sides
Is to their Difference,
So is the Tangent of the Half-Sum of the two oppoſite Angles
To the Tangent of Half the Difference of thoſe two Angles.

Which Half-Difference, being adeed to the Half-Sum, gives the greater of the two Angles; and, being ſubtracted from the Half-Sum, leaves the leſs of the two unknown Angles.

Rule 3. When the three Sides are given to find the Angles.

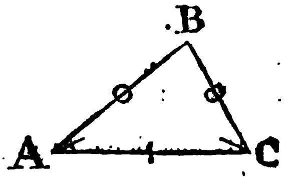
As the Baſe of any Triangle
Is to the Sum of the other two Sides,
So is the Difference of thoſe Sides
To the Difference of the two Segments of the Baſe, made by letting fall a Perpendicular to the Baſe from the Angle oppoſite to it.

The Learner being now a little uſed to this Sort of Proportions, it will be proper to ſhew him how to perform them without ſubtracting the firſt Number from the Sum of the ſecond and third; which is done by ſetting down the arithmetical Complement of the firſt Term inſtead of the Logarithm.

This may be readily done thus: Subtract the firſt Figure of the Logarithm to the right Hand from 10, and ſet down the Remainder; then ſubtract each of the other Figures, Index and all, from 9, ſetting down the Remainders, and place a Dot before the Index, as in the Caſe of the Logarithm. Thus the arithmetical Complement (uſually marked Co. Ar.) of the Logarithm 9.778455 is 0.221545; and ſo of any other. When the arithmetical Complement of the firſt Term is uſed inſtead of the Logarithm, add all the three Numbers together, and reject ten out of the Index of their Sum, as in thoſe Caſes where the Radius is the firſt Term.

Prob. 26. *In the oblique-angled Triangle* A B C, *given the Angle* A 36°. 40′, *the Angle at* C 60°. 51′, *and the Baſe* A C 85.6, *to find the other two Sides.*

Conſtruction. Draw the Baſe A C at Pleaſure, and ſet off from the diagonal Scale, by the general Rule, Prob. 19, the given Length 85.6, from A to C; then, by Prob. 8, make an Angle of 36°. 40′ at A, and an Angle of 60°. 51′ at C, and the Meeting of theſe two Lines in B completes the Triangle.



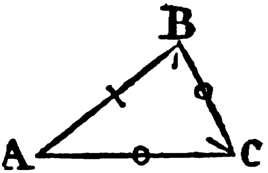
When two Angles of any Triangle are given, their Sum, being ſubtracted from 180°, leaves the third Angle. Therefore, in this Caſe, 97°. 31′, being ſubtracted from 180°, leaves 82°. 29′ for the Angle at B: Hence we have the Angle at B oppoſite the given Baſe, and the Angles at A and C oppoſite the two required Sides; which may be found by the firſt Rule, as follows:

As the Sine of the Angle at B, 82°. 29′, Co. Ar. 0.003748
Is to the Baſe A C, 85.6, —— 1.932474
So is the Sine of the Angle at A, 36°. 40′, — 9.776090
To the Side B C, 51.55, —— —— 1.712312

As the Sine of the Angle at B, 82°. 29′, Co. Ar. 0.003748
Is to the Baſe A C, 85.6, —— —— 1.932474
So is the Sine of the Angle at C, 60°. 51′, — 9.941187
To the Side A B, 75.4, —— —— 1.877409

The Sides B C and A B may be meaſured by the general Rule, Prob. 19.

But in this Caſe, of two Angles and the included Side given, if the Included Side is not the Baſe, it will make ſome Difference in the Conſtruction; therefore, in the Triangle ABC, the Angle at A 36°. 40′, the Angle at B 80°. 29′, and the Side AB 75.4 given, to find the other two Sides.



Conſtruction. Draw the Baſe AC at Pleaſure, and on any Point aſſumed, as A, make an Angle of 36°. 48′, by Prob. 8; take 75.4 from the diagonal Scale, by the general Rule, Prob. 19, in the Compaſſes, and ſet it from A to B; then, by the ſame Prob. 8, make an Angle of 82°. 29′ at B, and draw BC 'till it interſects the Baſe in C, and the Triangle is completed.

Here we have two Angles given, whoſe Sum, being ſubtracted from 180°, gives 60°. 51′ for the Angle at C; by which the unknown Sides may be found by the firſt Rule, as follows:

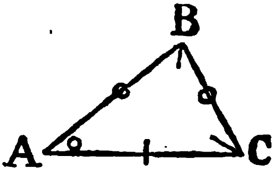
As the Sine of the Angle at C, 60°. 51′, Co. Ar. 0.058813
Is to the Side AB, 75.4, ——— 1.877371
So is the Sine of the Angle at A, 36°. 40′, — 9.776090
To the Side BC, 51.55, ——— 1.712274

And,

As the Sine of the Angle at C, 60°. 51′, Co. Ar. 0.058813
Is to the Side AB, 75.4, —— —— 1.877371
So is the Sine of the Angle at B, 82°. 29′, — 9.996252
To the Baſe AC, 85.6, —— —— 1.932436

The Side BC and Baſe AC may be meaſured by the general Rule, Prob. 19.

Prob. 27. *In the oblique-angled Triangle* ABC, *given the Baſe* AC 85.6, *the Angle at* B 82°. 29′, *and the Angle* C 60°. 51′, *to find the other two Sides.*



Conſtruction. Draw the Baſe AC, and from the diagonal Scale ſet off 85.6 from A to C; at C, by Prob. 8, make an Angle of 60°. 51′; then the Angle at A, being the Supplement of the Sum of the other two Angles to 180°, will be 36°. 40′; by the ſame Prob. 8, make an Angle of 36°. 40′

at A, and draw the Line A B 'till it interſects C B in the Point B, and the Triangle is completed,

Here having the Angle B oppoſite the known Side A C, and the Angles A and C oppoſite the required Sides, they may be found by the firſt Rule, as follows:

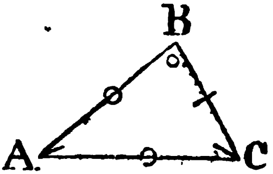
As the Sine of the Angle at B, 82°. 29′,	Co. Ar.	0.003748
Is to the Baſe A C, 85.6,	———	1.932474
So is the Sine of the Angle at A, 36°. 40′,	—	9.776090
To the Side B C, 51.55,	———	1.712312

And,

As the Sine of the Angle at B, 82°. 29′,	Co. Ar.	0.003748
Is to the Baſe A C, 85.6,	———	1.932474
So is the Sine of the Angle at C, 60°. 51′,	—	9.941187
To the Side A B, 75.4,	——— ———	1.877409

The Sides B C and A B may be meaſured by the general Rule, Prob. 19.

In this Caſe, of two Angles and an oppoſite Side given, if the given Side is not the Baſe, the Conſtruction will be ſomewhat different; therefore, in the oblique-angled Triangle A B C, given the Side B C 51.56, the Angle at A 36°. 40′, and the Angle at C 60°. 51′, to find the other Sides.



Conſtruction. Draw the Baſe A C at Pleaſure, and on any Point aſſumed, as C, by Prob. 8, make an Angle of 60°. 51′; by the general Rule, Prob. 19, ſet off 51.56 from C to B; at B, by the ſame Prob. 8, make an Angle of 82°. 29′, the Supplement of the two given Angles to 180°, and draw B A 'till interſects the Baſe in A, and the Triangle is completed.

Here we have the Angle A oppoſite the known Side B C, the Angle C oppoſite the Side A B, and the Angle B, the Supplement of the other two to 180°, oppoſite the Baſe A C, the other unknown Side; which may be both found, as in the former Example, by the following Proportion:

As the Sine of the Angle at A, 36°. 40′, Co. Ar. 0.223909
Is to the Side B C, 51.55, —— . 1.712229
So is the Angle at C, 60°. 51′, —— 9.941187

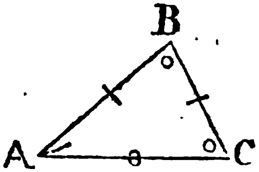
To the Side A B, 75.4, —— 1.877325

And,
As the Sine of the Angle at A, 36°. 40′, Co. Ar. 0.223909
Is to the Side B C, 51.55, —— 1.712229
So is the Sine of the Angle at B, 82°. 29′, — 9.996252

To the Baſe A C, 85.6, —— —— 1.932390

The Side A B and Baſe A C may be meaſured by the general Rule, Prob. 19.

Prob. 28. *In the oblique-angled Triangle* A B C, *given the Side* B C 51.56, *the Side* A B 75.4, *and the Angle at* A 36°. 40′, *to find the Baſe* A C, *and the Angles at* B *and* C.



Conſtruction. Draw the Baſe A C at Pleaſure, and on any Point aſſumed, as A, by Prob. 8, make an Angle of 36°. 40′; take 75.4 from the Scale, by the general Rule, Prob. 19, and ſet from A to B; then take 51.56 from the Scale, and ſet one Foot of the Compaſſes in B, and with the other interſect the Baſe in C; laſtly, draw B C, and the Triangle is completed.

Here we have the Side B C oppoſite the known Angle at A, and the Side A B oppoſite the unknown Angle at C, which may be ſound by the following Proportion, according to the firſt Rule:

As the Side B C, 51.55, —— Co. Ar. 8.287770
Is to the Sine of the Angle at A, 36°. 40′, — 9.776090
So is the Side A B, 75.4, —— 1.877371

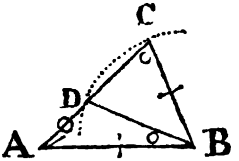
To the Sine of the Angle at C, 60°. 51′, — 9.941231

Having found the Angle at C, the Angle at B is found to be 82°. 29′, by ſubtracting the Sum of the other two Angles from 180°, both which may be meaſured by Prob. 9; then, for the Baſe A C,

As the Sine of the Angle at A, 36°. 40′,	Co. Ar.	0.223910
Is to the Side B C, 51.55,	———	1.712229
So is the Sine of the Angle at B, 82°. 29′,	—	9.996252
To the Baſe A C, 85.6,	———	1.932391

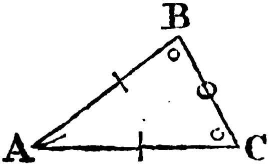
The Baſe A C may be meaſured by the general Rule, Prob. 19.

But here it is to be obſerved, that this Caſe, where two Sides and an Angle oppoſite one of them are given, is an *ambiguous Caſe*, and admits of two Solutions, unleſs, as in theſe Examples, the Side oppoſite the given Angle is longer than the other given Side: This will appear by the Conſtruction: For, if, in the Triangle A B C, the Baſe A B 45.3, the Side B C 34.6, and the Angle at A 40°. 20′, were given to find the Side A C, and the two Angles B and C, it admits of two Solutions.



Conſtruction. Draw the Baſe A B, and ſet off 45.3, by the Scale, from A to B; then, by Prob. 8, make an Angle at A of 40°. 20′, and draw the Line A C to a convenient Length; take 34.6 from the Scale, by the general Rule, Prob. 19, in the Compaſſes, and ſet one Foot in B, and with the other interſect the Line A C, which it will do both in C and D; whence it becomes uncertain whether the Side A D or A C is the true Solution, which can only be determined by the Circumſtances of the Triangle: For, if the Angle at C was to be acute, it will be the Side A C; but, if the Angle was to be obtuſe, as the Angle D in the Triangle A D B, then A D would be the Side ſought.

Prob. 29. *In the oblique-angled Triangle* A B C, *given the Side* A B 75.4, *the Baſe* A C 85.6, *and the included Angle at* A, *to find the other two Angles* B *and* C, *and the Side* B C.



Conſtruction. Draw the Baſe-Line A C at Pleaſure, and from the diagonal Scale ſet off 85.6 from A to C; make an Angle at A of 36°.40′, by Prob. 8, and draw A B; then take 75.4 from the Scale, and ſet it from A to B; laſtly, draw the Line B C, and the Triangle is completed.

Here we have given the two Sides A C and A B, with the Angle included between them; and, therefore, this Caſe cannot be done by the firſt general Rule, as neither of the Sides is oppoſite the given Angle; but muſt be ſolved by the ſecond Rule, Page 224. Now, as the three Angles of every Triangle are equal to two right Angles, or 180°, the Angle at A 36°.40′, being ſubtracted from 180°, leaves 143°.20′, the Sum of the two unknown Angles B and C, Half of which is 71°.40′; and Half their Difference may be found by the following Proportion, according to the ſaid ſecond Rule:

As the Sum of the two Sides, 161, Co. Ar. 7.793174
Is to their Difference, 10.2, —— 1.008600
So is the Tangent of the Half-Sum of the unknown Angles, 71°.40′, —— 10.479695

To the Tangent of Half their Difference, 10°.1′, 9.281469

If to the Half-Sum of the unknown Angles B and C Half their Difference is added, the Sum is the greater Angle; and, if from the Half-Sum the Half-Difference is ſubtracted, the Remainder is the leſſer of the two unknown Angles:

71° . 40′ Half-Sum.
10 . 49 Half-Difference.

82 . 29 greater Angle.

60 . 51 leſſer Angle.

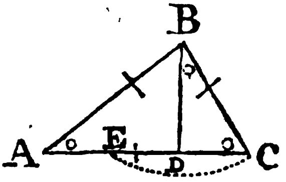
Now, becauſe the greateſt Side is always oppoſite to the greateſt Angle, the Angle at B muſt be greater than the Angle at C: And, having found the two Angles B and C, the Side B C may be found by the following Proportion, as at Prob. 27:

As the Sine of the Angle at B, 82°.29′, Co. Ar. 0.003748
Is to its oppoſite Side A C, 85.6, —— 1.932474
So is the Sine of the Angle at A, 36°.40′, — 9.941187

To its oppoſite Side B C, 51.56, —— 1.877409

Which may be meaſured by the general Rule, Prob. 19; and the Angles B and C may be meaſured by Prob. 9.

Prob. 30. *In the oblique-angled Triangle* A B C, *given the Side* A B 75.4, *the Side* B C 51.56, *and the Baſe* A C 85.6, *to find the Angles.*



Conſtruction. Draw the Baſe A C, and ſet off 85.6 from A to C; take 75.4 in the Compaſſes, and ſet one Foot in A, and with the other draw an Arch; then take 51.56 from the ſame Scale, and ſet one Foot of the Compaſſes in C, and with the other interſect the former Arch in B; from B draw Lines to A and C, and the Triangle is completed.

Here being no Angle given, this Problem muſt be ſolved by the third Rule, in the following Manner: Place one Foot of the Compaſſes in B, and extend the other ſo as to take in the ſhorteſt Side B C, and draw the Arch C E; from B let fall a Perpendicular on the Baſe A C, which will divide it into two Segments, C D the leſs, and A D the greater, whoſe Difference is A E; then by the following Proportion, according to the ſaid third Rule,

As the Baſe A C, 85.6,	——	Co. Ar. 8.067526
Is to the Sum of the two Sides, 126.96,	——	2.103667
So is the Difference of the Sides, 23.84,	——	1.377306
To the Difference of the Segments of the Baſe, or A E, 35.359,	—— ——	1.548499

Half the Difference of the Segments, being added to Half the Baſe, gives the greater Segment A D; and Half the Difference, being ſubtracted from Half the Baſe, gives the leſſer Segment C D:

42.8 Half the Baſe.
17.679 Half the Difference.

60.479 greater Segment A D.

25.121 leſſer Segment C D.

Or thus, The Baſe A C is — 85.6
Subtract A E —— 35.359

Remains E C —— 50.241

And as E D is equal to D C, therefore the Half of 50.24, or 25.12, is D C, the leſſer Segment: And as E D is the ſame as D C, that is, —— —— 25.12
If to that we add A D, or —— 35.359

The Sum is A D, the greater Segment, — 60.479 as before.

Thus is the oblique-angled Triangle A B C divided into two right-angled Triangles, A B D and C B D, both right-angled at D, in each of which are given the Baſe and Hypothenuſe to find the other Parts, which may be done in the following Manner: Firſt, for the Angle at B in the right-angled Triange A B D,

As the Hypothenuſe A B, 75.4, — Co. Ar. 8.122619
Is to the Radius, —— —— 10.
So is the Baſe A D, 60.48, —— 1.781612

To the Sine of the Angle at B, 53°.20′, — 9.904241

The Angle at A is 36°.40′, being the Complement of the Angle at B to 90°.

Then for the Angle at B in the right-angled Triangle CBD,

As the Hypothenuſe B C, 51.56, — Co. Ar. 8.287770
Is to the Radius, —— —— —— 10.
So is the Baſe D C, 25.12, —— 1.400020

To the Sine of the Angle at B, 29°.9′, — 9.687790

Whence the Angle at C is 60°.51′, the Complement of 29°.9′ to 90°; and the Angle at B in one Triangle, being added to the Angle at B in the other, is 82°.29′: The Angle at B in the oblique-angled Triangle A B C and the Angles at A and C being found before, the Solution of the Problem is finiſhed.

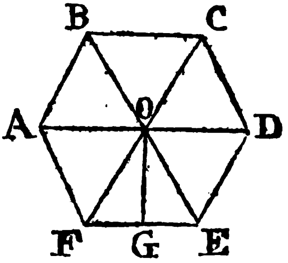
To prove the Truth of the Calculation, the Angles may be meaſured by Prob. 9.

Having, in this plain and eaſy Manner, explained to the Reader how to conſtruct the Triangles from the Parts given, and to find the required Parts by Calculation, I ſhall now ſhew the Uſe and Application of Trigonometry; and firſt in finding the Multipliers for computing the Areas of regular Polygons, as was promiſed in Art. 60.

Prob. 31. *To find the Area of any regular Polygon.*

Rule. Firſt find the Center of the Polygon, (which may readily be done by Prob. 17) and draw Lines from the Center to each Angle of the Polygon; thus will the given Polygon be divided into as many equal Triangles as there are Sides: A Perpendicular, being let fall from the Center to the Middle of one of the Sides, will divide the Triangle into two right-angled Triangles, in which will be given the Angles and one Side to find the other Side, which may be done by Prob. 19 or 24; and then the Area may be found by Art. 56.

Exam. 1. The Side of the Hexagon ABCDEF being 10, to find the Area?



Divide 360, the Degrees in a Circle, by the Number of Sides in the given Polygon, and the Quotient is the Angle at the Center of each Triangle, which in this Example is 60°, the Half of which is the Angle F O G 30°. In the right-angled Triangle F O G, right-angled at G, whoſe oppoſite Side F G is 5, being Half the Side F E, to find the Perpendicular G O, oppoſite the Angle at F 60°, being the Complement of the Angle at O to 90°.

As the Sine of the Angle at O, 30°, Co. Ar. 0.301030
Is to the Side F G, 5, —— 0.698970
So is the Sine of the Angle at F, 60°, —— 9.937531

To the Perpendicular G O, 8.66025, —— 0.937531

Having found the Perpendicular G O, if that is multiplied by the Side F G, the Product will be double the Area of the Triangle F O G, by Art. 56, that is, it will be the Area of the Triangle F O E, which, being multiplied by 6, gives the Area of the given Hexagon.

8.66025 the Length of G O.
5 the Length of F G.

43.30125 Area of the Triangle F O E.
6

259.80750 Area of the Hexagon.

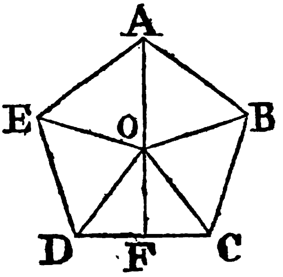
This is the Area of an Hexagon whoſe Side is 10; therefore, to find the Area of an Hexagon whoſe Side is 1, the Proportion muſt be, As the Square of 10 is to the Square of 1, ſo is the Area of the given Hexagon to the Area of an Hexagon whoſe Side is 1; ſince all ſimilar Figures are as the Squares of their Sides.

As 100 is to 1, ſo is 259.8075.

100) 259.8075 (2.598075 the Multiplier for finding the Area of an Hexagon in the Table, Art. 60.

Thus the Area of any regular Figure, whoſe Side is 1, being found, is a Multiplier, by which the Square of the Side of a like Figure being multiplied, the Product is the Area of the Figure ſought.

Exam. 2. The Side of the Pentagon A B C D E being 10, to find the Area?



For the Angles at the Center, 360° being divided by 5, the Number of Sides, give 72°, the Half of which is the Angle D O F 36°, whoſe Complement to 90° is the Angle O D F 54°; therefore, in the right-angled Triangle D O F, right-angled at F, are given the Side D F 5, and the Angle at D 54°, to find the Perpendicular F O, oppoſite the Angle at D: Whence,

As the Sine of the Angle at O, 36°, Co. Ar. 0.230781
Is to the Side D E, 5, ——— 0.698970
So is the Sine of the Angle at D, 54°, — 9.907958

To the Perpendicular F O, 6.88191, — 0.837709

Which, being multiplied by the Side D F 5, gives the Area of the Triangle D O C; and that, being multiplied by 5, the Number of Triangles, gives the Area of the Pentagon:

6.88191 Length of F O.
5 Length of D F.

34.40955 Area of the Triangle D O C.
5

172.04775 Area of the Pentagon.

Now, as 100, the Square of 10, the given Side, is to the Square of 1, ſo is the Area of the given Pentagon to the Area of a Pentagon whoſe Side is 1.

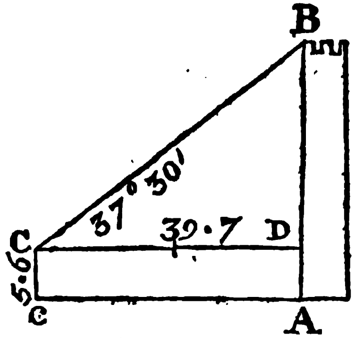
As 100 is to 172.04775.

100) 172.04775 (1.7204775 the Multiplier for the Pentagon in the Table, Art. 60.

In the next Place, I ſhall ſhew the Uſe of Trigonometry in meaſuring the Heights of Objects, or the Diſtance between them.

Prob. 32. *To meaſure the Height of the Tower* A B.

At any convenient Diſtance from the Tower A B, as at C, with a Quadrant take the Angle to the Top of the Tower, which let be 37°.30′; meaſure the Diſtance C D, which let be 39.7 Feet; and, when you have taken the Angle at C, meaſure how high the Center of the Quadrant was from the Ground, which let be 5.6 Feet.



To conſtruct the Figure: Draw the Line C D at Pleaſure, and ſet off the meaſured Diſtance 39.7 from your Station at C to the Tower; at C make the given Angle 37°.30′, by Prob. 8, and draw C B; at D erect the Perpendicular D B, 'till it meets the Hypothenuſe in B, and continue it downward to A; make D A 5.6 Feet the Height of the Center of the Quadrant; draw C *c* parallel and equal to D A, and draw *c* A equal to C D, 'till it meets B D continued in A; thus will A B be the Height of the Tower.

Now, in the right-angled Triangle C B D, there is given, beſides the right Angle, the Baſe C D 39.7 Feet, the Angle at C 37°.30′, and the Angle at B, the Complement of 37°.30′ to 90°, *viz.* 52°.30′, to find the Side B D, oppoſite the Angle at A; which may be found by the following Proportion:

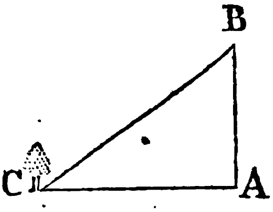
As the Sine of the Angle at B, 52°. 30′, Co. Ar. 0.100533
Is to the Side C D, 39.7, ——— 1.598790
So is the Sine of the Angle at A, 37°. 30′, —— 9.784447

To the Side B D, 30.46, ——— 1.483770

To which adding the Height of the Center of the Quadrant, 5.6, it gives 36.06 Feet for the Height of the Tower A B from the Ground:

30.46 B D, by Calculation.
5.6 C *c*, the meaſured Height of the Quadrant, equal to D A.
———
36.06 Height of the Tower.

Prob. 33. *Having the Height of any given Object, A B 59.4, Feet, to meaſure how far it is to a Tree at C, growing on the ſame Level with the Baſe of the given Object.*



Keep the Center of the Quadrant even with the Top of the Object, and meaſure the Angle B 51°. 45′, and the Angle C, being the Complement of it to 90°, will be 38°. 15′; ſo that we have given the Angle C, oppoſite the known Side A B, and the Angle B, to find the Side C A.

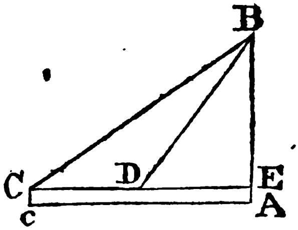
Conſtruction. Draw the Baſe-Line C A at Pleaſure, on which, at any Point, as A, erect A B perpendicular to C A; lay 59.4 from A to B, by the general Rule, Prob. 19; at B make an Angle of 51.45, by Prob. 8, and draw B C, which completes the right-angled Triangle C A B. Then,

As the Sine of the Angle at C, 38°. 15′, Co. Ar. 0.208244
Is to its oppoſite Side A B, 59.4, ——— 1.773786
So is the Sine of the Angle at B, 51°. 45′, —— 9.895045

To the Side C A, or Diſtance of the Object, 75.35, 1.877075

Prob. 34. *To meaſure the Height of an inacceſſible Object, A B; that is, where the Diſtance between the Station and the Baſe of the Object cannot be meaſured.*

At



At any convenient Diſtance, as at C, on the ſame Plain with the Object, meaſure the Angle at C to the Top of the Object, which let be 31°. 25′; then meaſure the Height of the Center of the Quadrant from the Ground, which ſuppoſe 4.9 Feet; and then meaſure, in a right Line from C towards the Object, any convenient Diſtance, as C D, which ſuppoſe 36.5 Feet; here, holding the Quadrant the ſame Height as before, meaſure the Angle D to the Top of the Object 52°. 10′.

Conſtruction. Draw the Baſe-Line C E at Pleaſure, on which lay 36.5 from C to D; at C, by Prob. 8. make an Angle of 31°. 25′, and draw C B at Pleaſure; at D make an Angle of 52°. 10′, and draw D B, meeting C B in B; from B let fall the Perpendicular B E; make C *c* perpendicular to C E, on which lay 4.9; then draw *c* A parallel to C E, and continue B E 'till it meets *c* A, and the Figure is conſtructed.

Now, in the oblique-angled Triangle C B D, are known the Side C D 36.5 Feet, and the Angle at C 31°. 25′, alſo the obtuſe Angle C D B 127°. 50′, being the Supplement of the Angle B D E, or 52°. 10′, to 180°; therefore the Angle C B D is found by deducting the Sum of the other two from 180°:

127° . 50′	Angle CDB.	180° . 00′	
31 . 25	Angle BCD.	159 . 15	
159 . 15		20 . 45	Angle CBD.

Having the Angle at B 20°. 45′ oppoſite the known Side C D 36.5, and the Angle at C 31°. 25′ oppoſite the Side D B; hence,

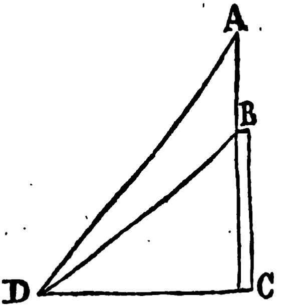
As the Sine of the Angle at B, 20°. 45′,	Co. Ar.	0.450640
Is to its oppoſite Side C D, 36.5,	——	1.562293
So is the Sine of the Angle at C, 31°. 25′,	—	9.717053
To its oppoſite Side B D, 53.7,	——	1.729986

Now we have, in the right-angled Triangle DBE, given the Hypothenuſe DB 53.7 oppoſite the right Angle at E, and the Angle at D 52°. 10′ oppoſite the required Side BE; therefore,

As the Radius,	10.
Is to the Hypothenuſe BD, 53.7,	1.729986
So is the Sine of the Angle at D, 52°. 10′,	9.897516
To its oppoſite Side BE, 42.41,	1.627502

Thus having found the Side BE, the Height of the Object above the Center of the Quadrant at the Time of Obſervation, if to that is added 4.9 Feet, the Height of the Quadrant, it gives BA 47.31 for the Height of the Object above the Plain *c* A.

Prob. 35. *To meaſure the Object* AB *above the Object* BC.



At any convenient Diſtance, as at D, with a Quadrant take the Angle ADC, which ſuppoſe 50°. 56′; take the Angle BDC 38°. 30′; and then meaſure the Diſtance from your Station at D to the Baſe of the Object at C, which ſuppoſe 48.6 Feet.

Conſtruction. Draw the Baſe-Line DC at Pleaſure; at D, by Prob. 8, make an Angle ADC of 50°. 56′, and draw AD; at the ſame Point D, make an Angle BDC of 38°. 30′, and draw DB; ſet off, by the diagonal Scale, on the Baſe-Line 48.6 from D to C; at C erect a Perpendicular, which, being drawn to meet DA and DB, completes the Figure; for AB is the Height of the Object above BC.

Now, in the Triangle DAC, right-angled at C, there are given the Angle CDA 50°. 56′, the Angle at A, being the Complement of that to 90°, 39°. 4′, and the Side DC 48.6, to find the Perpendicular AC.

As the Sine of the Angle at A, 39° . 4′, Co. Ar. 0.200505
Is to the Side D C, 48.6, ——— 1.686636
So is the Sine of the Angle at D, 50° . 56′, — 9.890093

To the Perpendicular A C, 59.87, —— 1.777234

And, in the Triangle D B C, there are given the Angle C D B 38° . 30′, the Angle D B C 51° . 30′, found by ſubtracting 38° . 30′ from 90°, and the Baſe D C 48.6, to find the Perpendicular C B:

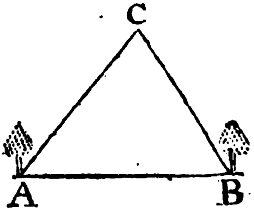
As the Sine of the Angle at B, 51° . 30′, Co. Ar. 0.106456
Is to the Side D C, 48.6, ——— 1.686636
So is the Sine of the Angle at D, 38° . 30′, — 9.794149

To the Perpendicular B C, 38.65, —— 1.587241

Now from 59.83 the Height of A C,
Take 38.65 the Height of B C.
(required.
21.18 Height of A B above B C, as was

This Object being acceſſible, that is, which could be come at, ſo as to meaſure the Diſtance of the Station from the Foot of the Object, one Station is ſufficient: But, if any Obſtacle lies in the Way, ſo as to make it inacceſſible, it is neceſſary to have two Stations, as in Prob. 34; and the Center of the Inſtrument ſhould be kept at the ſame Height, while each of the Angles is meaſured at each Station.

Prob. 36. *To meaſure the Diſtance between two Objects,* A *and* B, *when you can meaſure from the Station to one of the Objects.*



Take a Station at a convenient Diſtance from the Object, as C, and obſerve the Angle C 70° . 56′: If either of the Objects is acceſſible, meaſure the Diſtance from your Station to the acceſſible Object, as ſuppoſe C B 49.7 Feet; if you cannot meaſure to the other Object A in a right Line, go to it, and take the Angle at A, which here will be 50° . 10′, and then the Angle at B, which is the Supplement of the other two to 180°, will be 58° . 54′.

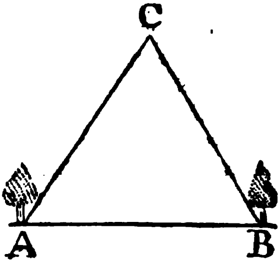
Conſtruction. Draw the right Line C B, on which lay from the diagonal Scale 49.7 from C to B; at C, by Prob. 8, make an Angle of 70°. 56′, and draw C A; at B make an Angle of 58°. 54′, and draw B A, meeting C A in A, which conſtructs the Triangle C A B, and A B is the Diſtance required.

In the Triangle A C B, there are given the three Angles, and the Side C B 49.7, to find the Side A B, or the Diſtance of the two Objects: Whence,

As the Sine of the Angle at A, 50°. 10′, Co. Ar.	0.114689
Is to the Side C B, 49.7, ——	1.696356
So is the Sine of the Angle at C, 70°. 56′, —	9.975496
To the Side A B, 61.17, the Diſtance of the Objects A and B, —— ——	1.786541

Another Example. *Suppoſe I am at C, and can meaſure from C to each of the Objects A and B, to find the Diſtance between the Objects themſelves.*

With the Inſtrument meaſure the Angle A C B 59°. 30′; and then meaſure the Diſtance C B 49.2 Feet, and the Diſtance C A 51.7 Feet.



Conſtruction. Draw the right Line C B, on which lay from C to B 49.2; at C, by Prob. 8, make an Angle of 59°. 30′, and draw C A, on which ſet off 51.7 from C to A; then draw A B, meeting C B in B, and the Triangle A B C is conſtructed, A B being the Diſtance required.

In the Triangle A C B, there are given the Side A C, the Side B C, and the included Angle at C, to find the Baſe A B; which may be done by firſt finding the other Angle, by Prob. 28, as follows:

51.7		180° . 00′	
49.2		59 . 30	Angle at C.
100.9	Sum of the Sides.	120 . 30	Sum of the Angles A and B.
		2	
2.5	Diff. of the Sides,	60 . 15	Half the Angles A and B.

Now,

As the Sum of the given Sides, 100.9, Co. Ar. 7.996109
Is to their Difference, 2.5, ——— 0.397940
So is the Tangent of the Half-Sum of the unknown Angles, 60°. 15′, —— } 10.242948

To the Tangent of Half their Difference, 2°. 29′, 8.636997

To 60°. 15′ the Half-Sum of A and B.
Add 2 . 29 the Half-Difference.

62 . 44 greater Angle B.

57 . 46 leſſer Angle A.

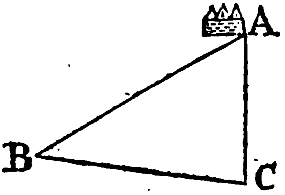
Then,

As the Sine of the Angle at A, 57°. 46′, Co. Ar. 0.072690
Is to the oppoſite Side B C, 49.2, —— 1.691965
So is the Sine of the Angle at C, 59°. 30′, 9.935220

To the Side A B, 50.14, the Diſtance of the Objects A and B, ——— } 1.699875

Prob. 36. *To meaſure the Diſtance between an Object at* A *from any Place, as* C, *where, by Reaſon of ſome Obſtacle, it cannot be come at.*

In any Place on the ſame Plain with C place a viſible Mark, as B, and meaſure the Angle A C B 88°. 20′; meaſure the Diſtance B C, which ſuppoſe 76.35; then with the Inſtrument take the Angle A B C 36°. 10′.



Conſtruction. Draw the Baſe-Line B C, on which lay 76.35 from C to B; at C, by Prob 8, make an Angle of 88°. 20′, and draw C A; at B, by the ſame Problem, make an Angle of 36°. 10′, and draw B A, meeting C A in A, which conſtructs the Triangle B C A, and A C is the Diſtance ſought.

Now, in the Triangle B C A, there is given

36° . 10′ the Angle at B.	180° . 00′
88 . 20 the Angle at C.	124 . 30
124 . 30	55 . 30 the Angle at A.

Whence, all the three Angles and the Diſtance B C being known, the Side A C, or the Diſtance of the Object A from C, may be found thus:

As the Sine of the Angle at A, 55° . 30′,	Co. Ar.	0.084006
Is to the Side B C, 76.35,	———	1.557507
So is the Sine of the Angle at B, 36° . 10′,	—	9.770952
To the Side A C, 25,85,	———	1.412465

Of SURVEYING.

SURVEYING, or Meaſuring of Land, conſiſts in taking the Dimenſions of a Piece of Ground, as a Field, laying it down in a Draught or Map, and finding the Content in Acres, Roods, &c.

The firſt Part of theſe, that is, taking the Dimenſions, conſiſts of two Parts; firſt, taking the Angles, and then meaſuring the Diſtances: To perform which ſeveral Inſtruments are neceſſary, as a *Plain-Table*, *Theodolite*, *Semicircle*, *Compaſs*, and a *Chain*, alſo a *Protractor* for plotting.

The Plain-Table is a Board, about fifteen Inches broad, and twelve long, encompaſſed with a Frame, jointed at the Corners: From a Braſs Center in the Middle are projected three hundred and ſixty Degrees of a Circle, each Degree being halved, and numbered at every tenth Degree, both Ways, to ſave Subtraction: On the contrary Side, a Semicircle is uſually projected, divided and numbered in the ſame Manner, to one hundred and eighty Degrees, both Ways. To the Table is uſually fitted a Compaſs, to ſet it North or South, and a Ruler or Index with Sights. The Table is fixed, by a Ball and Socket, to a three-legged Staff for a Stand, on which it may be turned round, or faſtened by a Screw.

Of Chains there are two Sorts, each divided into one hundred Links, for the Conveniency of ſetting down the Links as Decimals of the Chain: That called *Gunter's Chain* is four Poles, or ſixty-ſix Feet, in Length; the other is but one Pole, that is, ſixteen Feet and an Half, but conſiſts of one hundred Links as the other.

A Protractor is an Inſtrument of Braſs, Silver, or the like, with a ſemicircular Limb, divided into one hundred and eighty Degrees, ſubtended by a Diameter, in which is a little Notch called the Center: Its Uſe is to lay down the Angles in plotting a Field.

To take the Dimenſions of a Field at one Station.

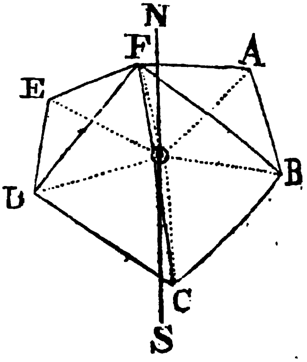
Place the Inſtrument in any Station, as at ⊙, from whence all the Angles may be ſeen; turn the Table about 'till the Needle ſtands over the North and South Points of the Compaſs, then ſcrew the Inſtrument faſt, and direct the Sights to the firſt Angle, A, to the right Hand of the North and South Line; obſerve the Degrees and Minutes contained between 360 of the Table and the Index, which is 56° . 20′: Direct the Sights, in the ſame Manner, to B, and obſerve the Degrees and Minutes that the Index now cuts on the Table, which is 95° . 40′: In the ſame Manner, turn the Index to C, and ſo on to D, E, and F, and the ſeveral Angles will be as under;

50° . 20′
95 . 40
170 . 10
251 . 20
290 . 40
341 . 10

Now take the Chain and meaſure the Diſtance ⊙ A ſeven Chains and thirty Links, or 7.30; the Diſtance ⊙ B 8.45; and the reſt as in the following Table, uſually called a *Field-Book*:

50° . 20′	7.30
95 . 40	8.45
170 . 10	6.54
251 . 20	7.15
290 . 40	7.47
341 . 10	5.60

To protract or lay down the Work.



Draw any right Line, as N S, for the North and South Line, any Point of which, as ⊙, aſſume for the Station; lay the Center of the Protractor on this Point, turn it 'till the Diameter of the Protractor falls upon the North and South Line, and prick off the Angle 50°. 20′, the Angle 95°. 40′, and ſo of the reſt; then draw the Lines ⊙ A, ⊙ B, ⊙ C, ⊙ D, ⊙ E, and ⊙ F: Now from the Diagonal take 7.30 for the ſeven Chains and thirty Links, and ſet them from ⊙ to A; do the ſame for the eight Chains and forty-five Links, and ſet it from ⊙ to B; and ſo of the reſt: Laſtly, draw the Lines A B, B C, C D, D E, and E F, which is the Plot or Plan of the Field.

To find the Number of Acres it contains.

Draw the right Lines F B, F C, and F D, which divides the Figure into four Triangles: Take F B in the Compaſſes, and apply it to the diagonal Scale, which meaſures 11.96 for the Baſe of the Triangle F A B; then ſet one Foot of the Compaſſes in A, and extend the other ſo as juſt to reach the Line F B without paſſing it, which, being meaſured on the diagonal Scale, is 3.48 for the Perpendicular of the ſame Triangle. The other Baſes and Perpendiculars, being meaſured, will be as follows, for the ſeveral Triangles:

	Baſe.	Perpend.
F A B	11.72	3.23
F B C	12.25	8.68
F C D	12.25	7.04
F D E	9.46	3.12

Multiply each Baſe by its Perpendicular, and cut off five Figures for Decimals in the Products: Add thoſe Products together, and divide their Sum by 2, and the Integers in the Quotient are Acres; multiply the Fraction by 4, and the Integers in the Product are Roods; and multiply this laſt Fraction by 40, and the Integers in the Product are ſquare Poles or Perches:

11.72
3.23
3516
2344
3516
3.78556

12.25
8.68
9800
7350
9800
10.63300

12.25
7.04
4900
85750
8.62400

9.46
3.12
1892
946
2838
2.95152

3.78556
10.633
8.624
2.95152
2) 25.99408
12.99704

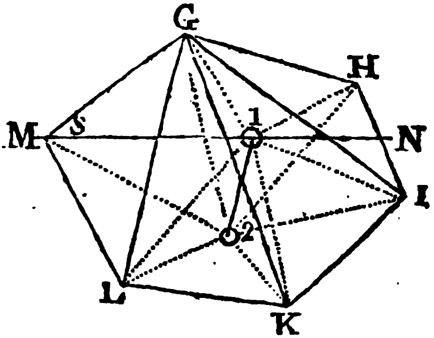
.99704
4
3.98816
40
39.52640

By which the Area appears to be twelve Acres, three Roods, and thirty-nine ſquare Poles or Perches.

The Reaſon of cutting off five Figures in the Products is, becauſe, the Chain being four Poles in Length, every ſquare Chain contains ſixteen ſquare Poles, which is the tenth Part of an Acre; and, as each of the Numbers has two Decimals, there muſt be four cut off, according to the Rule in Multiplication of Decimals, Art. 42; and then the Integers in the Product are Tenths of an Acre, therefore cutting off one Place more reduces them to Acres.

But then, as we multiplied the Baſes and Perpendiculars together, the Products are double the Area of the Triangles, for which Reaſon we divide their Sum by 2.

To ſurvey the Field GHIKLM *at two Stations.*



Having ſet the Inſtrument, at the firſt Station ⊙ 1, North and South by the Needle, direct your Sights to I, the firſt Angle to the right Hand of the Inſtrument, and obſerve the Degrees and Minutes cut by the Index, which is 20° . 49′ : In the ſame Manner, view all the ſeveral Angles, which will be as follows:

N ⊙ I	20° . 49′
N ⊙ K	76 . 25
N ⊙ L	130 . 49
N ⊙ M	180 . 00
N ⊙ G	234 . 10
N ⊙ H	335 . 30

Then chooſe another Station in the Field, from whence all the Angles may be ſeen, as at ⊙ 2, at which ſet up a Staff for a Mark; direct the Sights of the Inſtrument to that Mark, and obſerve the Angle cut by the Ruler, which is 97° . 55′ ; then meaſure the Diſtance of the two Stations, which is two Chains and thirty-two Links, or 2.32.

Carry the Inſtrument to the ſecond Station, and leave a Staff, or ſome other viſible Mark, at the firſt Station ; ſet the Center of the Inſtrument over the Place where the Staff ſtood at ⊙ 2, and put the Sights to the Angle they were at from ⊙ 1 to ⊙ 2, that is, to 97° . 55′ ; then unſcrew the Table, keep the Index and Sights fixed, turn the Inſtrument and Index 'till thro' the back Sight you ſee the Mark at ⊙ 1, and then ſcrew the Inſtrument, which will then ſtand in the ſame Poſition as at ⊙ 1.

Direct the Sights to the Angle I, the Angle obſerved firſt at the firſt Station, and obſerve the Degrees and Minutes cut by the Index; then direct the Sights to K and L, and ſo on to the other Angles, and the Degrees and Minutes cut from [illegible] Station will be as follows:

I	345°	. 20′
K	52	. 25
L	162	. 45
M	209	. 22
G	258	. 45
H	314	. 24

To protract or lay down the Work.

Draw a Line at Pleaſure for the North and South Line, on which aſſume any Point, as ⊙ 1, for the firſt Station; lay the Center of the Protractor on this Point, and the Diameter on the Line N S; prick off the Angles, and draw the Lines ⊙ 1 I, ⊙ 1 K, *&c.* to a convenient Length: Then finding, by the Field-Book, that the Angle to ⊙ 2 is 97°.55′, lay down that Angle, and draw the Line ⊙ 1, ⊙ 2; take from the diagonal Scale 2.32, the Diſtance from ⊙ 1 to ⊙ 2, and lay from ⊙ 1 to ⊙ 2, which is the ſecond Station: Lay the Protractor to the Point ⊙ 2, and turn it about 'till the right Line drawn from ⊙ 1 to ⊙ 2 cuts the Angle 97°.55′; then lay down the ſeveral Angles at ⊙ 2, beginning with that which was firſt taken, and proceeding 'till you have taken down all the Angles; draw the Lines ⊙ 2 K, *&c.* 'till they interſect the former Lines in the Points K, L, M, G, H, and I, which Interſections will repreſent the ſeveral Angles of the Field: Then join the ſeveral Angles, by the Lines G H, H I, I K, K L, L M, and M G, and you have the Plot of the Field; which may be divided, by Diagonals drawn from any Angle, as G, into four Triangles, whoſe Baſes and Perpendiculars, being meaſured on the diagonal Scale, as in the laſt Example, will be as under:

	Baſe.	Perpend.
M G L	6.48	3.32
L G K	8.01	3.45
K G I	8.01	3.68
G I H	7.11	1.84

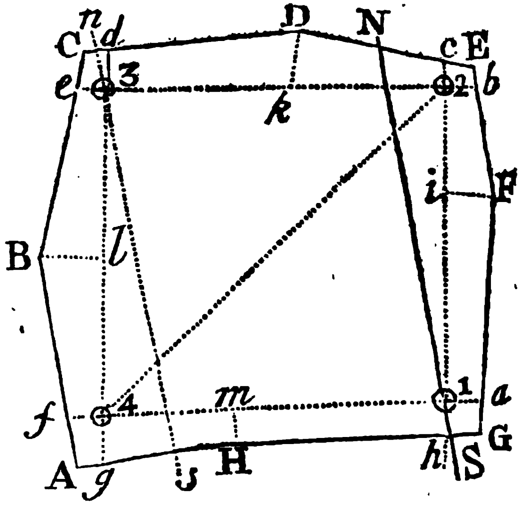
Multiply the Baſe and Perpendicular of each Triangle, and cut off five Figures; then add the ſeveral Products, and divide their Sum by 2, as in the other Example.

```
  6.48        8.01        8.01        7.11
  3.32        3.45        3.68        1.84
 ------      ------      ------      ------
  1296        4005        6408        2844
 1944        3204        4806        5688
1944        2403        2403         711
-------     -------     -------     -------
2.15136     2.76345     2.94768     1.30824


        2.15136          .585365
        2.76345                4
        2.94768         ---------
        1.30824         2.34146|0
       ---------             40
      2)9.17073         ---------
       ---------        13.65840
        4.585365
```

The Area is four Acres, two Roods, and thirteen Poles.

To ſurvey a Field by going round it at ſeveral Stations.



Chooſe any Place for the firſt Station, as at ⊙ 1, place the Inſtrument there, by the Needle, North and South, and ſcrew it faſt; then chooſe a ſecond Station, as at ⊙ 2, and there ſet up a viſible Mark; turn the Index of your Inſtrument about, till thro' the Sights you ſee the Mark at ⊙ 2, and it will cut 10°. 30′ N. E. for the Poſition of the Line you are about to meaſure, which enter in the middle Column of your Field-Book, as you ſee below.

Next lay the Chain from ⊙ 1 in a right Line towards ⊙ 2, the ſecond Station; which being done, let it lie 'till you have entered the Obſervations in that Chain's Length, that is, meaſure the Diſtance from ⊙ 1 to *a*, the Hedge or Ditch, fifty-two Links; which you muſt enter in the outſide Column of your Field-Book, on the right Hand, the Hedge being on the right Hand of the Courſe; having entered ⊙ 1, for the firſt Station, in the middle Column, againſt it put .52, the Offset, ſignifying, that, at the Inſtrument, the Hedge at the right Hand is fifty-two Links from the Chain: Then meaſuring on towards ⊙ 2, at the Diſtance 3.51 you come directly oppoſite to the Angle F, and the Diſtance or Offset from the Chain to F is .61; therefore enter 3.51 in the middle Column of your Field-Book, and .61 in the right-hand Column: Then meaſuring on to ⊙ 2, you will find it to be 4.78, and the Offset .46; both which enter in your Field-Book, as before: So is your Work relating to the firſt Station finiſhed.

	10° . 30′	Offsets.
		N. E.
	⊙ 1	.52
	3.51	.61
	4.78	.46
	279° . 45′	
		N. W.
	⊙ 2	.35
	2.31	.63
	5.32	.46
	191° . 50′	
		S. W.
	⊙ 3	.53
	2.78	.95
	5.21	.54
	265° . 12′	
		N. W.
	⊙ 4	.71
	2.06	.39
	5.44	.42

Then bring your Inſtrument to ⊙ 2, place the Center directly over where the Staff ſtood, and ſend a Staff forward to ⊙ 3; unſcrew the Inſtrument, lay the Index on 10° . 30′, the Angle cut from ⊙ 1 to ⊙ 2, and turn the Inſtrument, with the Index in that Poſition, 'till thro' the back Sight you ſee the Staff left at ⊙ 1; ſcrew the Inſtrument faſt, then turn the Index about 'till thro' the Sights you ſee the Staff at ⊙ 3, and it will cut 279° . 45′, which enter in your Field-Book: Then obſerve, that at ⊙ 2 the Offset to *c* is .35; and in meaſuring towards ⊙ 3, at 2.31 Diſtance from ⊙ 2, the Offset is .63; and the whole Diſtance from ⊙ 2 to ⊙ 3 is 5.32, where the Offset is .46; all which enter in your Field-Book, as before.

In the ſame Manner carry the Inſtrument to ⊙ 3, and proceed in all Reſpects as at ⊙ 2, and the Angle to ⊙ 4 will be found 191°. 50′: Then meaſure the Diſtance from ⊙ 3 to ⊙ 4, and alſo from ⊙ 4 to ⊙ 1; together with the ſeveral Offsets; place them in the Field-Book, which will then ſtand as in the foregoing Page.

To protract the Work.

Draw any Line at Pleaſure for the North and South Line, on which aſſume the firſt Station ⊙ 1, and with the Protractor lay down the Angle 10°. 30′; draw the Line ⊙ 1, ⊙ 2, and ſet the Offset .52 from ⊙ 1 to *a*; take 3.51 and ſet from ⊙ 1 to *i*, where ſet off the Offset .61 from *i* to F; take 4.78 and ſet from ⊙ 1 to ⊙ 2; take .46, the Offset at ⊙ 2, and ſet from ⊙ 2 to *b*; the Offsets are to be ſet off perpendicular to the ſtationary Line: Then lay the Protractor on ⊙ 2, with the Limb on the contrary Side of the Line; and, as the Degrees cut were 279°. 45′, take 279°. 45′ from 360°, the Remainder is 80°. 15′, thro' which Angle draw the Line ⊙ 2, ⊙ 3; then from the Scale lay 2.31 from ⊙ 2 to *k*, where ſet off .35 the Offset from *k* to D; then lay 5.32 from ⊙ 2 to ⊙ 3, and ſet off .46 from ⊙ 3 to *d*.

Thro' ⊙ 3 draw a Meridian-Line, as *n s*, parallel to the former Meridian-Line N S, by Prob. 6; lay the Center of the Protractor on ⊙ 3, and, as the Degrees cut on the Inſtrument were 191°. 50′, mark off 11°. 50′, the Exceſs above 180°, for the Angle that the Line ⊙ 3, ⊙ 4 is to make with the pricked Meridian-Line, and draw the Line ⊙ 3, ⊙ 4, on which lay 2.78 from ⊙ 3 to *l*, where ſet off the Offset .95; and lay 5.21 from ⊙ 3 to ⊙ 4, where ſet off the Offset .54. From ⊙ 4 draw a Line to ⊙ 1, on which ſet 2.06 to *m*, and the Offsets as in the Table. Laſtly, laying down 5.44, it will reach from ⊙ 4 to ⊙ 1, where ſet off .42 from *i* to *b*; and draw Lines from the Offset *a* to F, from F to *b*, &c. which will be the true Repreſentation of the Field.

To find the Number of Acres.

At ⊙ 1 I obſerve, ⊙ 1 *a* is .52, and ⊙ 1 *b* .42, which are Sides of the Parallelogram ⊙ 1, *a*, G, *b*, for in this Caſe the Error is inſenſible: Multiply the two Numbers together, and cut off five Places for Decimals in the Product, theſe will be the Decimals of an Acre; if there are not five Places of Figures in the Product, prefix a Cypher to make five Places: Then, for the Parallelogram ⊙ 1, *a*, *i*, F, add .52, the

Width at one End, to .61 the Offset at *i*, the Width at the other End; the Half of that Sum is the mean Width, which, being multiplied into 3.51, the Length, gives the Area, remembering to cut off five Places for Decimals in the Product.

In the ſame Manner may the Areas of all the Parallelograms be computed, remembering to ſupply the Places with Cyphers where they are deficient:

$$\begin{array}{r} .52 \\ .42 \\ \hline 104 \\ 208 \\ \hline .02184 \end{array} \qquad \begin{array}{r} .61 \\ .52 \\ \hline 2)\,1.13 \\ \hline .56 \end{array} \qquad \begin{array}{r} 3.51 \\ .56 \\ \hline 2106 \\ 1755 \\ \hline .19656 \end{array}$$

$$\left.\begin{array}{l} \odot\ 1,\ a,\ b,\ G, \\ \odot\ 1,\ a,\ i,\ F, \\ \odot\ 2,\ b,\ i,\ F, \\ \odot\ 2,\ b,\ c,\ E, \\ \odot\ 2,\ c,\ k,\ D, \\ \odot\ 3,\ d,\ k,\ D, \\ \odot\ 3,\ d,\ e,\ C, \\ \odot\ 3,\ e,\ l,\ B, \\ \odot\ 4,\ f,\ l,\ B, \\ \odot\ 4,\ f,\ g,\ A, \\ \odot\ 4,\ g,\ m,\ H, \\ \odot\ 1,\ h,\ m,\ H, \end{array}\right\}\text{is}\left\{\begin{array}{l} .02184 \\ .19656 \\ .19133 \\ .0161 \\ .11319 \\ .16254 \\ .02438 \\ .20572 \\ .17982 \\ .03834 \\ .1133 \\ .1352 \end{array}\right.$$

$$\begin{array}{r} \hline 1.39832 \end{array}$$

Draw a Diagonal from $\odot$ 2 to $\odot$ 4, to divide the Part of the Field within the ſtationary Lines into two Triangles; then ſet one Foot of the Compaſſes in $\odot$ 1, and extend the other to the Diagonal ſo as not to paſs it; this being meaſured on the Scale will be 3.37 for the Perpendicular of the Triangle $\odot$ 4, $\odot$ 2, $\odot$ 1; then ſetting one Foot in $\odot$ 3, and extending the other to the Diagonal in the ſame Manner, it gives 3.66 for the Perpendicular of the Triangle $\odot$ 4, $\odot$ 3, $\odot$ 2; and the Diagonal is 7.48 for the Baſe of each Triangle.

```
      7.48                        7.48
      3.37                        3.66
    -------                     -------
      5236                        4488
     2244                        4488
    2244                         2244
   --------                    --------
   2.52076                     2.73768
                               2.52076
                              --------
                            2) 5.25844
                              --------
                               2.62922
Area of the Parallelograms add 1.39832
                              --------
                               4.02754
                                     4
                              --------
                                .11016
                                    40
                              --------
                               4.40640
```

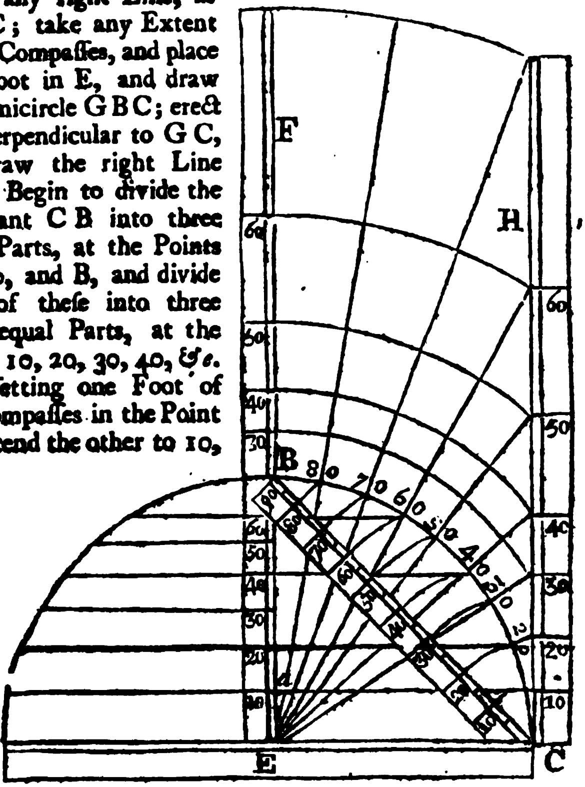
The Area is four Acres and four Poles.

Surveying being by much the beſt learned by Practice, I have been the ſhorter in this Article: And, the Practice of Surveying requiring divers Inſtruments, I would recommend my Readers, who may have Occaſion to purchaſe any, to Meſſrs. *Heath* and *Wing*, Mathematical Inſtrument Makers, near *Exeter-'Change*, in the *Strand*; where all Sorts of Mathematical and Philoſophical Inſtruments are made, to a great Degree of Exactneſs and Perfection.

The Conſtruction of the Line of Chords, Sines, Tangents, *and* Secants.

Of the Line of Chords.

EVERY Circle being divided into 360 Degrees, each Quadrant contains 90 Degrees: A Line of Chords, therefore, is the Chord of 90 Degrees, or a Quadrant to any Radius. The Way to project a Line of Chords is, Draw



Draw any right Line, as G E C; take any Extent of the Compaſſes, and place one Foot in E, and draw the Semicircle G B C; erect E F perpendicular to G C, and draw the right Line C B: Begin to divide the Quadrant C B into three equal Parts, at the Points 30, 60, and B, and divide each of theſe into three more equal Parts, at the Points 10, 20, 30, 40, *&c.* then ſetting one Foot of the Compaſſes in the Point C, extend the other to 10, 20, 30, 40, *&c.* and transfer it to the Chord-Line C B; ſo will this Line be divided in the Points 10, 20, 30, 40, *&c.* the Chords of thoſe Degrees. If you continue to divide the Arch C B into ſtill ſmaller Diviſions, and transfer them in the ſame Manner, theſe Diviſions, where they interſect the right Line C B, will be the Chords of thoſe Degrees; which, in the Figure, are divided but to every fifth Degree.

Of the Line of Sines.

If thro' the Points 10, 20, 30, *&c.* of the Quadrant C B, Lines are drawn parallel to the Diameter G C, they will cut the Radius B E, and the Part cut off will be the Sine of the Number of Degrees which ſuch Parallel paſſes thro' in the *Periphery* of the Circle: Thus E *a* is the Sine of 10 Degrees, becauſe the Parallel which cuts the Radius E B in the Point *a* paſſes thro' 10 Degrees in the Periphery. Thus, by drawing Parallels thro' each Degree of the Quadrant, the Radius B E would be divided into a Line of Sines to all the 90 Degrees of the Quadrant.

Of the Lines of Tangents *and* Secants.

A Perpendicular being raiſed, by Prob. 3, on the End of the Diameter G C, as C H, is a Tangent-Line; to which, Lines drawn from the Center E, thro' the ſeveral Degrees of the Quadrant, will divide it into a Line of Tangents to the Radius B E; and the Lines ſo drawn from the Center are the Secants of the ſeveral Degrees thro' which they paſs, which may be transferred to the Line B F, by placing one Foot of the Compaſſes in E, and extending the other to 10, 20, 30, 40, *&c.* on the Tangent-Line, and drawing the reſpective Arches 'till they interſect the Line E F, which Diſtances from the Center will be the Secants of their reſpective Degrees.

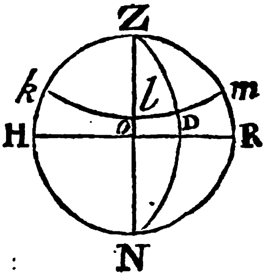
If the Reader takes the Chord of 60 Degrees from his Scale, and proceeds as directed, he may compare the Truth of his Conſtruction with the Lines on the Scale.

Of Spheric GEOMETRY.

SPHERIC GEOMETRY conſiſts in projecting the Circle of the Sphere on a Plain: In which Projection there are great Circles of three different Sorts, *viz.* primitive, right, and oblique Circles.

Definitions.

The *primitive Circle* is that on which the Projection is made, as the Center H Z R N.



A *right Circle* is a Circle which cuts the primitive Circle at right Angles, and divides it into two equal Parts; therefore, in the Projection, this is a right Line, and becomes the Diameter of the primitive Circle; as H O R.

An *oblique great Circle* is a Circle which cuts the primitive Circle in two oppoſite Points, but does not paſs thro' the Center, as the Circle Z D N.

An *oblique leſs Circle* is a Circle parallel to a right Circle, as the Circle *k l m*, parallel to H O R.

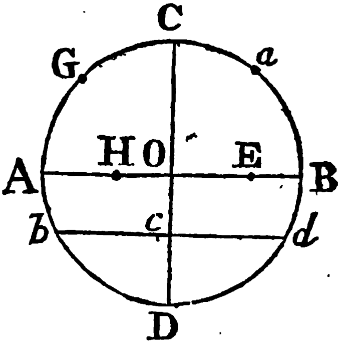
A *ſpherical Angle* is the Meeting of two Circles in a Point; thus, by the Meeting of the primitive Circle H Z R N with the oblique Circle Z D N, is made the Angle R Z D.

The *Pole* of any Circle is that Point which is ninety Degrees from it every Way.

When any Circle paſſes thro' the Pole of another Circle, it interſects it at right Angles, or is perpendicular to it.

Prob. 37. *To find the Pole of the primitive Circle.*

Draw any right Line within the primitive Circle, as *b c d*; biſect this Line, by Prob. 1, with the Line C D, and biſect this Line by the right Line A B; the Point O, where A B and C D meet, is the Center or Pole of the primitive Circle.



Prob. 38. *To find the Poles of any right Circle,* A B.

The Poles of every right Circle are, in the Periphery of the primitive Circle, ninety Degrees from it; therefore, draw C D perpendicular to A B, paſſing thro' the Center O; then C and D are the Poles of the given right Circle A B.

Prob. 39. *To lay any Number of Degrees, as* 51°. 30′, *on the primitive Circle.*

Take 51°. 30′ from the Line of Chords, and ſet it on the primitive Circle, as from B to *a*.

Prob. 40. *To meaſure any Part, as* C G, *of the primitive Circle.*

Take C G in the Compaſſes, and, applying it to the Line of Chords, it will reach from the Braſs Center to 46°. 36′, the Anſwer.

Prob. 41. *To lay any Number of Degrees, as* 68°, *on a right Circle.*

This Problem admits of two Caſes:

Caſe 1. If the Degrees are to be laid from the Center towards the Periphery of the primitive Circle, take 34°, the Half of 68°, from the Line of Tangents, or 68° from the Line of Half-Tangents, and lay them from O to E.

Caſe 2. When the Degrees are to be laid from the Periphery towards the Center or Pole of the primitive Circle; As, to lay 43°. 24′ on the right Circle A B from A towards O, take the Complement of 43°. 24′ to 90°, which is 46°. 36′, from the Scale of Half-Tangents, or 23°. 18′, the Half of it, from the Line of Tangents, and lay it from O to H; then is A H 43°. 24′, becauſe O A is 90°. Or, ſet one Foot of the Compaſſes on 90° in the Line of Half-Tangents, and extend the other 'till you have 43°. 24′ between the Compaſſes; with this Extent, place one Foot of the Compaſſes in A, and the other will fall on H; thus will A H be 43°. 24′, as required.

Prob. 42. *To meaſure any Part of a right Circle.*

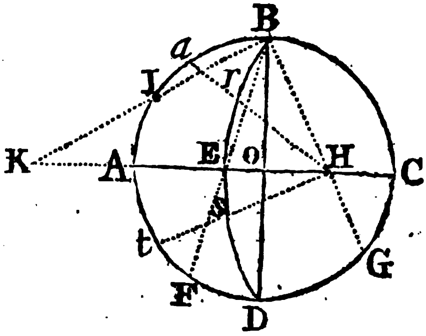
This, like the laſt Problem, admits of two Caſes:

Caſe 1. To meaſure the Part O E of the right Circle A B, take O E in the Compaſſes, and, applying it to the Line of Half-Tangents, it will reach to 68°; or, applying it to the Line of Tangents, it will reach to 34°, which, being doubled, gives 68°, as before.

Caſe 2. To meaſure a Part, as A H, of the right Circle A B, take O H in the Compaſſes, and, applying it to the Line of Half-Tangents, it will reach to 46°. 36′, the Complement of which to 90° is 43°. 24′, the Meaſure of A H; or, take A H in the Compaſſes, and ſetting one Foot on 90° in the Line of Half-Tangents, and turning the other towards the Beginning of the Scale, they will be found to include 43°. 24′, the Meaſure of A H, as before.

Prob. 43. *To find the Pole of an oblique Circle,* B E D.

Draw the Diameter B O D, and the Diameter A O C perpendicular to B O D; then draw a Line from B thro' E, meeting



meeting the primitive Circle in F; from F ſet off 90° to G, and draw B G interſecting A O C in H, the Pole of the oblique Circle B E D, which lies betwixt the primitive Circle and its Center. If the other Pole be required, ſet off 90° on the primitive Circle from F to I, extend A C towards K, and draw B I meeting A C in K, the other Pole of B E D.

From what has been ſhewn, of finding the Poles of the ſeveral Circles, the Learner may perceive, that the Pole of every Circle is ninety Degrees diſtant from the Circle itſelf: Thus, if E H, the Diſtance between the oblique Circle B E D and its Pole, be meaſured, by Prob. 42, Caſe 2, the Part E O will be found thirty-four Degrees, and O H fifty-ſix Degrees, which together are ninety Degrees.

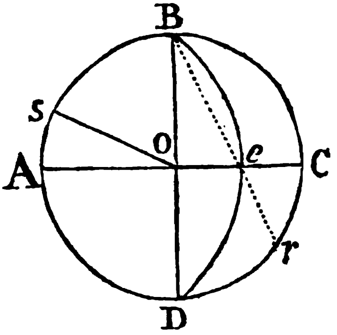
Prob. 44. *To lay any Number of Degrees, as* 37°, *on the oblique Circle*, B E D. See the laſt Figure.

Find H, the Pole of the oblique Circle, by Prob. 43; take 37° from the Line of Chords, and ſet from B to *a*; then draw H *a*, cutting the oblique Circle in *r*; thus will B *r* be 37°, as was required.

Prob. 45. *To meaſure a Part, as* B *r*, *of an oblique Circle*, B E D.

Find H, the Pole of the oblique Circle, by Prob. 43; then draw H *r*, cutting the primitive Circle in *a*; and B *a*, being meaſured on the Line of Chords, will give 37°, the Meaſure of B*r*. Again, if it were required to meaſure the Part *r s* of the ſame oblique Circle, firſt draw H *r*, cutting the primitive Circle in *a*; then draw H *s*, cutting the primitive Circle in *t*; and the Diſtance *a t*, being meaſured on the Line of Chords, gives 87°, the Meaſure of *r s*, required.

Prob. 46. *To make an Angle of* 23°. 29′ *at the Center or Pole of the primitive Circle.*



Draw the right Circle A C; then take 23°. 29′ from the Line of Chords, and ſet from A to *s*, and draw O *s*; thus will the Angle at O be 23°. 29′, as was required.

Prob. 47. *To make an Angle of* 36°. 40′ *at the Point* B *in the primitive Circle.*

From the Point B draw the Diameter B D, and at right Angles to it draw A C; from C ſet off 36°. 40′, by the Line of Chords, to *r*, and draw B *r*, which will interſect A C in *e*; thro' B *e* D, by Prob. 17, draw B *e* D, making an Angle of 36°. 40′ with the primitive Circle in the Point B. But here it will be proper to obſerve, that, when an Angle is ſignified by three Letters, the middle Letter is that at the angular Point; as, here it is the Angle *e* B C that is the Angle required, for the Angle O B *e* is 53°. 20′, being the Complement of the other to 90°.

Prob. 48. *To draw a right Circle perpendicular to the primitive Circle.*

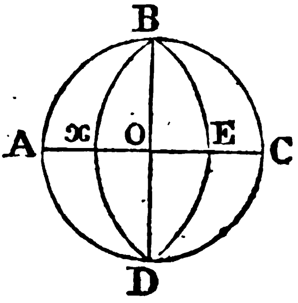
This is no more than to draw a Diameter, as A C in the laſt Figure, thro' O, the Center or Pole of the primitive Circle.

Prob. 49. *To draw a right Circle perpendicular to the right Circle* A C.

Find B and D, the Poles of A C, by Prob. 38; thro' which drawing the right Circle B D, it will be perpendicular to A C, and paſs thro' the Pole of the primitive Circle.

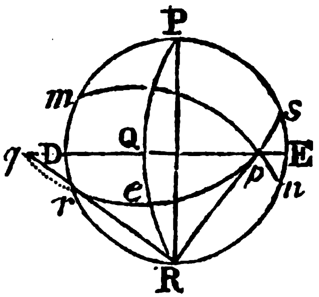
Prob. 50. *To draw an oblique Circle perpendicular to the right Circle* A C.

Having drawn the primitive Circle, and the right Circle A C, draw B D perpendicular to A C, by Prob. 49; ſet one Foot of the Compaſſes any where in A C, except in O, extend the



the other Foot to B or D, and draw B *x* D, which is perpendicular to A C, becauſe it paſſes thro' its Poles, B and D. But, when it is required to draw a Perpendicular to a right Circle, to make a given Angle with the primitive Circle, as of 49°, in this Caſe, take 41°, the Complement of the given Angle, from the Scale of Half-Tangents, and lay from O to E; then, by Prob. 17, draw an oblique Circle thro' the three Points B, E, and D, which will be perpendicular to A C, and make an Angle, C B E, of 49° with the primitive Circle.

Prob. 51. *To draw an oblique Circle, m p n, perpendicular to another oblique Circle,* P Q R.

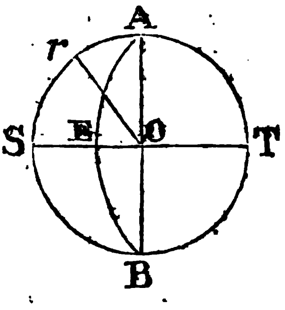


By Prob. 43, find *p*, the Pole of the oblique Circle P Q R; then taking any two oppoſite Points in the primitive Circle not in a right Line with *p*, as *m* and *n*, draw a Circle, by Prob. 17, to paſs thro' the three Points *m*, *p*, and *n*, which will be perpendicular to the given oblique Circle P Q R.

Prob. 52. *To draw an oblique Circle, r e p s, perpendicular to the oblique Circle* P Q R, *to paſs thro' e, any given Point in the oblique Circle* P Q R.

Draw the Diameter P R, alſo D E perpendicular to it; find *p*, the Pole of the oblique Circle, as before, and draw R *p*; at R erect *q* R perpendicular to R *p*, and extend it 'till it interſects the right Circle D E produced in *q*; thro' *q*, *e*, and *p*, a Circle being drawn, by Prob. 17, will be perpendicular to P Q R, and paſs thro' the given Point *e*.

Prob. 53. *To meaſure a ſpherical Angle, r* O *s, when the angular Point* O *is at the Center or Pole, of the primitive Circle.*

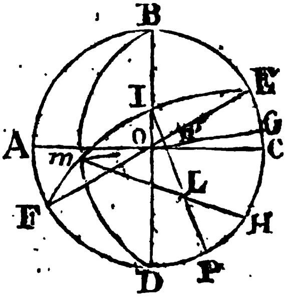


Take *r* S in the Compaſſes, and, applying it to the Line of Chords, it will reach to 47°. 30′, the Meaſure of the given Angle; this being the ſame as meaſuring a common Angle.

Prob. 54. *To meaſure a ſpherical Angle,* S A E, *when the angular Point* A *is in the Periphery of the primitive Circle.*

Draw the right Circle A O B paſſing thro' the Pole of the primitive Circle, alſo S O T perpendicular to A O B; then meaſure O E on the Scale of Half-Tangents 41°. 28′, which, being ſubtracted from 90°, leaves S E 48°. 32′, the Meaſure of the Angle S A E.

Prob. 55. *To meaſure a ſpherical Angle,* B I E, *made by the Interſection of the oblique Circle* E I F *with the right Circle* B D *at the Point* I.

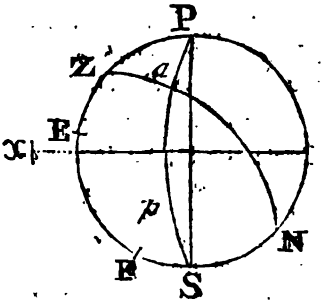


Draw A C perpendicular to B D paſſing thro' the Point O; then draw the right Circle E O F; by Prob. 43, find L, the Pole of E I F; and draw I L, meeting the primitive Circle in P; then C P, being meaſured on the Line of Chords, is 64°. 25′, the Meaſure of the given Angle B I E.

Prob. 56. *To meaſure a ſpherical Angle,* E *m* B, *made by the Interſection of two oblique Circles,* E *m* F *and* B *m* D, *in the Point m.*

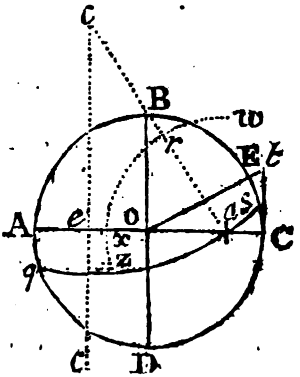
Find *x*, the Pole of the oblique Circle B *m* D, and L, the Pole of E *m* F, as before; then draw *m x* G and *m* L H; and G H, being meaſured on the Line of Chords, is 48°. 30′, the Meaſure of the given Angle E *m* B.

But



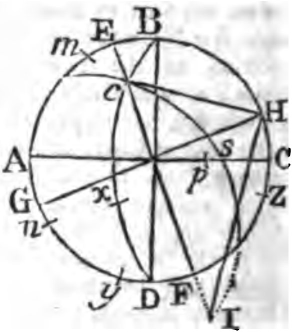
But this Rule holds only when the two oblique Circles, E *m* F and B *m* D, have their convex Parts the ſame Way: For, if it were required to meaſure the Angle S *a* N, made by the Interſection of the two oblique Circles P *a* S and Z *a* N; having, by Prob. 43, found *p*, the Pole of the oblique Circle Z *a* N, by the ſame Prob. find *x*, the Pole of the oblique Circle P *a* S, which is on the ſame Side the Center of the primitive Circle with *p*, the before-found Pole of Z *a* N; a Ruler, being laid from the angular Point *a* to *p*, will cut the primitive Circle in F, and, being laid from the ſame Point *a* to *x*, will cut the primitive Circle in E; then the Diſtance E F, being meaſured on the Line of Chords, is 72°. 23′, the Meaſure of the given Angle S *a* N.

Prob. 57. *To draw an oblique Circle, q a s, to make an Angle,* A *a q*, *of* 39° *with a right Circle,* A C, *and to paſs thro' a given Point, a, in any Part of the right Circle.*



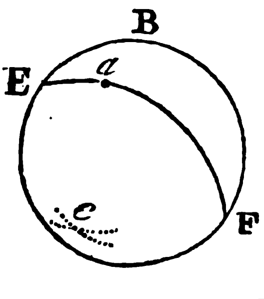
Draw BD perpendicular to AC, thro' O, the Center of the primitive Circle; lay a Ruler from D to *a*, which will cut the primitive Circle in E, and draw O E to a convenient Length; at C erect C *t* perpendicular to A C, 'till it meets E O in *t*; take C *t* in the Compaſſes, and ſet it from the Center O to *e*; at *e* erect *c e c* perpendicular to A C, and extend it both Ways to *c c*; take 60° from the Line of Chords, ſet one Foot of the Compaſſes in *a*, and with the other draw an Arch, *w x z*; then take 51°, the Complement of the given 39°, ſet it from *x* to *r*, and draw *a r*, 'till it meets *c e c* in *c*; on the Point *c*, with the Radius *c a*, draw the oblique Circle *q a s*, and the Angle A *a q* will be 39°, as was required.

Prob. 58. *To draw an oblique Circle, c s* I, *to make an Angle of* 80° *with another oblique Circle,* B *c* D, *in c, any given Point in* B *c* D.



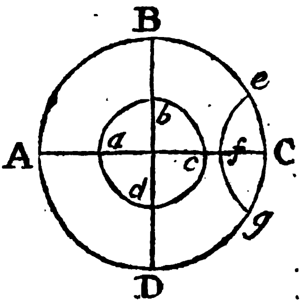
Find *p*, the Pole of the oblique Circle B *c* D, by Prob. 43; lay a Ruler from *p* to *c*, which will cut the primitive Circle in *m*, and from *m* ſet off 90° to *n*; a Ruler, being laid from *p* to *n*, will cut the oblique Circle B *c* D in *x*; a Ruler, being laid from *c* to *x*, will cut the primitive Circle in *y*, from whence lay the given 80° to *z*; a Ruler, being laid from *c* to *z*, will cut the right Circle A C in *s*; thro' either of the Points *c* or *s*, as *c*, draw a right Circle, E F, which croſs at right Angles with the right Circle G H; then draw the Line *c* H, and at H erect H I perpendicular to *c* H, which continue 'till it meets E F produced in I; laſtly, thro' the three Points *c*, *s*, and I, by Prob. 17, draw the oblique Circle *c s* I, paſſing thro' *c*, and making an Angle, *s c* D, of 80° with the oblique Circle B *c* D, as was required.

Prob. 59. *To draw an oblique Circle,* F *a* E, *thro' a given Point, a, ſo as to make an Angle of* 40° *with the primitive Circle.*



Take the Tangent of 40° in the Compaſſes, ſet one Foot in the Center of the primitive Circle, and draw an Arch, *e*; then, with the Secant of the ſame Angle of 40°, ſet one Foot of the Compaſſes in the given Point *a*, and deſcribe an Arch interſecting the former in *e*, which is the Center of the required Circle; upon which, with the Extent *e a*, deſcribe E *a* F, which will meet the primitive Circle in E, and make an Angle, *a* E B, of 40°, as required.

Prob. 60. *To draw a leſſer Circle parallel to the primitive Circle, at a given Diſtance of 46° from the Center of the primitive Circle.*

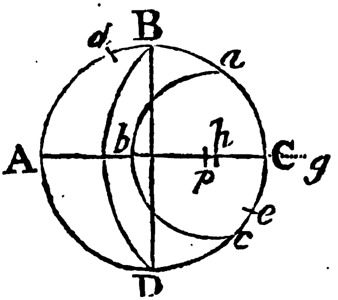


With the Half-Tangent of the given 46° in the Compaſſes, ſet one Foot in the Pole of the primitive Circle, and draw the leſſer Circle *a b c d*, which will be parallel to the primitive Circle A B C D.

Prob. 61. *To draw a leſſer Circle, e f g, parallel to the right Circle* B D, *at* 53° *from it.*

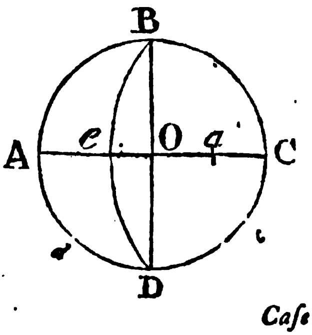
Take 53° from the Line of Chords, and lay them from B to *e*, and from D to *g*; then lay a Ruler from B to *g*, and it will cut the right Circle A C in *f*; thro' the three Points *e*, *f*, and *g*, by Prob. 17, draw *e f g*, the Parallel required.

Prob. 62. *To draw a leſſer Circle, a b c, parallel to an oblique Circle,* B *e* D, *at* 22° *diſtant from it.*



Find *p*, the Pole of the oblique Circle B *e* D, by Prob. 43; a Ruler, being laid from D to *p*, will cut the primitive Circle in *a*; ſet off 68°, the Complement of the given Diſtance, from *a*, both Ways, to *d* and *e*; a Ruler, being laid from D to *d* and *e*, will interſect the right Circle A C in *b* and *g*; then, by Prob. 1, biſect *b g* in *h*, the Center, on which, with the Radius *h b*, or *h g*, draw the Arch *a b c*, the Parallel required.

Prob. 63. *Any Point being given as a Pole, to draw its Circle.*



This Problem has three Caſes:

Caſe 1. When the given Point is the Center, as O, then is the Circle a primitive Circle, A B C D; which may be drawn with any Radius, at Pleaſure.

Caſe 2. When the given Point, as A, is in the Periphery of the primitive Circle, ſet off 90° from A, both Ways, to B and D; then draw the right Circle B O D, the Circle required.

Caſe 3. When the given Point, as *a*, is neither in the Center or Periphery of the primitive Circle, thro' *a* and the Center of the primitive Circle draw a right Circle, A C, and another at right Angles to it, as B D; a Ruler, being laid from B to *a*, will cut the primitive Circle in *c*; ſet off 90° from *c* to *d*, and lay a Ruler from B to *d*, which will cut the right Circle A C in *e*; then thro' the three Points B, *e*, and D, by Prob. 17, draw the Circle B *e* D, which will be the required oblique Circle to the Pole *a*.

Spherical TRIGONOMETRY.

PLAIN or right-lined Triangles are formed by the Interſection of right Lines: Spherical Triangles are formed by the Interſection of Circles: The Sides of theſe are Parts of thoſe Circles; and ſpherical Angles are meaſured by the Arch of a great Circle, intercepted between the Arches which conſtitute the Angle.

The Properties of ſpherical Triangles are as follows:

1. The Sum of the three Angles of any ſpherical Triangle are greater than two right Angles, and leſs than ſix right Angles.
2. The Side of a ſpherical Triangle is leſs than a Semicircle.
3. The three Sides of a ſpherical Triangle are leſs than a Circle or 360°.
4. The greateſt Side always ſubtends the greateſt Angle.
5. In a right-angled ſpherical Triangle, if the Legs or Sides containing the right Angle are, each of them, greater or leſs than 90°, then will the Hypothenuſe be leſs than 90°.
6. But, if the two Legs are, one greater, and the other leſs, than 90°, then will the Hypothenuſe be greater than 90°.
7. In a right-angled ſpherical Triangle, if the Legs containing the right Angle are, each of them, greater than 90°, each of the Angles oppoſite will be greater than 90°; but, if each

each of the Legs are leſs than 90°, each of the oppoſite Angles will be leſs than 90°.

8. If the Hypothenuſe be more than 90°, the Sides containing the Angles will be, one leſs, the other more, than 90°; and the Angles oppoſite to thoſe Sides will be, one leſs, the other more, than 90°.

9. If the Hypothenuſe is leſs than 90°, the other Sides will be each more than 90°, or each leſs than 90°; and the Angles oppoſite thoſe Sides will alſo be each more, or each leſs, than 90°, as were the Sides.

The Solution of right-angled ſpherical Triangles.

The Lord *Nepier*, the noble Inventor of Logarithms, diſcovered a Method for ſolving right-angled ſpherical Triangles, which is called *the Catholick Propoſition*: This being beſt adapted for the Eaſe of the Memory, I ſhall endeavour to explain it ſo that the Reader may meet with as little Difficulty as may be in this Part of the Work.

The Hypothenuſe, Baſe, Perpendicular, and the other two Angles, that is, the five Parts of a ſpherical right-angled Triangle, excluſive of the right Angle, are called the *five circular Parts*.

Of theſe *five circular Parts*, if the two of them which are given, and the third which is to be found, are ſo ſituated that no other Part of the Triangle lies between them, it is called an *Extreme conjunct*: The right Angle is not reckoned as any Part, it not being one of the five circular Parts: And the Part that then lies in the Middle of theſe three Parts is called the *middle Part*, and the other two the *Extremes conjunct*.

But, if between the two Parts that are given, and the third which is to be found, there lies any other Part except the right Angle, it is called an *Extreme disjunct*; and *that Part*, between which and the other Parts there lies either a Side or an Angle, is called the *middle Part*; and the two Parts lying together are called the *Extremes disjunct*. This Diſtinction I adviſe the Reader to be careful to obſerve.

Rule 1. When it is an Extreme conjunct, and the middle Part is to be found:

As the Radius
Is to the Tangent of one Extreme,
So is the Tangent of the other Extreme
To the Sine of the middle Part.

Note, That, if the *middle Part*, or either of the *Extremes conjunct*, be the Hypothenuſe, or either of the oblique Angles, then, in the above Proportion, inſtead of the Tangents and Sines, uſe the Co-tangents and Co-ſines; but, if the Baſe, or Perpendicular, ſhould be the middle Part, or the Extremes conjunct, then uſe only the Tangents and Sines.

Rule 2. When it is an Extreme conjunct, and one of the Extremes is to be found:

As the Tangent of the given Extreme
Is to the Radius,
So is the Sine of the middle Part
To the Tangent of the required Extreme.

The ſame *Note* muſt be obſerved here as in the firſt Rule.

Rule 3. When it is an Extreme disjunct, and the middle Part is to be found:

As the Radius
Is to the Co-ſine of one Extreme,
So is the Co-ſine of the other Extreme
To the Sine of the middle Part.

Note, That, if either of the Extremes disjunct be the Hypothenuſe, or either of the oblique Angles, in the above Proportion, inſtead of the Co-ſine, uſe the Sine; but, if the Baſe, or Perpendicular, ſhould be the Extremes disjunct, then uſe the Co-ſine.

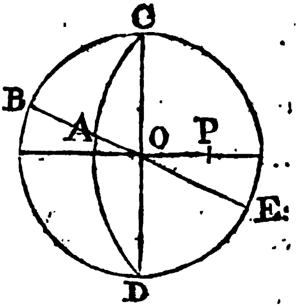
Rule 4. When it is an Extreme disjunct, and one of the Extremes is to be found:

As the Co-ſine of the given Extreme
Is to the Radius,
So is the Sine of the middle Part
To the Co-ſine of the required Extreme.

The ſame *Note* is to be obſerved here as in the third Rule.

Prob. 64. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Hypothenuſe* A C 75°. 20′, *and the Angle* B C A 57°. 16′, *to find the Perpendicular* B C.

Conſtruction. With the Chord of 60°, draw the primitive Circle E D B C; draw the oblique Circle C A D, by Prob. 48, ſo as to make an Angle at C of 57°.16′; by Prob. 44, find P, the Pole of this oblique Circle, and, by Prob. 45, lay on it 75°.20′ from C to A; then draw the right Circle B E thro' the Point A and Center or Pole of the primitive Circle O, which interſects the primitive Circle in B, and forms the Triangle A B C, right-angled at B.



In the ſpherical Triangle A B C there is given, beſides the right Angle at B, the Hypothenuſe A C 75°.20′, and the Angle at C 57°.16′, to find the Perpendicular B C: Theſe Parts having no Parts betwixt them, it is an *Extreme conjunct*, and A C and B C are the two Extremes, and the Angle at C is the middle Part; hence the preſent *Caſe* is *an Extreme conjunct, and one of the Extremes is to be found*: Therefore, to find the Perpendicular B C, by the ſecond Rule, and the *Note*, as the middle Part is an oblique Angle, and one of the Extremes is the Hypothenuſe,

As the Co-tangent of A C, 75°.20′, Co. Ar. 0.582158
Is to the Radius, ——— ——— 10.
So is the Co-ſine of the middle Part BCA, 57°.16′, 9.732980

To the Tangent of the Perpendicular BC, 64°.10′, 10.315138

The Perpendicular B C, being Part of the primitive Circle, may be meaſured with a Line of Chords, by Prob. 41; by which the Learner will ſee the Agreement between the Conſtruction and Calculation.

Prob. 65. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Hypothenuſe* A C 75°.20′, *and the Angle at* C 57°.16′, *to find the other Leg, or Baſe*, A B.

The ſame Things being given as in the laſt Problem, the Conſtruction is the ſame: And here I adviſe the Learner to conſtruct the Figure by the Directions there given, and to mark the Parts given with a —, and the Part required with an o; the Reader will find the Advantage of doing this at every Problem.

The Perpendicular or Leg B C lying between the given Angle at C and the Baſe A B, it is an *Extreme disjunct*, and the Baſe A B, the Part to be found, is the middle Part; therefore, by the third Rule, and its *Note*,

As the Radius ——— ——— 10.
Is to the Sine of one Extreme A C, 75°. 20′, — 9.985613
So is the Sine of the other Extreme B C A, 57°. 16′, 9.924897
To the Sine of the middle Part A B, 54°. 28′, 9.910510

The Baſe A B, being Part of a right Circle, may be meaſured by Prob. 43, Caſe 2.

Prob. 66. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Hypothenuſe* A C 75°. 20′, *and the Angle at* C 57°. 16′, *to find the Angle at* A. See the Figure at Prob. 64, the Conſtruction being the ſame.

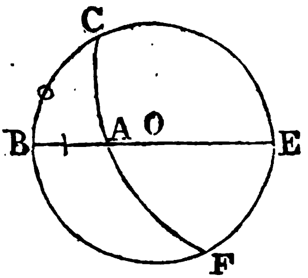
The two Parts given and the Part required lying together, with no Part between them, this is an *Extreme conjunct*, and B A C, one of the Extremes, is to be found; which may be done by the ſecond Rule, and its *Note*, as follows:

As the Co-tangent of B C A, 57°. 16′, one of the Extremes, —— } Co. Ar. 0.191917
Is to the Radius, ——— ——— 10.
So is the Co-ſine of A C, 75°. 20′, the middle Part, 9.403455
To the Co-tangent of B A C, 68°. 30′, the other Extreme, ——— ——— } 9.595372

The Angle B A C, made by the Interſection of the right Circle B O E and the oblique Circle C A D, may be meaſured by Prob. 55.

Prob. 67. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Side* A B 54°. 28′, *and the Angle at* A 68°. 30′, *to find the Leg* B C.

Conſtruction. With the Chord of 60°, draw the primitive Circle B C E F; draw the right Circle B E, which will be perpen-



perpendicular to the primitive Circle, becauſe it paſſes thro' its Pole O; lay 54°. 28′ from B to A, by Prob. 42, Caſe 2; then, by Prob. 57, draw an oblique great Circle, C A F, to make an Angle of 68°. 30′ with the right Circle B E, in the Point A, which completes the ſpherical Triangle A B C, right-angled at B.

Here nothing lying between the Side B C, the Side B A, and the Angle at A, it is an *Extreme conjunct*, and one of the Extremes is to be found; wherefore, by the ſecond Rule, and its *Note*,

As the Co-tangent of A, 68°. 30′, the given Extreme, ——— } Co. Ar. 0.404602
Is to the Radius, ——— ——— 10.
So is the Sine of A B, 54°. 28′, the middle Part, 9.910506

To the Tangent of B C, 64°. 10′, the other Extreme, ——— ——— } 10.315108

The Perpendicular B C may be meaſured on the Line of Chords, by Prob. 41.

Prob. 68. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Baſe or Side* A B 54°. 28′, *and the Angle at* A 68°. 30′, *to find the Angle at* C. See the Figure in the laſt Problem, the Conſtruction being the ſame.

The Hypothenuſe A C lying between the Angles A and C, it is an *Extreme disjunct*, and the middle Part C is to be found; hence, by the third Rule, and its *Note*,

As the Radius ——— ——— 10.

Is to the Co-ſine of the Extreme A B, 54°. 28′, 9.764308
So is the Sine of the Angle at A, 68°. 30′, the other Extreme, ——— ——— } 9.968678

To the Co-ſine of the Angle at C, 57°. 16′, — 9.732986

The Angle at C may be meaſured by Prob. 54.

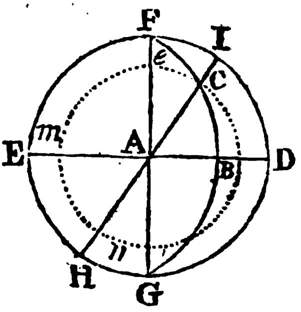
Prob. 69. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Side* A B 54°. 28′, *and the Angle at* A 68°. 30′, *to find the Hypothenuſe* A C. See the Figure at Prob. 67, the Conſtruction being the ſame.

This is an *Extreme conjunct*, and one of the Extremes is to be found; therefore, by the ſecond Rule, and its *Note*,

As the Tangent of the given Extreme AB, 54°. 28′,	10.146198
Is to the Radius ——— ———	10.
So is the Co-ſine of the middle Part BAC, 68°. 30′,	9.564075
	19.564075
	10.146198
To the Co-tang. of the Hypothenuſe AC, 75°. 20′,	9.417877

The Logarithm-Tangent in the firſt Number being greater than the Radius, the arithmetical Complement could not be had. The Truth of the Work may be found by meaſuring the Hypothenuſe, by Prob. 46.

Prob. 70. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Hypothenuſe* A C 75°. 20′, *and the Baſe* A B 54°. 28′, *to find the Perpendicular* B C.



Conſtruction. With the Chord of 60°, draw the primitive Circle E F G D, thro' the Pole or Center A; draw a right Circle, E A D, and croſs it at right Angles with the right Circle F A G; take the Half-Tangent of 75°. 20′, and, with one Foot of the Compaſſes in the Center of the primitive Circle, draw the leſſer Circle *m n e*, parallel to the primitive Circle; lay 54°. 28′ on the right Circle E D, from A to B, by Prob. 42, Caſe 1; then thro' the three Points F, B, and G, by Prob. 17, draw the oblique Circle F B G, interſecting the leſſer Circle *m n e* in C, thro' which and the Pole or Center of the primitive Circle draw the right Circle H A C I, and A B C is the Triangle required.

The Angle B A C falling between the given Sides A B and A C, it is an *Extreme diſjunct*, and the Extreme B C is to be found; whence, by the fourth Rule, and its *Note*,

As the Co-ſine of the given Extreme A B, 54°. 28′, } Co. Ar. 0.235692
Is to the Radius, —— —— 10.
So is the Co-ſine of the middle Part A C, 75°. 20′, 9.403455

To the Co-ſine of the other Extreme B C, 64°. 10′, 9.639147

The Perpendicular or Leg B C, being a Part of the oblique Circle F B G, may be meaſured by Prob. 46.

Prob. 71. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Baſe* A B 54°. 28′, *and the Hypothenuſe* A C 75°. 20′, *to find the Angle at* A.

The Conſtruction is the ſame as in the laſt Problem.

The three Parts lying together, with no Part between, this is an *Extreme conjunct*, and the Angle B A C, required, is the middle Part; whence, by the firſt Rule, and its *Note*,

As the Radius —— —— 10.

Is to the Co-tangent of the Extreme A C, 75°. 20′, 9.417842
So is the Tang. of the other Extreme A B, 54°. 28′, 10.146198

To the Co-ſine of the middle Part B A C, 68°. 30′, 9.564040

The Angle at A, being at the Center of the primitive Circle, is meaſured as a plain Angle, by Prob. 53.

Prob. 72. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Baſe* A B 54°. 28′, *and the Hypothenuſe* A C 75°. 20′, *to find the Angle at* C.

The Conſtruction is the ſame as in the two laſt Problems. See the Figure at Prob. 70.

The Angle B A C lying between the Baſe and the Hypothenuſe, this is an *Extreme diſjunct*, and the Angle at C, which is to be found, is one of the Extremes; therefore, by the fourth Rule, and its *Note*,

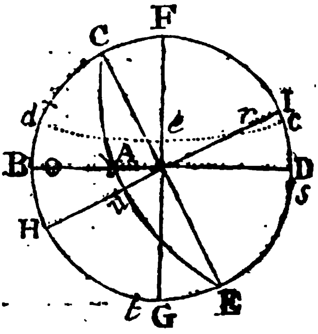
As the Sine of the given Extreme A C, 75° . 20′, } Co. Ar. 0.014387
Is to the Radius, ——— ——— 10.
So is the Sine of the middle Part A B, 54° . 28′, 9.910506

To the Sine of the required Extreme ACB, 57°.16′, 9.924893

The Angle A C B, being made by the Interſection of an oblique Circle and a right Circle, may be meaſured by Prob. 55.

Prob. 73. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Perpendicular* B C 64° . 10′, *and the Angle at* A 68° . 30′, *to find the Baſe* A B.

Conſtruction. With the Chord of 60°, deſcribe the primitive Circle B F D G; thro' which draw, at Pleaſure, the right Circle B D, and croſs it at right Angles with the right Circle F G; from B ſet off 64° . 10′ to C, and draw C E, and at right Angles to it draw H I; from F, both Ways, ſet off 68° . 30′ to *c* and *d*; a Ruler, being laid from B to *c*, or from D to *d*, will cut F G in *e*; thro' the three Points *d*, *e*, and *c*, by Prob. 17, draw the parallel Circle *c e d*, interſecting the right Circle H I in *r*; a Ruler, being laid from C to *r*, cuts the primitive Circle in *s*, from whence lay 90° to *t*; a Ruler, being laid from C to *t*, will cut the right Circle H I in *u*; then, by Prob. 17, draw a Circle thro' the three Points C, *u*, and E, which will interſect the right Circle B D in A, making an Angle, B A C, of 68° . 30, and complete the required Triangle A B C.



The three Parts lying together, with no Part between them, (except the right Angle, which is always excepted) this is an *Extreme conjunct*, and the middle Part A B is to be found; then, by the firſt Rule, and its *Note*,

As the Radius ——— ——— 10.

Is to the Tangent of the Extreme B C, 64° . 10′, 10.315032
So is the Co-tang. of the other Extreme BAC, 68°.30′, 9.595398

To the Sine of the middle Part A B, 54° . 28′, 9.910430

The Baſe A B may be meaſured by Prob. 43, Caſe 2.

Prob. 74. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Perpendicular* B C 64°.10′, *and the Angle at* A 68°.30′, *to find the Hypothenuſe* A C.

See the Figure of the laſt Problem, the Conſtruction being the ſame.

The Baſe lying between the two given Parts, B C and the Angle B A C, this is an *Extreme disjunct*, and the Hypothenuſe A C, the Part to be found, is one of the Extremes; therefore, by the fourth Rule, and its *Note*,

As the Sine of the Extreme BAC, 68°.30′, Co. Ar. 0.031322
Is to the Radius, —— —— 10.
So is the Sine of the middle Part B C, 64°.10′, 9.954274

To the Sine of the Hypothenuſe A C, 75°.20′, 9.985596

The Hypothenuſe A C may be meaſured by Prob. 46.

Prob. 75. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Perpendicular* B C 64°.10′, *and the Angle at* A 68°.30′, *to find the Angle at* C.

The ſame Parts being given as in the two laſt Problems, the Conſtruction is the ſame. See the Figure at Prob. 73.

The Hypothenuſe A C lying between the two Angles B A C and B C A, this is an *Extreme disjunct*, and the required Angle at C is one of the Extremes; therefore, by the third Rule, and its *Note*,

As the Co-ſine of the Extreme B C, 64°.10′, Co. Ar. 0.360758
Is to the Radius, —— —— 10.
So is the Co-ſine of the middle Part BAC, 68°.30′, 9.564075

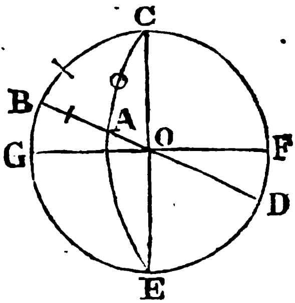
To the Sine of the Angle at C, 57°.16′, — 9.924833

The Angle at C, made by the oblique Circle C A E interſecting the primitive Circle at the Point C, may be meaſured by Prob. 54.

Prob. 76. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Perpendicular* B C 64°.10′, *and the Baſe* A B 54°.28′, *to find the Hypothenuſe* A C.

Conſtruction. With the Chord of 60°, draw the primitive Circle B C D E; thro' the Pole or Center draw the right

Circle B D; take 64° . 10′ from the Line of Chords, and lay it from B to C; draw the right Circle C O E, and croſs it at right Angles with the right Circle F O G; by Prob. 42, Caſe 2, lay on the right Circle B D 54° . 28′, from B to A; then, by Prob. 17, draw an oblique Circle thro' the three Points C, A, E, and the Triangle A B C is completed.



The Angles lying between the two given Parts and the Part required, it is an *Extreme disjunct*, and the middle Part A C is to be ſound; therefore, by the third Rule, and its *Note*,

As the Radius ——— ———	10.
Is to the Co-ſine of the Extreme A B, 54° . 28′,	9.764308
So is the Co-ſine of the other Extreme BC, 64°.10′,	9.639242
To the Co-ſine of the middle Part A C, 75° . 20′,	9.403550

The Hypothenuſe may be meaſured by Prob. 46.

Prob. 77. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Perpendicular* B C 64° . 10′, *and the Baſe* A B 54° . 28′, *to find one of the Angles, as* C.

See the laſt Figure, the Conſtruction being the ſame as in the laſt Problem.

No Part lying between the two given Sides and the Angle required, it is an *Extreme conjunct*, and the Angle at C, one of the Extremes, is to be ſound; whence, by the ſecond Rule, and its *Note*,

As the Tangent of the Extreme A B, 54° . 28′,	10.146198
Is to the Radius, ——— ———	10.
So is the Sine of the middle Part B C, 64° . 10′,	9.954274
To the Co-tangent of B C A, 57° . 16′, —	9.808076

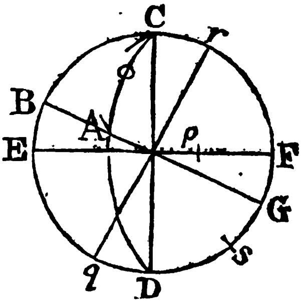
The Angle at C, made by the oblique Circle C A E interſecting the primitive Circle at the Point C, may be meaſured by Prob. 54.

If the ſame Things are given, and the Angle at A required, it is ſtill an *Extreme conjunct*, and the Baſe A B is now the middle Part; to find which, by the firſt Rule, and its *Note*, the Proportion is as follows:

As the Tangent of B C, 64°. 10′, ——	10.315032
Is to the Radius, —— ——	10.
So is the Sine of A B, 54°. 28′, ——	9.910506
	19.910506
	10.315032
To the Co-tangent of B A C, 68°. 30′, ——	9.595474

The Angle at A, made by the right Circle B A D and oblique Circle C A E interſecting each other at the Point A, may be meaſured by Prob. 55.

Prob. 78. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Angle at* A 68°. 30′, *and the Angle at* C 57°. 16′, *to find the Hypothenuſe* A C.



Conſtruction. With the Chord of 60°, draw the Circle E C F D; and, thro' the Pole or Center, draw the right Circle C D, at Pleaſure, and croſs it at right Angles with the right Circle E F; by Prob. 48, draw an oblique Circle, C A D, to make an Angle of 57°. 16′, with the primitive Circle, at the Point C; by Prob. 40, find *p*, the Pole of the oblique Circle C A D; a Ruler, being laid from C to *p*, will cut the primitive Circle in *s*; on the primitive Circle, from *s* to *q*, ſet 68°. 30′, the given Angle at A; then draw the right Circle *q r*, perpendicular to which draw the right Circle B G, interſecting the oblique Circle C A D in A, making an Angle, C A B, of 68°. 30′, and A B C is the Triangle required.

The three Parts lying together, this is an *Extreme conjunct*, and the Hypothenuſe A C is the middle Part; therefore, by the firſt Rule, and its *Note*,

As the Radius ——— ——— 10.

Is to the Co-tangent of B A C, 68°. 30′, — 9.595398
So is the Co-tangent of B C A, 57°. 16′, — 9.808083

To the Co-Sine of the middle Part A C, 75°. 20′, 9.403481

The Hypothenuſe A C may be meaſured by Prob. 46.

Prob. 79. *In the right-angled ſpherical Triangle* A B C, *right-angled at* B, *given the Angle at* A 68°. 30′, *and the Angle at* C 57°. 16′, *to find one of the Legs, as the Baſe* A B.

The Conſtruction is the ſame as in the laſt Problem. See the Figure there.

The Hypothenuſe A C lying between the given Angles, it is an *Extreme disjunct*, and the Baſe A B is one of the Extremes; therefore, by the fourth Rule, and its *Note*,

As the Sine of the Extreme BAC, 68°. 30′, Co. Ar. 0.031322
Is to the Radius, ——— ——— 10.
So is the Co-ſine of the middle Part BCA, 57°. 16′, 9.732980

To the Co-ſine of the Baſe A B, 54°. 28′, — 9.764302

The Baſe A B may be meaſured by Prob. 43, Caſe 2.

If the Perpendicular B C had been required, it would ſtill have been an *Extreme disjunct*; but the Angle at A would then have been the middle Part, and the Perpendicular might be found by the fourth Rule, and its *Note*; thus,

As the Sine of the Extreme BCA, 57°. 16′, Co. Ar. 0.075102
Is to the Radius, ——— ——— 10.
So is the Co-ſine of the middle Part, BAC, 68°. 30′, 9.564075

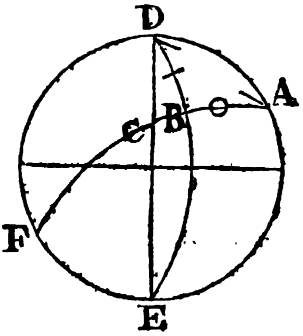
To the Co-ſine of the Perpendicular B C, 64°. 10′, 9.639177

Of

Of Oblique Spherical Trigonometry.

The two firſt Caſes of oblique-angled ſpherical Triangles are ſolved by this Axiom: *The Sines of the Sides of all ſpherical Triangles are in Proportion to the Sines of their oppoſite Angles.*

Prob. 80. *In the oblique-angled ſpherical Triangle* A B D, *given the Angle* B D A 64°. 15′, *the Angle* B A D 59°. 47′, *and the Side* B D 47°. 21′, *oppoſite to one of the given Angles, to find the Side* B A, *oppoſite the other given Angle.*



Conſtruction. With the Chord of 60°, deſcribe the primitive Circle A E F D; at any Point, as D, of the Periphery, by Prob. 48, draw the oblique Circle DE, ſo as to make an Angle with the primitive Circle, at D, of 64°. 15′, and lay 47°. 21′ on this oblique Circle, by Prob. 45, from D to B; thro' B, by Prob. 48, draw an oblique Circle, ACF, to make an Angle, DAB, of 59°. 47′ with the primitive Circle, and form the oblique Triangle A B D.

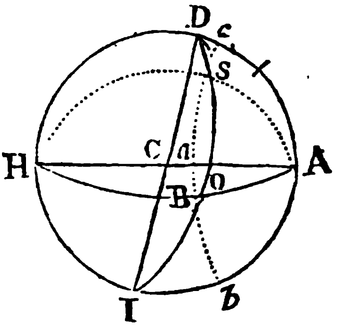
Here being given the Angle at A, oppoſite the given Side BD, and the Angle at C, oppoſite the required Side B A, this may be found by the following Proportion, founded on the above Axiom:

As the Sine of B A D, 59°. 47′, —— Co. Ar.	0.063422
Is to the Sine of the oppoſite Side B D, 47°. 21′,	9.866586
So is the Sine of B D A, 64°. 15′, ——	9.954579
To the Sine of the oppoſite Side A B, 50°. 3′,	9.884587

The Side A B, being Part of the oblique Circle A B F, may be meaſured by Prob. 46.

Prob. 81. *In the oblique-angled ſpherical Triangle* A B D, *given the Side* A D 74°. 20′, *the Side* A B 65°. 34′, *and the Angle at* D 59°. 50′, *to find the Angle at* B.

Conſtruction. With the Chord of 60°, deſcribe the primitive Circle H D A I; on which lay 74°. 20′ from any Point aſſumed



aſſumed, as A, to D, and draw the right Circles D C I and A C H; at the Point D, by Problem 48, draw an oblique Circle, D B I, ſo that it may make an Angle, I D A, of 59°. 50′ with the primitive Circle; take 65°. 34′ from the Line of Chords, and ſet from A, both Ways, to *b* and *c*; by Prob. 42, Caſe 2, lay 65°. 34′ from A to *a*, on the right Circle A C H; thro' the three Points *b*, *a*, and *c*, by Prob. 17, draw the parallel Circle *b a c*, interſecting the oblique Circle D I in B; laſtly, by Prob. 17, draw an oblique Circle thro' the Points H, B, and A, and the Triangle A B D is the Triangle required.

But here it is to be obſerved, that the parallel Circle *b a c* cuts the oblique Circle D I in two Places, *viz.* in B and *s*, whence the Caſe is ambiguous; for it can't be determined whether the Triangle A B D or the Triangle A *s* D be the true Solution, ſince the Side A *s* is 65°. 34′ as well as the Side A B, as may be found by meaſuring them, as taught in Prob. 46; and this is always the Caſe when the Side oppoſite the given Angle is leſs than the other given Side.

In the oblique-angled Triangle A B D, we have the Side A B oppoſite the given Angle at D, and the Side A D oppoſite the required Angle at B; therefore,

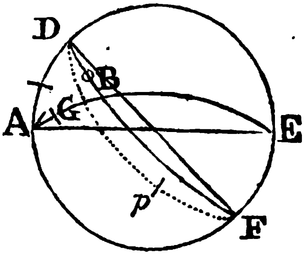
As the Sine of the Side A B, 65°. 34′, Co. Ar.	0.040747
Is to the Sine of the oppoſite Angle A D B, 59°. 50′,	9.936799
So is the Sine of the Side A D, 74°. 20′, —	9.983558
To the Sine of its oppoſite Angle A B D, 66°. 7′,	9.961104

The Angle at B, being made by the Interſection of the two oblique Circles H B A and D B I, may be meaſured by Prob. 56.

Prob. 82. *In the oblique-angled ſpherical Triangle* A B D, *given the Side* A D 45°. 36′, *the Side* A B 48°. 50′, *and the included Angle at* A 54°. 19′, *to find the Side* D B.

Conſtruction. With the Chord of 60°, draw the primitive Circle A D E F; from any Point aſſumed, as A, ſet off

45°.36′ to D, and draw D F and A E; by Prob. 48, draw an oblique Circle, E B A, to make an Angle of 54°.19′ with the primitive Circle, in the Point A; and, by Prob. 45, lay 48°.50′ on this oblique Circle, from A to B; laſtly, thro' the Points D, B, and F, by Prob. 17, draw the oblique Circle D B F, and the Triangle A B D will be the Triangle required.



In order to ſolve this Problem, and moſt of the following Caſes, it is neceſſary to let fall a Perpendicular, which muſt always fall *from the End of a given Side oppoſite to a given Angle.* The Side on which the Perpendicular falls is uſually called the Baſe: And, if the Angles at the Baſe are, either both more, or both leſs, than 90°, the Perpendicular will fall within the Triangle, and divide it into two right-angled Triangles: But, if one of the Angles of the Baſe is leſs than 90°, and the other greater, the Perpendicular will fall without the Triangle, and, by that Means, add a right-angled Triangle to the given oblique Triangle: In both Caſes, the Parts may be found by the Rules laid down in the ſeveral Caſes of right-angled ſpherical Triangles, from Prob. 64 to Prob. 79, incluſive.

In the preſent Caſe, a Perpendicular, being let fall from D on the Baſe A B, will divide the Triangle A B D into two right-angled Triangles, A D G and B D G, both right-angled at G. To do this, find *p*, the Pole of the oblique Circle E B A; then, by Prob. 17, draw the oblique Circle D *p* F, which falls from D, the End of the given Side A D, oppoſite the given Angle at A, and is perpendicular to E B A, the Circle on which it is to fall, becauſe it paſſes thro' its Pole *p*.

Now, in the right-angled Triangle A D G, we have the Side A D 45°.36′, and the Angle at A 54°.19′, to find the Perpendicular D G; and, the Angle A D G lying between the given Side A D and the required Side D G, it is an *Extreme disjunct*, and D G is the middle Part; whence, by the third Rule, and its *Note*,

As the Radius	10.
Is to the Co-ſine of A D, 45°. 36′,	9.844889
So is the Sine of DAG, 54°. 19′,	9.909692
To the Sine of D G, 34°. 38′,	9.754581

To find the Side A G in the ſame Triangle A D G; the ſame Things being given, it is an *Extreme conjunct*, and the Side A G is one of the Extremes; whence, by the ſecond Rule, and its *Note*,

As the Co-tangent of A D, 45°. 36′,	Co. Ar. 0.009097
Is to the Radius,	10.
So is the Co-ſine of D A G, 54°. 19′,	9.765896
To the Tangent of A G, 30°. 13′,	9.774993

Then ſubtract A G 30°. 13′ from A B 48°. 50′, and the Remainder is B G 18°. 37′: Then, to find the Hypothenuſe D B, in the right-angled Triangle B D G, we have the Perpendicular D G 34°. 38′, found by the firſt Proportion, and the Side B G, by the laſt: Now, the Angle at B lying between B G and B D, it is an *Extreme disjunct*, and the Hypothenuſe B D, the middle Part, is required; therefore, by the third Rule, and its *Note*,

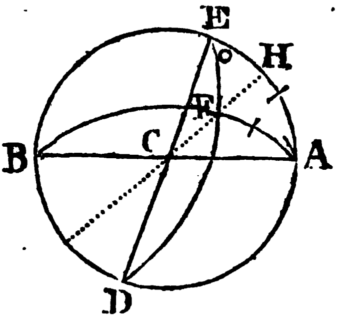
As the Radius	10.
Is to the Co-ſine of D G, 34°. 38′,	9.915297
So is the Co-ſine of B G, 18°. 37′,	9.976660
To the Co-ſine of D B, 39°. 47′,	9.891957

The Side D B, being Part of the oblique Circle D B F, may be meaſured by Prob. 46.

A Perpendicular might have been let fall from the angular Point to the Side A D, by drawing a right Circle thro' the Pole of the primitive Circle, as the Side A D, on which it would fall, is a Part of the primitive Circle: By this Means, the Side D B might have been found in the ſame Manner.

Prob. 83. *In the oblique-angled ſpherical Triangle* A E F, *given the Side* A E 72°. 34′, *the Side* A F 53°. 12′, *and the included Angle at* A 47°. 32′, *to find the Angle at* E.

There being the ſame Things given as in the laſt Problem, the Conſtruction is the ſame.



If a Perpendicular is let fall from the angular Point at F, on the Side A E, which is done by drawing a right Circle thro' C, the Pole of the primitive Circle, and F, then will the oblique Triangle E F A be divided into two right-angled Triangles, A F H and E F H, both right-angled at H: In the Triangle A F H there is given the Hypothenuſe A F 53°. 12′, and the Angle at A 47°. 32′, to find the Perpendicular F H.

The Angle A F H lying between the given Part A F and the required Part F H, it is an *Extreme disjunct*, and the Perpendicular F H is the middle Part; therefore, by the third Rule, and its *Note*,

As the Radius ——— ———	10.
Is to the Sine of A F, 53°. 12′, ——	9.903487
So is the Sine of F A H, 47°. 32′, ——	9.867862
To the Sine of the middle Part F H, 36°. 12′,	9.771349

To find the Side A H, the ſame Parts being ſtill given as in the laſt Calculation, it is now an *Extreme conjunct*, and A H, one of the Extremes, is required; by the ſecond Rule, and its *Note*,

As the Co-tangent of the Hypothenuſe AF, 53°. 12′, ——— ———	} Co. Ar. 0.126043
Is to the Radius, ——— ———	10.
So is the Co-ſine of the middle Part FAH, 47°. 32′,	9.829407
To the Tangent of A H, 42°. 4′, ———	9.955450

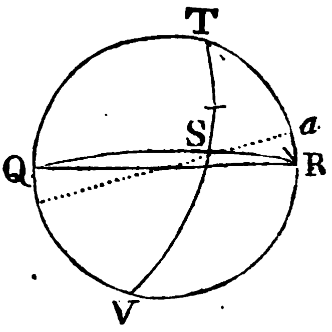
Which being ſubtracted from A E 72°. 34′, there remains H E 30°. 30′; therefore, in the right-angled Triangle E F H, we have given the Side F H 36°. 12′, and the Side H E 30°. 30′, to find the Angle at E; which is an *Extreme conjunct*, and the Angle at E, being one of the Extremes, is found by the ſecond Rule, and its *Note*:

As the Tangent of F H, 36°. 12′, —	Co. Ar. 0.135555
Is to the Radius, —— ——	10.
So is the Sine of the middle Part H E, 30°. 30′,	9.70469
To the Co-tangent of the Angle at E, 55°. 15′,	9.841024

The Angle at E, being made at the Periphery of the primitive Circle by the oblique Circle D F E, may be meaſured by Prob. 54.

Prob. 84. *In the oblique ſpherical Triangle* R S T, *given the Angle at* R 82°. 19′, *the Angle at* T 59°. 10′, *and the Side* T S, *oppoſite one of the given Angles*, 69°. 18′, *to find the third Angle at* S.

The ſame Parts being given here as in Prob. 80, the Conſtruction will be the ſame.



And, if a Perpendicular is let fall from either End of the given Side T S, as S, on the oppoſite Side T R, which Perpendicular will be a right Circle, drawn thro' the Pole of the primitive Circle and the Angle at S, the oblique Triangle R S T will be divided into two right-angled Triangles, T S *a* and R S *a*, both right-angled at *a*: In the Triangle T S *a*, there is given the Side T S, and the Angle at T, to find the Side S *a*; therefore it is an *Extreme disjunct*, and the Side S *a*, which is the middle Part, is found by the third Rule, and its *Note*:

As the Radius —— ——	10.
Is to the Sine of T S, 69°. 18′, ——	9.971018
So is the Sine of S T *a*, 59°. 10′, ——	9.933822
To the Sine of the middle Part S *a*, 53°. 26′,	9.904840

To find the Angle T S *a*: In the ſame Triangle T S *a*, we have given the ſame Parts as before, to find the Angle T S *a*; which is an *Extreme conjunct*, and the Angle at S is one of the Extremes; therefore, by the ſecond Rule, and its *Note*,

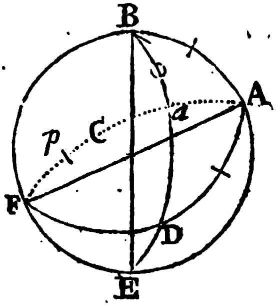
As the Co-tangent of S T *a*, 59°. 10′,		Co. Ar. 0.224092
Is to the Radius,	——— ——	10.
So is the Sine of the middle Part T S, 69°. 18′,		9.548359
To the Co-tangent of T S *a*, 59°. 22′,	—	9.772451

Having found the Angle T S *a*, we are next to find the Angle R S *a*, the other Part of the required Angle R S T; to find which, we have, in the Triangle R S *a*, right-angled at *a*, given the Side S *a*, before found, and the given Angle T R S, or, which is the ſame, the Angle *a* R S; which is an *Extreme disjunct*, and, the Angle at R being the middle Part, the Angle T S *a*, which is to be found, is one of the Extremes; therefore, by the fourth Rule, and its *Note*,

As the Co-ſine of S *a*, 53°. 26′,	—	Co. Ar. 0.224930
Is to the Radius,	—— ———	10.
So is the Co-ſine of *a* R S, 82°. 19′,	——	9.126125
To the Sine of R S *a*, 12°. 58′,	———	9.351055

To this adding the Angle T S *a*, found before, it makes the Angle T S R 72°. 20′, which, being made by the Interſection of two oblique Circles, may be meaſured by Prob. 56, Caſe 2.

Prob. 85. *In the oblique-angled ſpherical Triangle* A D B, *given the Side* A D 79°. 25′, *the Side* A B 72°. 36′, *and the Angle at* B 54°. 29′, *to find the* third *Side* D B.



The ſame Parts being given here as in Prob. 81, the Conſtruction will be the ſame. And, to let fall a Perpendicular from the Angle at A, on the Side B D, find *p*, the Pole of the Circle B D E; then, thro' the three Points F, *p*, A, draw the oblique Circle F *p* A, which will be perpendicular to the oblique Circle B D E, and divide the Triangle B A D into two right-angled Triangles, A *a* B and A *a* D, both right-angled at *a*: In the

Triangle A *a* B, there is given the Side A B, and the Angle at B, to find the Perpendicular A *a*; which being ſeparated from the two other Parts, it is an *Extreme disjunct*, and the Part required is the middle Part; therefore, by the third Rule, and its *Note*,

As the Radius —— ——	10.
Is to the Sine of the Hypothenuſe A B, 72°.36′,	9.979658
So is the Sine of *a* B A, 54°.29, ——	9.910596
To the Sine of the middle Part *a* A, 50°.58′,	9.890254

To find B *a*: With the ſame Parts given, the Part of the Baſe *a* B may be found, this being an *Extreme conjunct*, and the required Side is one of the Extremes; therefore, by the ſecond Rule, and its *Note*,

As the Co-tangent of A B, 72°.16′,	Co. Ar. 0.495146
Is to the Radius, —— ——	10.
So is the Co-ſine of the middle Part *a* BA, 54°.29′,	9.764131
To the Tangent of *a* B, 61°.10′, ——	10.259277

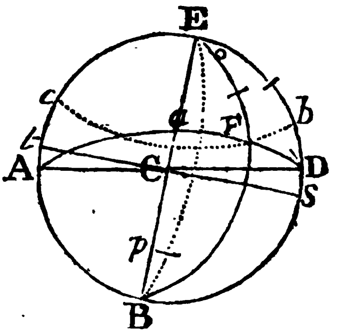
In the right-angled Triangle A *a* D, we have given the Hypothenuſe A D, and the Perpendicular *a* A, found before, to find *a* D, the other Part of the required Side B D, which is an *Extreme disjunct*, and the required Side is one of the Extremes; therefore, by the fourth Rule, and its *Note*,

As the Co-ſine of A *a*, 50.58′, ——	Co. Ar. 0.200816
Is to the Radius, —— ——	10.
So is the Co-ſine of A D, 79°.25′, ——	9.264027
To the Co-ſine of *a* D, 73°.2′, ——	9.464843

This, being added to *a* B 61°.10′, before found, is 134°.12′, the Side B D, ſought, which may be meaſured by Prob. 46.

Prob. 86. *In the oblique-angled ſpherical Triangle* D E F, *given the Side* D E 79°.56′, *the Side* E F 66°.29′, *and the Angle at* D 62°.40′, *to find the Angle at* E.

Conſtruction. With the Chord of 60°, deſcribe the primitive Circle A E D B; draw a right Circle, A D, at Pleaſure; from



from D lay 79°. 56′ to E, and draw the right Circle E B, which croſs at right Angles with the right Circle *s t*; by Prob. 48, draw the oblique Circle A F D, to make an Angle of 62°. 40′ at the Point D; by Prob. 61, draw the parallel Circle *b* F *c* 23°. 31′, the Complement of E F, diſtant from *s t*, interſecting A F D in F; laſtly, thro' the three Points E, F, B, by Prob. 17, draw the oblique Circle E F B, which forms the required Triangle D E F. Draw an oblique Circle from the Angle at E, thro' *p*, the Pole of A F D, to B, which will be perpendicular to A F D, and fall without the Triangle on the Baſe D F, produced at the Point *a*; by which is found the right-angled Triangle *a* E D, right-angled at *a*; in which is given the Side D E, and the Angle at D, to find the Perpendicular *a* E: This is an *Extreme disjunct*, and *a* E is the middle Part; therefore, by the third Rule, and its *Note*,

As the Radius —— —— 10.

Is to the Sine of the Hypothenuſe E D, 79°. 56′, 9.993262
So is the Sine of *a* D E, 62°. 40′, —— 9.948584

To the Sine of the middle Part *a* E, 61°, — 9.941846

The ſame Parts being given, in the ſame right-angled Triangle E *a* D, to find the Angle *a* E D, will be an *Extreme conjunct*, and the required Angle at E is one of the Extremes; therefore, by the ſecond Rule, and its *Note*,

As the Co-tangent of *a* D E, 62°. 40′, Co. Ar. 0.286614
Is to the Radius, —— —— 10.
So is the Co-ſine of the middle Part E D, 79°. 56′, 9.242526

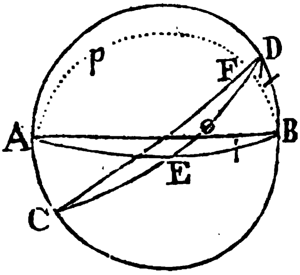
To the Co-tangent of *a* E D, 71°. 19′, — 9.529130

Having found the Angle *a* E D, to find the Angle *a* E F, in the Triangle E *a* F, right-angled at *a*, given the Hypothenuſe E F 66°. 29′, and the Perpendicular *a* E 61°, before found; whence this is an *Extreme conjunct*, and the middle Part is required; therefore, by the firſt Rule, and its *Note*,

As the Radius ——— ———	10.
Is to the Co-tangent of E F, 66°. 29′, —	9.638647
So is the Tangent of *a* E, 61°, ———	10.256248
To the Co-ſine of *a* E F, 38°. 17′, ———	9.894895

This, being ſubtracted from the Angle *a* E D, before found, leaves the required Angle F E D 33°. 2′; which, being made at the Periphery of the primitive Circle by the oblique Circle E F B, may be meaſured by Prob. 54.

Prob. 82. *In the oblique-angled ſpherical Triangle* B D E, *given the Side* B D 35°. 27′, *the Side* B E 82°. 10′, *and the Angle at* D 79°. 44′, *to find the third Side* E D.



The ſame Parts being given as in Prob. 81, the Conſtruction of the Triangle E B D is the ſame. And, to let fall a Perpendicular from the Angle at B, on the Side D E, find *p*, the Pole of the oblique Circle B E A; then, thro' the three Points A, *p*, B, draw, by Prob. 17, the oblique Circle A *p* F B, which will be perpendicular to D E C, and divide the Triangle B D E into two right-angled Triangles, E B F and B D F, both right-angled at F: In the Triangle D B F, is given the Side B D, and the Angle at D, to find the Perpendicular B F; which, being an *Extreme disjunct*, and the required Part B F being the middle Part, may be found by the third Rule, and its *Note*,

As the Radius ——— ———	10.
Is to the Sine of the Hypothenuſe B D, 35°. 27′,	9.763422
So is the Sine of B D F, 79°. 44′, ———	9.992990
To the Sine of the middle Part, B F, 34°. 48′,	9.756412

And, in the ſame Triangle, from the ſame Parts given, to find the Side D F, will be an *Extreme conjunct*, and D F, the Part required, is one of the Extremes; therefore, by the ſecond Rule, and its *Note*,

As the Co-tangent of B D, 35° . 27′, — 10.147534

Is to the Radius, ——— ——— 10.
So is the Co-ſine of B D F, 79° . 44′, ——— 9.250980

To the Tangent of D E, 7° . 14′, ——— 9.103446

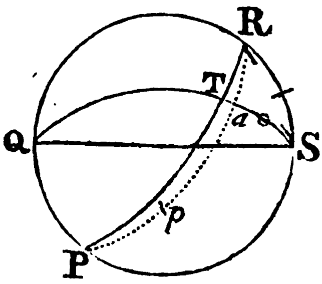
Now, in the Triangle B F E, right-angled at F, having found B F, and the Side B E being given, the Side E F may be found by the fourth Rule, and its *Note*, ſince it is an *Extreme disjunct*, and E F is one of the Extremes; therefore,

As the Co-ſine of B F, 34° . 48′, — Co. Ar. 0.085578
Is to the Radius, ——— ——— 10.
So is the Co-ſine of BE, 82° . 10′, ——— 9.134470

To the Co-ſine of E F, 80° . 26′, ——— 9.220048

Then D F and F E, being added together, make 87° . 40′ for the required Side E D; which, being a Part of an oblique Circle, may be meaſured by Prob. 46.

Prob. 88. *In the oblique-angled ſpherical Triangle* R S T, *given the Angle at* R 73° . 42′, *the Angle at* S 45° . 36′, *and the Side* R S 52° . 28′, *to find an oppoſite Side, as* S T.



Conſtruction. With the Chord of 60°, draw the primitive Circle P Q R S; from any Point, as S, ſet off 52° . 28′ from S to R; then, by Prob. 48, draw an oblique Circle, R T P, to make an Angle at R of 73° . 42′; and, by the ſame Problem, draw another oblique Circle, Q T S, to make an Angle at S of 45° . 36′, interſecting the former oblique Circle in T, and thereby forming the Triangle T R S. To let fall a Perpendicular from the Angle at R, on the Side S T, find *p*, the Pole of the oblique Circle; then, thro' the three Points R, *p*, P, draw, by Prob. 17, the oblique Circle R *p* P, which will be perpendicular to Q T S, and divide the Triangle R S T into two right-angled Triangles, *a* R T and *a* R S, both right-angled at *a*: In the Triangle R S *a*, given the Side R S, and the Angle at S, to find the Perpendicular *a* R; which, being an *Extreme diſjunct*, and the Side *a* R the middle Part, may be found by the third Rule, and its *Note:*

As the Radius —— —— 10.
Is to the Sine of the Hypothenuſe R S, 52° . 28′, 9.899273
So is the Sine of *a* S T, 45° . 36′, —— 9.853986

To the Sine of the middle Part *a* R, 34° . 31′, 9.753259

In the ſame Triangle, the ſame Parts being given, the Side *a* S may be found by the ſecond Rule, and its *Note*, ſince it is an *Extreme conjunct*, and the required Side *a* S is one of the Extremes; therefore,

As the Co-tangent of R S, 52° . 28′, — Co. Ar. 0.114496
Is to the Radius, —— —— 10.
So is the Co-ſine of the middle Part *a* S R, 45° . 36′, 9.844889

To the Tangent of *a* S, 42° . 19′, —— 9.959385

In the ſame Triangle, given the Side R S, oppoſite the right Angle at *a*, and the Side *a* S, found by the laſt Proportion, oppoſite the Angle *a* R S, the next Part to be found; this may be obtained by the general Axiom; thus,

As the Sine of R S, 52° . 28′, —— Co. Ar. 0.100727
Is to the Radius, —— —— 10.
So is the Sine of *a* S, 42° . 19′, —— 9.828162

To the Sine of *a* R S, 58° . 6′, —— 9.928889

This being taken from the given Angle T R S 73° . 42′, there remains 15° . 36′, for the Angle T R *a*; therefore, in the right-angled Triangle *a* R T, we have the Angle T R *a* 15° . 36′, and the Side *a* R 34° . 31′, to find the Side *a* T; which lying together, with nothing betwixt them but the right Angle, it is an *Extreme conjunct*, and *a* T is one of the Extremes; therefore, by the ſecond Rule, and its *Note*,

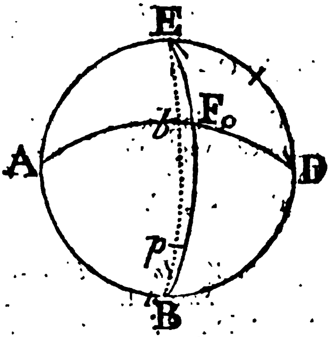
As the Co-tangent of T R *a*, 15°.36′, — 10.554077

Is to the Radius, ——— ——— 10.
So is the Sine of *a* R, 34°.31′, the middle Part, 9.753312

To the Tangent of *a* T, 8°.59′, ——— 9.199235

Which being added to the Side S *a* 42°.19′, before found, makes 51°.18′, the Side T S, required.

Prob. 89. *In the oblique-angled ſpherical Triangle* D E F, *given the Angle at* D 63°.45′, *the Angle at* E 71°.49′, *and the Side* D E 86°.25′, *to find the third Angle at* F.



The ſame Parts being given here as in the laſt Problem, the Conſtruction is the ſame. The Perpendicular E *b* B, being let fall from the Angle at E, on A F D, as in the laſt Problem, will interſect the Baſe F D, produced in *b*, making the two right-angled Triangles *b* E D and *b* E F: In the Triangle *b* E D, given the Side E D, and the Angle at D, to find the Perpendicular *b* E; which is an *Extreme disjunct*, and the Side *b* E is the middle Part; therefore, by the third Rule, and its *Note*,

As the Radius ——— ——— 10.

Is to the Sine of the Hypothenuſe E D, 86°.25′, 9.999150
So is the Sine of *b* D E, 63°.45′, ——— 9.952731

To the Sine of the middle Part *b* E, 63.31′, — 9.951881

And, in the ſame Triangle, from the ſame Parts given, to find the Angle *b* E D, is an *Extreme conjunct*, and the required Angle is one of the Extremes; therefore, by the ſecond Rule, and its *Note*,

As the Co-tangent of *b* D E, 63°.45′, Co. Ar. 0.307025
Is to the Radius, ——. —— 10.
So is the Co-ſine of the middle Part E D, 86°.25′, 8.795881

To the Co-tangent of *b* E D, 82°.47′, — 9.102906

From which ſubtract the given Angle F E D 63°.45′, and the Remainder is the Angle *b* E F 19°.2′. Then, in the Triangle *b* E F, right-angled at *b*, given the Side *b* E, and the Angle *b* E F, to find the Angle *b* F E, the Supplement of the required Angle D F E to 180°; this is an *Extreme diſjunct*, and the Angle *b* F E is the middle Part; therefore, by the third Rule, and its *Note*,

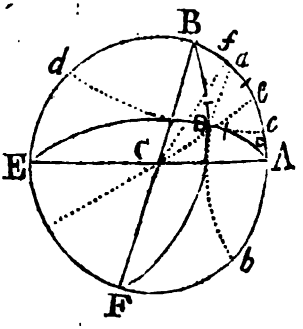
As the Radius —— —— 10.

Is to the Co-ſine of *b* E, 63°.31′, —— 9.649781
So is the Sine of *b* E F, 19°.2′, —— 9.513375

To the Co-ſine of the middle Part *b* F E, 81°.21′, 9.163156

This being ſubtracted from 180°, the Remainder is the required Angle D F E 98°.39′; which, being made by the Interſection of two oblique Circles at F, may be meaſured by by Prob. 56, Caſe 2.

Prob. 86. *In the oblique-angled ſpherical Triangle* A B D, *given the Side* A B 74°.20′, *the Side* B D 58°.46′, *and the Side* A D 47°.38′, *to find an Angle, as the Angle at* A.



Conſtruction. With the Chord of 60°, draw the Circle ABEF; draw, at Pleaſure, the right Circle E C A; lay 74°.20′ from A to B, and draw B F; by Prob. 61, draw a Circle, *a* D *b*, at 47°.38′ diſtant from A; alſo, by the ſame Problem, draw *d* D *c*, at 58°.46′ diſtant from B, which will interſect the former in D; laſtly, thro' the Points A, D, and E, by Prob. 17, draw the oblique Circle E D A; and, thro' the Points B, D, and F, by the ſame Problem, draw the oblique Circle B D F, which will complete the Triangle A B D.

Here being no given Angle, the Perpendicular muſt be let fall oppoſite to the required Angle at A; therefore, thro' D, and C, the Pole of the primitive Circle, draw the right Circle C D *e*, which will be perpendicular to the primitive Circle: Make *e f* equal to A *e*, then will A B be the Baſe, and *f* B the alternate Baſe; to find which, add the two Sides A D and B D together, and take the Half of their Sum; then ſubtract the leſſer Side A D from the greater Side B D, and take Half the Difference; alſo take Half the Baſe A B:

58° . 46′ B D.
47 . 38 . A D.
106 . 24 Sum.
53 . 12 Half-Sum.
11 . 8 Difference.
5 . 34 Half-Difference.

74° . 20′ Baſe A B.
37 . 10 Half.

Having the Half of the Baſe A B, and Half the Sum and Difference of the Sides A D and D B, the alternate Baſe B *f* is thus found:

As the Tang. of Half the Baſe AB, 37°. 10′, Co. Ar. 0.120259
Is to the Tangent of Half the Sum of the Sides, 53°. 10′, ——— ——— } 10.126043
So is the Tangent of Half the Difference of the Sides, 5° . 34′, ——— ——— } 8.988842

To the Tangent of Half B *f*, the alternate Baſe, 9° . 45′, ——— ——— } 9.235144

Which, being added to Half the Baſe A B 37° . 10′, is the Side B *e* 46° . 55′, the Baſe of the right-angled Triangle B D *e*; and, being ſubtracted from the Half-Baſe, the Remainder is A *e* 27° . 25′, the Baſe of the right-angled Triangle A D *e*; in which there is given the Side A D 47° . 38′, and the Side A *e* 27° . 25′, to find the Angle at A; which is an *Extreme conjunct*, and the Angle at A is the middle Part; therefore,

As the Radius	——— ———	10.
Is to the Co-tangent of A D, 47°. 38′,	—	9.960023
So is the Tangent of A *e*, 27°. 25′,	——	9.714933
To the Co-ſine of the Angle at A, 61°. 46′,	—	9.674956

This Problem may be ſolved another Way, without drawing a Perpendicular: Thus, Add the three Sides together, and take Half the Sum; then ſubtract each of the two Sides containing the Angle from the Half-Sum:

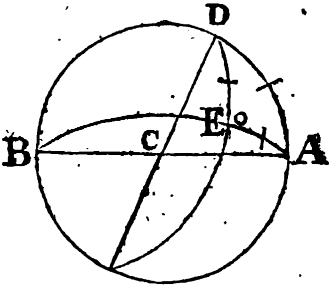
74°. 20′ A B.	90°. 22′ Half-Sum.	90°. 22′
58 . 46 B D.	74 . 20 A B ſubtract.	47 . 38 A D.
47 . 38 A D.		
	16 . 2 Remainder.	42 . 44
180 . 44 Sum.		
90 . 22 Half-Sum.		

Then add the Sines of theſe Remainders, *viz.* 16°. 2′, the Difference between Half the Sum of the Sides and A B, and 42°. 44′, the Difference between Half the Sum of the Sides and the Side A D, to the arithmetical Complements of the Sines of the two Sides containing the required Angle; Half that Sum is the Sine of Half the required Angle, which, being doubled, gives the required Angle at one Operation:

Sine of 16°. 2′	——	9.441218
Sine of 42°. 44′	——	9.831606
Sine of 74°. 20′	——	Co. Ar. 0.016442
Sine of 47°. 38′	——	Co. Ar. 0.131446
	Sum —	19.420712
Sine of 30°. 53′	——	9.710356
2		
61 . 46 Angle B A D, as before.		

The Angle at A, being made at the Periphery, may be meaſured by Prob. 54: The other Angles may now be found by the general Axiom, Page 277; by which the Angle B D A will be found to be 67°. 28′, and the Angle D B A 49°. 35′, which the Learner may try for his Improvement.

Prob.



Prob. 91. *In the oblique-angled ſpherical Triangle* A D E, *given the Angle* A D E 37°. 19′, *the Angle* A E D 62°. 25′, *and the Angle* D A E 49°. 54′, *to find the Sides.*

The Way to conſtruct this Problem is to conſtruct, by the laſt Problem, a Triangle whoſe Sides ſhall be equal to the given Angles, as the Triangle A D E; where A E is 37°. 19′, A D is 62°. 25′, and E D is 49°. 54′: Now, by either of the Rules in the laſt Problem, find the required Angle, as A E D; thus,

62° . 25′	A D.	74° . 49′	Half-Sum.	74° . 49′	
49 . 54	E D.	49 . 54	E D ſubtract.	37 . 19	A E.
37 . 19	A E				
		24 . 55	Remainder.	37 . 30	
149 . 38	Sum.				
74 . 49	Half.				

Sine of 24° . 55′		9.624591
Sine of 37° . 30′		9.784447
Sine of 49° . 54′	Co. Ar.	0.116383
Sine of 37° . 19′	Co. Ar.	0.217369
	Sum	19.742790
Sine of 4[illegible]° . 3′		9.871395

2

96 . 6 Angle A E D.

The Angle at E is thus found to be 96°. 6′; then the Angle at A will be found 59°. 6′, and the Angle at D 42°. 50′, in the common Manner; and the Angles thus found are the Sides of a Triangle whoſe Angles are the ſame with thoſe in the Problem, the greateſt Side being oppoſite to the greateſt Angle; for, if an oblique-angled ſpherical Triangle be conſtructed by the laſt Problem, whoſe Sides are 96°. 6′, 59°. 6′, and 42°. 50′, the Angles will be the ſame with the Angles given in this Problem: This I leave to the Learner's Exerciſe.

Of ASTRONOMY.

BY ASTRONOMY is commonly meant the Knowledge of the Stars, and the Laws of their Motions: But what is here intended is the Doctrine of the Sphere, and its ſeveral Circles.

In order to this, it will be neceſſary, in the firſt Place, to give an Account of the ſeveral imaginary Circles of the Sphere.

Definitions.

1. A *Sphere*, in the common Acceptation of the Word, is a Globe or Ball; but by the *Sphere, in Aſtronomy*, we mean the concave Surface of the Heavens, in which the Planets have their Motions in their ſeveral Orbs, and in which the Sun and Planets, as well as the fixed Stars, appear to move round the Earth in twenty-four Hours; this laſt being only an apparent Motion, ſince it is occaſioned by the Rotation of the Earth upon its own Axis. On this Sphere are contained divers Circles, both great and ſmall.

2. A great Circle of the Sphere is that which divides it into two equal Hemiſpheres.

3. A ſmall Circle of the Sphere is any Circle which does not divide the Sphere into equal Parts.

Of the great Circles of the Sphere.

4. The *Horizon* is that Circle which divides the viſible Part of the Heavens from the inviſible, being the Circle which bounds our Sight when ſtanding on a Plain and viewing the viſible Part of the Heavens. The Point directly over our Heads is called the *Zenith*, and the Point oppoſite to that, directly under our Feet, is called the *Nadir*: Theſe two Points are the Poles of the Horizon, being 90° diſtant from it. The Horizon is diſtinguiſhed into *rational* and *ſenſible*: The former is that which diſtinguiſhes Day from Night; for, when the Sun is above this Circle, it is Day, and, when below it, the Sun is then ſaid to be ſet, and it would be quite dark if there was no Twilight: The *ſenſible* Horizon is the Circle that bounds our Sight.

5. The *Meridian* is the Circle in which the Sun is at Noon. This Circle cuts the Horizon in the North and South Points, and paſſes thro' the Zenith and Nadir of the Place and the two Poles of the World: When the Sun is upon it, and above the Horizon, it is Mid-noon; and, when he is upon it below the Horizon, it is Mid-night.

6. The *Poles of the World* are two Points diametrically oppoſite one another; one is called the North, the other the South Pole. A Line, being drawn from the North to the South Pole, will paſs thro' the Center of the Earth, and be as an *Axis*; the diurnal Motion of the Earth being made on this Line, like the Rotation of a Wheel on its Axis.

7. The *Æquinox*, or *Æquinoctial Circle*, is a Circle which divides the Heavens into two equal Parts, at 90° Diſtance from the Poles of the World, (which are therefore the Poles of this Circle). When the Sun is in this Circle, the Days and Nights are equal in all Latitudes; only directly under either of the Poles, or in 90° of Latitude, there the Sun is in their Horizon either riſing or ſetting.

8. The *Ecliptic*, in which the apparent Motion of the Sun is always performed, is a great Circle which interſects the Æquinoctial in two oppoſite Points, and makes with it an Angle of 23° . 30′, or, as later Improvements in Aſtronomy make it, 23° . 29′, commonly called the Inclination of the Ecliptic to the Æquator. The Ecliptic is divided into twelve equal Parts, called the twelve Signs; the firſt ſix are called the Northern Signs, the laſt ſix the Southern Signs: Their Names, Characters, and the Times when the Sun enters into each of them, are as follows:

Aries ♈	*Taurus* ♉	*Gemini* ♊	*Cancer* ♋
March 20.	*April* 20.	*May* 21.	*June* 21.
Leo ♌	*Virgo* ♍	*Libra* ♎	*Scorpio* ♏
July 23.	*Auguſt* 23.	*September* 23.	*October* 23.
Sagittarius ♐	*Capricorn* ♑	*Aquarius* ♒	*Piſces* ♓
November 22.	*December* 21.	*January* 20.	*Feb.* 18 or 19.

Theſe are called the Signs of the *Zodiac*, which is a great Circle encompaſſing the Heavens as a Girdle, (whence its Name) in which the Planets move. The Breadth of the Zodiac is uſually eſteemed about ſixteen or eighteen Degrees, in the Middle of which is the Ecliptic, from which the apparent Motion of the Sun never deviates.

9. The *Hour-Circles* are Meridians drawn thro' the Poles of the World and every fifteenth Degree of the Æquinoctial, of which the Meridian of the Place is the Hour-Line of twelve of the Clock.

10. The Meridian-Circle that interſects the Ecliptic in the firſt Point of ♈ and ♎ is called the *Æquinoctial Colure*; and that which cuts the Ecliptic in the Solſtitial Points, that is, the Beginning of ♋ and ♑, is called the *Solſtitial Colure*.

11. Circles drawn from the Zenith to the Nadir, cutting the Horizon at right Angles, are called the *Azimuth-Circles*; and that which interſects the Horizon in the Eaſt and Weſt is called the *Prime Vertical*, or the *Azimuth* of Eaſt and Weſt.

Of the ſmaller Circles.

12. The ſmall Circles of the Sphere are ſuch as are Parallels to any of the greater Circles before-mentioned; ſuch are,

13. The Tropics of *Cancer* and *Capricorn*, which are parallel to the Æquinoctial, at the Diſtance of 23°. 29′, the Sun's greateſt Declination: The Tropic of *Cancer* is on the North, and the Tropic of *Capricorn* on the South Side of the Æquinoctial.

14. The two Polar Circles are Parallels to the Æquinoctial, at the Diſtance of 66°. 31′: That near the North Pole is called the *Arctic* Circle, and that near the South Pole is called the *Antarctic* Circle.

15. Circles drawn parallel to the Horizon are called Circles of *Altitude*, becauſe the Altitude, or Height of any Object above the Horizon, is determined by thoſe Parallels.

16. Parallels of *Latitude* are Parallels to the Æquinoctial; and are alſo the Parallels of the Declination of the Sun or Planets.

17. The *Latitude* of a Place is the Elevation of the Pole above the Horizon, or the Diſtance of the Zenith from the Æquator: If the North Pole is elevated, it is North Latitude; but, if the South Pole is elevated, it is South Latitude. The Latitude never exceeds 90°. The *Longitude* of Places is meaſured on the Æquinoctial, and uſually is reckoned from Eaſt to Weſt round the Globe, from different Points of which the Longitude is computed by different Geographers.

18. The *Longitude* of the Sun is his Diſtance from the firſt Point of ♈, and is therefore the ſame with his Place in the Ecliptic.

19. The *Declination* of the Sun we have already ſhewn to be his Diſtance from the Æquinox: When he is in any of the ſix Northern Signs, his Declination is North; and, when he is in any of the ſix Southern Signs, his Declination is South.

20. A *right Sphere* is when the Poles of the World are in the Horizon: And the *right Aſcenſion* of the Sun is the Degree of the Æquator which riſes with the Sun in a right Sphere; or it is the Degree of the Æquator which comes to the Meridian of any Place with the Sun. The right Aſcenſion is reckoned on from the firſt Point of ♈ to 360°.

21. An *oblique Sphere* is when one of the Poles of the World is above, and the other is below the Horizon: And the *oblique Aſcenſion* of the Sun is that Degree of the Æquinox which riſes in an oblique Sphere with the Sun.

When the Poles of the World are in the Zenith and Nadir, it is called a *parallel Sphere*.

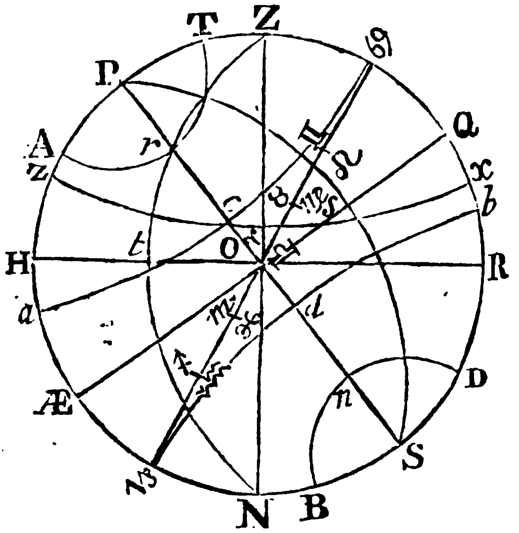
22. The *aſcenſional Difference* is the Difference between the right and oblique Aſcenſion of any Point of the Ecliptic; that is, the Difference between the Degree of the Æquinoctial that riſes with the ſame Point of the Ecliptic in a right and an oblique Sphere.

23. The *Amplitude* of the Sun is the Diſtance of the Sun's Riſing or Setting from the Eaſt or Weſt Points of the Horizon: When he riſes and ſets North of the Eaſt and Weſt Points of the Horizon, his Amplitude is North; but, when he riſes and ſets South to the Eaſt or Weſt Points of the Horizon, his Amplitude is South.

24. The Sun[illegible] *Azimuth* is his Diſtance from the Meridian, either Eaſt or Weſt: 'Till he has paſſed the Meridian, his Azimuth is Eaſt; and, when he has paſſed it, the Azimuth is Weſt.

25. The *Altitude* of the Sun is his Height above the Horizon: The *Meridian-Altitude* is his Altitude at Noon; which, when he is in the Tropic of ♋, is 61°. 57′; but, in the Tropic of ♑, is no more than 14°. 59′, in the Latitude of *London*, which is 51°. 32′ North.

Prob. 92. *To project the Circles of the Sphere on the Plain of the Meridian for the Latitude of* London 51°. 32′ *North.*



With the Chord of 60°, draw the primitive Circle HZRN for the Meridian of the Place; draw the right Circle HOR for the Horizon, and ZON at right Angles for the Prime Vertical, or Azimuth of Eaſt and Weſt, in which Z will be the Zenith and N the Nadir; take 51°. 32′, the Latitude of the Place, and ſet it from H to P, and from R to S; then will P repreſent the North Pole of the World elevated above the Horizon, and S the South Pole of the World depreſſed below the Horizon, equal to the Latitude of the Place; from P to S draw a Diameter POS, which repreſents the Axis of the World; at right Angles to POS draw ÆOQ, which will be the Æquinoctial; take from the Line of Chords 23°. 29′, and lay it, both Ways, from Æ to *a*, and from Æ to ♑; lay it alſo from Q to ♋, and from Q to *b*; draw the Ecliptic ♋ O ♑; take the Half-tangent of 23°. 29′, and lay it, both Ways, on the Axis of the World to *c* and *d*; then, thro' the three Points *a*, *c*, ♋, draw, by Prob. 17, the Parallel *a c* ♋ for the *Tropic of Cancer*; and, by the ſame Problem, draw ♑ *d b*, thro' the three Points ♑ *d b*, for the *Tropic of Capricorn*; or, if you take from the Scale of

Tangents 66°. 31′, the Complement of 23°. 29′, and ſet one Foot of the Compaſſes in *c* or *d*, letting the other fall on the Axis of the World produced to a ſufficient Length, that will be the Center, on which the Tropic may be drawn, as before.

The Center O, where the Ecliptic interſects the Æquator, is the Beginning of ♈; 30° being taken from the Scale of Half-Tangents, and laid on the Ecliptic from O towards ♋, is the Beginning of ♉; 60° being taken from the ſame Scale, and laid from the Center towards ♋, is the Beginning of ♊; and the Beginning of ♋ is at the Interſection of the Tropic with the Meridian: Now, reckoning back 30° from ♋ towards O, we ſhall come to the Beginning of ♌, the ſame Point ♊ is; reckoning back 60° from ♋ towards O, we ſhall come to the Beginning of ♍, the ſame Point where ♉ began; and, reckoning back 90°, we come to the Center, or Beginning of ♎, the firſt of the Southern Signs: Then take 30° from the Scale of Half-Tangents, and ſet from ♎ to ♏ for the Beginning of that Sign; take 60° from the ſame Scale, and ſet from ♎ to ♐ for the Beginning of that Sign; the Beginning of ♑ is at ♑, the Interſection of the Tropic with the Meridian; reckoning back from ♑ 30° will be the Beginning of ♒; and reckoning back 60° for the Beginning of ♓ will be the Beginning of ♏.

Take from the Line of Chords 23°. 29′, and lay it, both Ways, from the North Pole P to A and T; then, by Prob. 42, Caſe 2, lay 23°. 29′ from P on the Axis of the World to *r*: Now take the Tangent of 23°. 29′, and, placing one Foot of the Compaſſes in *r*, let the other fall on the Axis of the World extended beyond P, this will be the Center, on which may be deſcribed the *Arctic* Circle A *r* T about the North Pole P: And in the ſame Manner may the *Antarctic* Circle B *n* D be deſcribed about S, the South Pole.

If one Foot of the Compaſſes be placed on the Æquator, either within, or continued without the primitive Circle, and oblique Circles are drawn, paſſing thro' the Poles of the World, as P *s* S, they will be Meridians, or Hour-Circles. In the ſame Manner, placing one Foot of the Compaſſes in the Horizon, either within the primitive Circle, or produced beyond it, and drawing oblique Circles, Z *t* N, thro' the Zenith and Nadir, they are Azimuth-Circles.

If ſmall Circles, ſuch as *x c z*, are drawn parallel to the Horizon H O R, by Prob. 61, at any Diſtance leſs than 90°, they are Parallels of Altitude.

Having explained the Manner of projecting the principal Circles of the Sphere, I ſhall now ſhew the young Aſtronomer their Uſe; and, as the Sun's Place in the Ecliptic is the Foundation of moſt of the other Problems in Aſtronomy, I ſhall begin with that.

Prob. 93. *To find the Sun's Place in the Ecliptic.*

Rule. If the given Day of the Month is more than the Day of the Sun's Entrance in that Month, ſubtract the Day of the Sun's Entrance from the given Day, and the Remainder will be the Degrees of the Sun's Place in the Sign he enters in that Month.

Exam. 1. The Sun's Place is required on the twenty-ſeventh of *May* 1753:

From *May*	——— ———	27
Subtract the Day of Entrance into ♊	—	21
		6

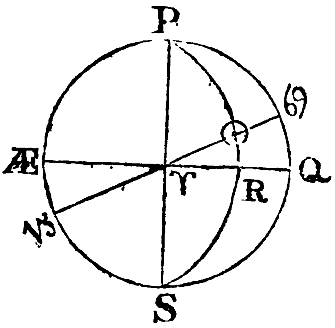
Therefore, on the twenty-ſeventh of *May*, the Sun is in ſix Degrees of ♊.

Exam. 2.			*Exam.* 3.		
September	——	30	*November*	—	26
Sun enters ♎ *Sep.*		23	Sun enters ♐ *Nov.*		22
		7 Deg. of ♎.			4 Deg. of ♏.

And, if the given Day is leſs than the Day of Entrance for that Month, add thirty to the given Day, and ſubtracting the Day of Entrance in the preceding Month from that Sum, the Remainder will be the Degrees of the Sun's Place in the Sign of the preceding Month:

Exam. 4.			*Exam.* 5.		
April	——	6	*July*	——	10
Add	——	30	Add	——	30
		36			40
Suns enters ♈		20	Sun enters ♋		21
		16 Deg. of ♈.			19 Deg. of ♋.

Prob. 94. *The Sun's Place* ♊ *7°. 13′ given, and his greateſt Declination 23°. 29′, (always known) to find his preſent Declination and right Aſcenſion.*



Conſtruction. With the Chord of 60°, draw the Meridian Æ P Q S; draw Æ ♈ Q the Æquator, and at right Angles to it P ♈ S the Axis; lay 23°. 29′, the Sun's greateſt Declination, from Æ to ♑, and from Q to ♋, and draw the Ecliptic ♑ ♈ ♋; by Prob. 43, Caſe 1, lay 67°. 13′, the Sun's Place in the Ecliptic, from ♈ to ☉; take the Secant of 22°. 40′, the Complement of the Sun's Place to 90°, place one Foot of the Compaſſes in ☉, and let the other fall on the Æquinoctial (continued, if neceſſary) for a Center, and draw the oblique Circle P ☉ R S, by which will be formed the right-angled Triangle ♈ R ☉, right-angled at R; in which we have given the Angle at ♈ 23°. 29′, the Sun's greateſt Declination, and the Hypothenuſe ♈ ☉ 67°. 13′, to find the Perpendicular ☉ R, the preſent Declination; this is an Extreme disjunct, and the Perpendicular is the middle Part; whence, by Lord *Nepier's* third Rule, and its *Note*,

As the Radius —— ——	10.
Is to the Sine of ☉ ♈ R, 23°. 29′, ——	9.600409
So is the Sine of ♈ ☉, 67°. 13′, ——	9.964720
To the Sine of R ☉, 21°. 33′, the Sun's Declinat.	9.565129

The Perpendicular R ☉, being a Part of the oblique Circle P ☉ S, may be meaſured by Prob. 46: Then, for the right Aſcenſion ♈ R, having the Angle at ♈ 23°. 29′, and the Hypothenuſe ♈ ☉ 67°. 13′, to find the Baſe ♈ R, is an Extreme conjunct, and one of the Extremes is required; whence, by Lord *Nepier's* ſecond Rule, and its *Note*,

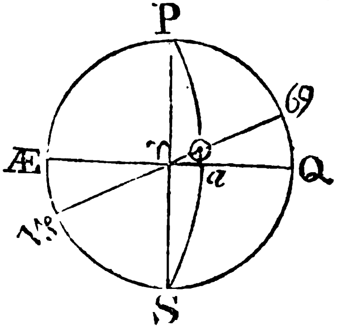
As the Co-tangent of ♈ ☉, 67°. 13′, Co. Ar.	0.376731
Is to the Radius, —— ——	10.
So is the Co-ſine of ☉ ♈ R, 23°. 29′, ——	9.962453
To the Tang. of ♈ R, 65°. 24′, Sun's right Aſcenſ.	10.339184

The Baſe ♈ R may be meaſured by Prob. 43. If the Declination were North, and increaſing, then the right Aſcenſion of the Sun is 65°.24′; but, if it were North, decreaſing, that is, if it were between the twentieth of *June* and the twenty-ſecond of *September*, then the Number found muſt be ſubtracted from 180°, and the Remainder 114°.36′ is the right Aſcenſion; becauſe, as was obſerved in the twentieth Definition, the right Aſcenſion is reckoned from the Point where the Æquinoctial interſects the Ecliptic in the firſt Point of ♈, 'till it returns to the ſame Point again: Therefore, when the Declination is South, increaſing, that is, from the twenty-ſecond of *September* to the twenty-firſt of *December*, 180° muſt be added to the Number found by this Problem; and, when the Declination is South, decreaſing, that is, from the twenty-firſt of *December* to the twentieth of *March*, then the Number found by the Calculation muſt be ſubtracted from 360°, and the Remainder is the right Aſcenſion ſought.

I ſhall now endeavour to give the young Aſtronomer ſome Idea of the Cauſe of the Inequality of natural Days, ariſing from the Obliquity of the Ecliptic to the Æquinoctial; by which the Sun's right Aſcenſion not increaſing exactly the ſame with his Place in the Ecliptic, it produces a Variation in the Length of the natural Days: For thoſe Meridians that divide the Æquinoctial into equal Portions, will not divide the Ecliptic in the ſame Manner, but into unequal Portions; by which the Length of the natural Day will be increaſed or diminiſhed; the Length of a natural Day being the Time between the Sun's leaving the Meridian of any Place and coming to it again; which, in different Revolutions of the Earth on its Axis, will be longer or ſhorter, as longer or ſhorter Portions of the Ecliptic paſs the Meridian. If the Reader finds any Difficulty in underſtanding the following Explanation, let him paſs it by, and review it when he has made a farther Progreſs.

Let it be required to find the Sun's Place in the Ecliptic when his right Aſcenſion is 15°.

With the Chord of 60°, draw Æ P Q S the Meridian; then draw Æ ♈ Q the Æquinoctial, and P ♈ S the Axis, or Hour-Circle of Six; from Æ to ♑, and from Q to ♋, ſet off 23°.29′, the Sun's greateſt Declination, and draw the Ecliptic ♋ ♈ ♑; then, from the Scale of Half-Tangents, lay 15° from ♈ to *a* on the Æquinoctial, and draw the Meridian P *a* S, which will interſect the Ecliptic in ☉, the Place of the Sun when his right Aſcenſion is 15°. In the right-angled Triangle ☉ ♈ *a*, right-angled at *a*, are given the Angle at ♈ 23° . 29′, and the Baſe ♈ *a* 15°, to find the Hypothenuſe ♈ ☉, the Longitude of the Sun from the firſt Point of ♈, or Place in the Ecliptic; which is an Extreme conjunct, and the Hypothenuſe is one of the Extremes; whence, by Lord *Nepier*'s ſecond Rule, and its *Note*,



As the Tangent of ♈ *a*, 15°, ——	Co. Ar. 0.571948
Is to the Radius, ——— ———	10.
So is the Co-ſine of ☉ ♈ *a*, 23° . 29′, ——	9.962453
To the Co-tangent of ♈ ☉, 16° . 17′, ——	10.534401

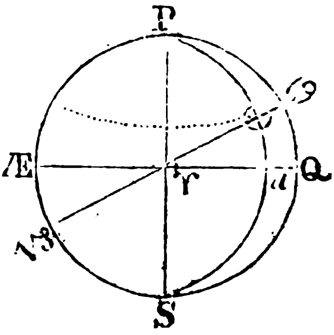
By which it appears, that, when the Sun's Place is 16° . 17′, his right Aſcenſion is but 15°; ſo that, in that Time, there is a Difference of 1° . 17′ of the Æquinoctial, which anſwers to five Minutes and eight Seconds of Time. Thus, as the Sun goes on towards the Tropic, he gains, 'till he comes to 15° of ♉, which is half Way; after which he loſes, 'till he arrives at the Tropic, when his Place and right Aſcenſion become equal. And, if the Æquinoctial were divided in every fifteenth Degree, and Meridians drawn thro' each, they would interſect the Ecliptic in the Sun's Place, for each 15° of right Aſcenſion, as below:

At	15°	of right Aſcenſion, the Longitude is	16° . 17′
	30		32 . 12
	45		47 . 28
	60		62 . 6
	75		75 . 15
	90		90

The Earth revolves round its Axis in twenty-four Hours, at the ſame Time moving forward in its Orbit about fifty-nine Minutes and eight Seconds. By this Means, the whole Æquinoctial, and fifty-nine Minutes, eight Seconds, paſſes the Meridian of any Place in twenty-four Hours: But the Ecliptic, interſecting the Æquinoctial at an Angle of 23°.29′, does not paſs the Meridian equally with the Æquinoctial. Hence, the Time from the Sun's Appearance on the Meridian of any Place, to the Time of his Return to it again, is ſometimes more, ſometimes leſs, than twenty-four Hours; but Clocks and Watches keep equal Time, when they go right; and this is one Reaſon why good Clocks and Watches go ſometimes faſter, ſometimes ſlower, than the Sun. The greateſt Variation, ariſing from this Cauſe, is when the Sun is in the Æquinoctial and Tropics; for, when the Sun is in the Æquinox, and going towards the Tropics, the Time from Noon to Noon will be but twenty-three Hours, fifty-nine Minutes, and forty Seconds; but, in returning from the Tropics towards the Æquinox, the Day will be twenty-four Hours and twenty-two Seconds.

There is another Cauſe of the Inequality of natural Days, ariſing from the annual Motion of the Earth, which moves round the Sun in an Ellipſis, in one of whoſe *Foci* the Sun is placed; and, from the Laws of Gravity, the Velocity of the Earth is continually changing in its Orb, by which the Earth moves thro' different Portions of the Orb in twenty-four Hours, which produces a Variation in the Length of natural Days. The Time marked by the Motion of the Sun is called *apparent Time*; the Time marked by Clocks, and other Time-keepers, is called *true Time*; and the Difference is called the *Equation of Time*.

Prob. 95. *The Sun's right Aſcenſion 75°.48′, and his greateſt Declination 23°.29′, (always known) to find his Place in the Ecliptic, and preſent Declination.*



Conſtruction. With the Chord of 60°, draw the Meridian ÆPQS; draw ÆQ the Æquinoctial, and PS the Axis of the World at right Angles to it; lay the Sun's greateſt Declination, 23°.29′, from Æ to ♑, and from Q to ♋, and draw the Ecliptic ♑ ♈ ♋; lay 75°.48′, the Sun's right Aſcenſion, by Prob. 42, Caſe 1, from ♈ to *a*; then, by Prob. 17, draw the Meridian P *a* S, cutting the Ecliptic in ☉, by which will be formed the right-angled Triangle ♈ ⊙ *a*, right-angled at *a*; in which is given the Angle at ♈ 23°. 29′, and the Baſe ♈ *a* 75°. 48′, to find the Hypothenuſe ♈ ⊙, or the Sun's Place in the Ecliptic; which is an Extreme conjunct, and the Hypothenuſe is one of the Extremes; wherefore, by Lord *Nepier*'s ſecond Rule, and its *Note*, Page 266,

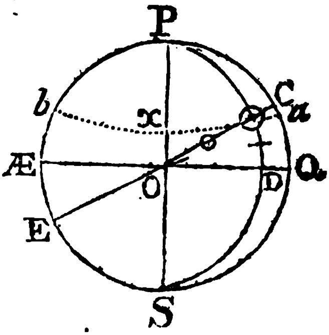
As the Tangent of ♈ *a*, 75°. 48′, ——	10.596813
Is to the Radius, —— ——	10.
So is the Co-ſine of ⊙ ♈ *a*, 23°. 29′, ——	9.962453
To the Co-tangent of ♈ ⊙, 76°. 56′, ——	9.365640

Therefore the Sun's Place is 16°. 56′ of ♊. To find the Declination, the ſame Things being given, it is ſtill an Extreme conjunct, and the Perpendicular *a* ⊙, being one of the Extremes, may be found by the ſame Proportion:

As the Co-tangent of ♈ ⊙, 23°. 29′, ——	10.362044
Is to the Radius, —— ——	10.
So is the Sine of ♈ *a*, 75°. 48′, ——	9.986523
	19.986523
	10.362044
To the Tangent of *a* ⊙, 22°. 50′, the Sun's present Declination, —— ——	9.624479

The Perpendicular *a* ⊙ may be meaſured by Prob. 46, and the Hypothenuſe ♈ ⊙ may be meaſured by Prob. 43.

Prob. 96. *The Sun's preſent Declination* 22°. 56′ *North, increaſing, and greateſt Declination* 23°. 29′, *given, to find his Place in the Ecliptic, and right Aſcenſion.*



Conſtruction. With the Chord of 60°, deſcribe the Meridian Æ P Q S; draw Æ Q the Æquinoctial, and P S the Axis of the World at right Angles to it; lay 23°. 29′, the Sun's greateſt Declination, from Æ to E, and from Q to C, and draw E O C the Ecliptic; lay 22°. 56′, the Sun's preſent Declination, from Q to *a*, and from Æ to *b*; and, by the Scale of Half-Tangents, lay, by Prob. 42, 22°. 56′ from O to *x*; then, by Prob. 17, draw *b x a*, the Parallel of the Sun's preſent Declination, interſecting the Ecliptic in ☉; thro' ☉, P, and S, by Prob. 17, draw the Meridian P ☉ S, interſecting the Equinoctial in D, and forming the right-angled Triangle O ☉ D, right-angled at D; in which is given the Angle at O 23°. 29′, and the Perpendicular D ☉ 22°. 56′, to find the Hypothenuſe O ☉, the Sun's Place in the Ecliptic; which is an Extreme disjunct, and the Hypothenuſe is one of the Extremes; wherefore, by Lord *Nepier*'s fourth Rule, and its *Note*, Page 266,

As the Sine of ☉ O D, 23°. 29′, — Co. Ar. 0.399591
Is to the Radius, ——— ——— 10.
So is the Sine of D ☉, 22°. 56′, ——— 9.590686

To the Sine of O ☉, 77°. 55′, the Sun's Place, 9.990277

That is 17°. 55′ of ♊. But, had the Declination been North, decreaſing, ſubtract 77°. 55′ from 180°, and the Remainder 102°. 5′ is the Sun's Place in the Ecliptic, that is, 12°. 5′ of ♋.

The Hypothenuſe O ☉ may be meaſured by Prob. 43.

Now, for the right Aſcenſion, the ſame Things being given, to find D O, is an Extreme conjunct, and D O is the middle Part; wherefore, by Lord *Nepier*'s firſt Rule, and its *Note*, Page 265,

As the Radius ——— ——— 10.

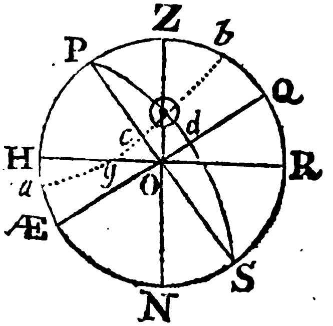
Is to the Co-tangent of ☉ O D, 23°. 29′, — 10.362044
So is the Tangent of D ☉, 22°. 56′, ——— 9.626445

To the Sine of D O, 76°. 51′, ——— 9.988489

Which is the Sun's right Aſcenſion. But, if the Declination had been North, decreaſing, ſubtract 76°. 51′ from 180°, and the Remainder, 103°. 9′, is the right Aſcenſion; and, if the Declination had been South, increaſing, 180° muſt be added to the Aſcenſion found by the Problem; laſtly, if the Declination given is South, decreaſing, take the Degrees found by the Problem from 360°, and the Remainder is the Sun's right Aſcenſion.

The Perpendicular DO may be meaſured by Prob. 46.

Prob. 97. *The Latitude of the Place (as* London) 51°. 32′, *and the Sun's preſent Declination* 23°. 4′ *North, given, to find the Altitude of the Sun, and Hour of the Day, when the Sun is due Eaſt or Weſt.*



Conſtruction. With the Chord of 60°, on O, as a Center, draw H Z R N the Meridian; draw H O R the Horizon, and Z O N the Prime Vertical, or Azimuth of Eaſt and Weſt, at right Angles to it; take 51°.32′, the Latitude, and ſet from H to P, and from R to S; draw P O S the Axis of the World, and Æ O Q at right Angles to it for the Æquinoctial; then lay 23°. 4′ from Æ to *a*, and from Q to *b*; alſo lay the Half-Tangent of 23°.4′ from O to *c*; and, by Prob. 17, draw the Parallel of Declination, *a c b*, interſecting the Prime Vertical in ⊙, the Sun's Place when due Eaſt or Weſt; laſtly, by Prob. 17, draw the Meridian P ⊙ S, interſecting the Æquinoctial in *d*, and forming the right-angled Triangle O ⊙ *d*, right-angled at *d*; in which we have given the Angle ⊙ O *d* 51°. 32′, the Diſtance of the Æquinoctial ftom the Zenith, equal to the Latitude of the Place, and the Side ⊙ *d* 23°.4′, the Sun's preſent Declination, to find the Hypothenuſe O ⊙, the Sun's Altitude above the Horizon when due Eaſt or Weſt; this is an Extreme disjunct, and the Hypothenuſe is one of the Extremes; wherefore, by Lord *Nepier*'s fourth Rule, and its *Note*, Page 266,

As the Sine of ⊙ O *d*, 51°. 32′,	——	Co. Ar. 0.106255
Is to the Radius	—— ——	10.
So is the Sine of ⊙ *d*, 23°.4′,	——	9.593067
To the Sine of O ⊙, 30°. 2′, the Sun's Altitude,		9.699322

Then, for the Hour of the Day, the ſame Things being given, to find the Baſe O *d* is an Extreme conjunct, and the Baſe O *d* is the middle Part; wherefore, by Lord *Nepier*'s firſt Rule, and its *Note*, Page 265, As the Radius ——— ——— 10.

Is to the Co-tangent of ⊙ O d, 51°.32′, —— 9.900087
So is the Tangent of ⊙ d, 23°.4′, —— 9.629255

To the Sine of O d, 19°.46′, ——— 9.529342

This being converted into Time, by allowing an Hour for every 15°, and four Minutes of Time for each Degree, gives one Hour and ſixteen Minutes for the 19°, and three Minutes and four Seconds for the 46′, which makes one Hour, nineteen Minutes, and four Seconds. Now, as the Sun riſes at y, where the Parallel of Declination cuts the Horizon, in his Aſcent to his Meridian Altitude, he will be on the Axis P O S at Six o'Clock in the Morning; therefore it will be paſt that Time one Hour, nineteen Minutes, and four Seconds, when he is due Eaſt, or at ⊙, that is, it will be nineteen Minutes and four Seconds paſt Seven o'Clock in the Morning: And, when he hath paſſed the Meridian, and is going towards the Horizon, to ſet at y, when he is at ⊙, or due Weſt, it will want one Hour, nineteen Minutes, and four Seconds of Six o'Clock, that is, it will be forty Minutes and fifty-ſix Seconds paſt Four in the Afternoon. Hence, the Degrees and Minutes, being converted into Time, and added to Six o'Clock, give the Time he is due Eaſt; and, being taken from Six, give the Time he is due Weſt in the Afternoon. But, if the Latitude of the Place and Declination be not both North, or both South, the Sun will be ſet before he is due Weſt, and will not riſe 'till after he is paſt the Eaſt Point of the Horizon.

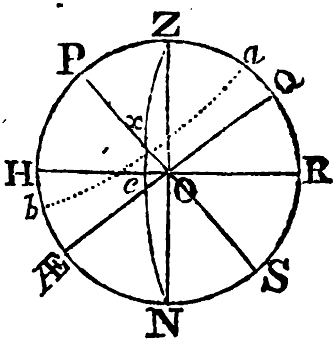
Prob. 98. *The Latitude of the Place 51°.32′ North, and the Sun's preſent Declination 23°.4′ North, given, to find the Altitude of the Sun at Noon.*

When the Latitude of the Place and the Declination of the Sun are both North, or both South, add the Declination to the Complement of the Latitude, and the Sum will be his Meridian Altitude; but, if the Declination and Latitude are, one North, the other South, ſubtract the Declination from the Complement of the Latitude, and the Remainder will be the Sun's Meridian Altitude:

From	90°
Subtract	51 . 32′
	38 . 28
Declination add	23 . 4
Meridian Altitude	61 . 32

In the ſame Latitude, if the Declination had been South 23°.4′, ſubtracting that from 38°.28′, leaves 15°.24′ for the Meridian Altitude of the Sun.

Prob. 99. *The Latitude of the Place* 51°.32′ *North, and the Sun's Declination* 18°.7′ *North, to find the Sun's Altitude and Azimuth at Six o'Clock.*



Conſtruction. With the Chord of 60°, draw H Z R N the Meridian of the Place; then draw H O R the Horizon, and Z O N the Prime Vertical at right Angles to it; ſet the Latitude, 51°.32′, from H to P, and from R to S; then draw the Axis P O S, and at right Angles to it draw Æ O Q the Æquinoctial; lay 18°.7′, the Sun's Declination, from Æ to *b*, and from Q to *a*, alſo the Half-Tangent of 18°.7′ from O to *x*; by Prob. 17, draw the Parallel of the Sun's Declination, *a x b*, interſecting the Axis P O S in *x*, the Place of the Sun at Six o'Clock; thro' *x*, and the Poles of the Horizon Z and N, by Prob. 17, draw the Azimuth-Circle Z *x* N, interſecting the Horizon in *e*, and forming the right-angled Triangle *e x* O, right-angled at *e*; in which is given the Angle at O 51°.32′, and the Hypothenuſe O *x* 18°.7′, to find the Perpendicular *e x*, the Sun's Altitude at Six o'Clock; this is an Extreme disjunct, and the required Perpendicular is the middle Part; whence, by Lord *Nepier*'s third Rule, and its *Note*, Page 266,

As the Radius	10.
Is to the Sine of *x* O *e*, 51°. 32′,	9.893745
So is the Sine of O *x*, 18°. 7′,	9.492695
To the Sine of *e x*, 14°. 5′, the Altitude of the Sun above the Horizon,	9.386440

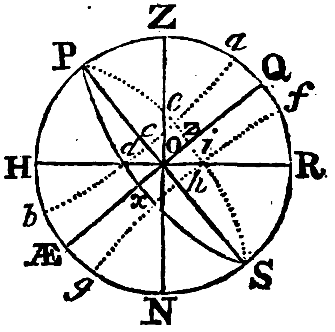
For the Sun's Azimuth: The ſame Things being given, to find the Baſe *e* O, the Sun's Azimuth from the Eaſt, Northward, is an Extreme conjunct, and the Baſe is one of the Extremes; whence, by Lord *Nepier*'s ſecond Rule, and its *Note*, Page 266,

As the Co-tangent of O *x*, 18°. 7′,	10.485223
Is to the Radius,	10.
So is the Co-ſine of *x* O *e*, 51°. 32′,	9.793832
To the Tangent of O *e*, 11°. 3′,	9.308609

Hence the Sun's Azimuth from the Eaſt or Weſt, at Six o'Clock, is 11°. 3′; and this, being added to 90°, or O R, gives *e* R 101°. 3′, the Sun's Azimuth from the Meridian towards the North Point of the Horizon.

Prob. 100. *The Latitude of the Place 51°. 32 North, and the Sun's preſent Declination 15°. 46′ North, given, to find the Sun's Amplitude, and the aſcenſional Difference; the Time of the Sun s Riſing and Setting, with the Length of the Day and Night; alſo the oblique Aſcenſion and Deſcenſion of the Sun.*

Draw the Meridian H Z R N, with the Chord of 60°; draw the Horizon H O R, and the Prime Vertical Z O N at right Angles to it; take the Latitude, 51°. 32′, and ſet it from H to P, and from R to S; draw the Axis P O S, and Æ O Q the Æquinoctial at right Angles to the Axis; take 15°. 46′, and ſet it from Æ to *b*, and from Q to *a*; ſet the Half-Tangent of 15°. 46′ from O to *c*, and draw, by Prob. 17, the



the Parallel of Declination *b c a*, interſecting the Horizon in *d*; thro' P, *d*, and S, by Prob. 17, draw the oblique Circle P *d* S, interſecting Æ O Q in *x*, and forming the right-angled Triangle O *d x*, right-angled at *x*; in which is given the Angle at O 38°. 28′, the Complement of the Latitude, and *d x* 15°. 46′, to find the Hypothenuſe O *d*, the Sun's Amplitude, or Diſtance of riſing from the Eaſt Point of the Horizon, Northward; which is an Extreme disjunct, and the Hypothenuſe is one of the Extremes; wherefore, by Lord *Nepier*'s fourth Rule, and its *Note*, Page 266,

As the Sine of *d* O *x*, 38°. 28′, —— Co. Ar. 0.206168
Is to the Radius, ———— ———— 10.
So is the Sine of *d x*, 15°. 46′, ———— 9.434122

To the Sine of *d* O, 25°. 54′, ———— 9.640290

The Hypothenuſe O *d* may be meaſured by Prob. 43.

If the Sun's Declination had been South 15°. 46′, the Parallel *g b f* being drawn, would interſect the Horizon in *i*, and an oblique Circle, P *i* S, would interſect Æ O Q in *z*, forming the right-angled Triangle O *i z*, right-angled at *z*, equal to the Triangle O *d x*; in which the Hypothenuſe O *i*, equal to O *d*, is the Sun's Amplitude, or the Diſtance of the Riſing and Setting from the Eaſt and Weſt Points of the Horizon, the ſame as before; only, in the former Caſe, the Amplitude is from the Eaſt or Weſt, Northward, and, in the latter, Southward; that is, in North Latitude, when the Sun's Declination is North, he riſes to the North of the Eaſt, and ſets to the North of the Weſt; but, when the Latitude is North and Declination South, he riſes to the South of the Eaſt, and ſets to the South of the Weſt.

Then, for the aſcenſional Difference, the ſame Parts being given, to find the Side O *x*, is an Extreme conjunct, and O *x* is the middle Part; whence, by Lord *Nepier*'s firſt Rule, and its *Note*, Page 265,

As the Radius ———— ———— 10.

Is to the Co-tangent of *d* O *x*, 38°. 28′, — 10.099914
So is the Tangent of *d x*, 15°. 46′, —— 9.450777

To the Sine of O *x*, 20°. 4′, ———— 9.550691

The Side O *x* may be meaſured by Prob. 43.

The aſcenſional Difference 20°.4′, being converted into Time, by allowing 15° for an Hour, four Minutes of Time for a Degree, and four Seconds of Time for each Minute, gives one Hour, twenty Minutes, and ſixteen Seconds; and ſo much the Sun riſes before Six in the Morning, and ſets after Six at Night; that is, the Sun riſes thirty nine Minutes and forty-four Seconds after Four o'Clock in the Morning, and ſets twenty Minutes and ſixteen Seconds after Seven at Night. But, if the Declination had been different, the one North, the other South, the one Hour, twenty Minutes, and ſixteen Seconds, being added to the Hour of Six, gives the Sun's Riſing twenty Minutes and ſixteen Seconds after Seven in the Morning; and, being ſubtracted from Six, gives thirty-nine Minutes and forty-four Seconds after Four at Night for the Time of his Setting.

If the Time of the Sun's Setting, ſeven Hours, twenty Minutes, and ſixteen Seconds, be doubled, it gives the Length of the Day, fourteen Hours, forty Minutes, and thirty-two Seconds; and thirty-nine Minutes and forty-four Seconds after Four, the Time of the Sun's Riſing, being doubled, gives nine Hours, nineteen Minutes, and twenty-eight Seconds for the Length of the Night.

To find the Oblique Deſcenſion.

When the Latitude and Declination are the ſame, add the aſcenſional Difference to the right Aſcenſion, and the Sum is the oblique Deſcenſion: But, when the Latitude and Declination are not the ſame, then ſubtract the aſcenſional Difference from the right Aſcenſion, and the Remainder is the oblique Deſcenſion; and, if you cannot ſubtract the aſcenſional Difference from the right Aſcenſion, add 90° to it, then ſubtract, and the Remainder is the oblique Deſcenſion.

To find the Oblique Aſcenſion.

When the Latitude and Declination are the ſame, ſubtract the aſcenſional Difference from the right Aſcenſion, and the Remainder is the oblique Aſcenſion; and, if the aſcenſional Difference cannot be taken from the right Aſcenſion, add 90° to it, then ſubtract, and the Remainder is the oblique Aſcenſion: But, when the Latitude and Declination are not the ſame, then the aſcenſional Difference and right Aſcenſion, being added together, is the oblique Aſcenſion.

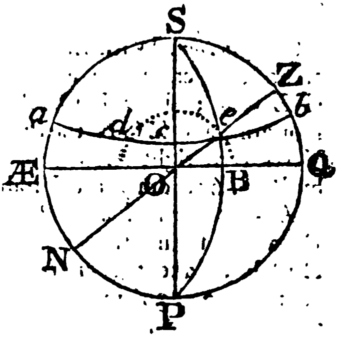
In the preſent Problem, the Declination being 15° . 46′ North, increaſing, the right Aſcenſion may be found, by Prob. 96, to be 40° . 38′; and, in the Latitude of 51° . 32′, we have found the aſcenſional Difference 20° . 4′; which, being added to, and ſubtracted from, the right Aſcenſion, gives the oblique Aſcenſion and Deſcenſion as follows:

To 40° . 38′ Right Aſcenſion. From 40° . 38′
Add 20 . 4 Aſcenſ. Diff. Subtract 20 . 4

60 . 42 Oblique Deſcenſion. 20 . 34 Oblique Aſcenſ.

Prob. 101. *The Declination of the Sun 21° . 36′ South, and his Altitude 42° . 27′ when due Eaſt or Weſt, given, to find the Latitude of the Place.*

Conſt. With the Chord of 60°, draw the Meridian Æ S Q P; draw Æ O Q the Æquinoctial, and S O P the Axis at right Angles to it; ſet off 21° . 36′, the Declination, from Æ to *a*, and from Q to *b*; and, by Prob. 61, draw the Parallel of Declination *a c b*, at the Diſtance of 21° . 36′ from the Æquinoctial; take the Half-Tangent of 42° . 27′ in the Compaſſes, ſet one Foot in the Center, and deſcribe an Arch interſecting the Parallel of Declination in *d* and *e*; a right Circle, N O Z, being drawn thro' either of theſe Points, as *e*, will be the Prime Vertical of the Place; then draw the Meridian S *e* P, by Prob. 17, interſecting the Æquinoctial in B, and forming the right-angled Triangle *e* O B, right-angled at B; in which is given the Hypothenuſe O *e* 42° . 27′, and the Perpendicular B *e* 21° . 36′; to find the Angle at O, the Latitude of the Place. The right Circle N O Z being the Prime Vertical, or Azimuth of Eaſt and Weſt, it paſſes thro' the Zenith of the Place; whence Z is the Zenith, and Z Q, or the Angle *e* O B, is the Latitude of the Place, being the Diſtance of the Zenith from the Æquinoctial.



This is an Extreme disjunct, and the Angle at O is one of the Extremes; whence, by Lord *Nepier*'s fourth Rule, and its *Note*, Page 266,

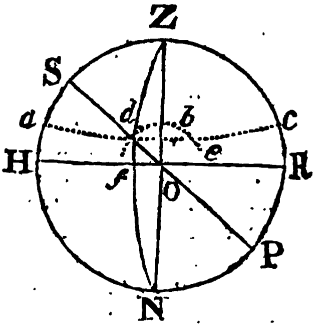
As the Sine of O *e*, 42°. 27′, —— Co. Ar. 0.170730
Is to the Radius, ——— ——— 10.
So is the Sine of B *e*, 21°. 36′, ——— 9.565995

To the Sine of B O *e*, 33°. 3′ South Latitude, 9.736725

The Declination being South, the Latitude is South; and the Angle B O *e*, being the ſame with the Angle Z O Q, may be meaſured by Prob. 53.

Prob. 102. *The Declination of the Sun* 22°. 36′ *South, and his Altitude* 16°. 32′ *at Six o'Clock, given, to find the Latitude of the Place.*

Conſtruction. With the Chord of 60°, deſcribe the Meridian H Z R N; draw H O R the Horizon, and Z O N the Prime Vertical at right Angles to it; by Prob. 61, draw the Parallel of Altitude, *a b c*, at 16°. 32′ Diſtance from the Horizon H O R; take the Half-Tangent of 22°. 36′ in the Compaſſes, ſet one Foot in O, and deſcribe an Arch interſecting the Parallel of Altitude in *d* and *e*; thro' either of which Points, as *d*, draw a right Circle, P *d* S, which will be the Axis, or Hour-Circle of Six o'Clock; laſtly, thro' the Points Z, *d*, and N, by Prob. 17, draw the Azimuth-Circle Z *d* N, interſecting the Horizon in *f*, and forming the right-angled Triangle *d* O *f*, right-angled at *f*, in which is given the Side *f d* 16°. 32′, and the Hypothenuſe O *d* 22°. 36′, to find the Angle at O, the Latitude of the Place, or the Elevation of S, the South Pole above the Horizon.



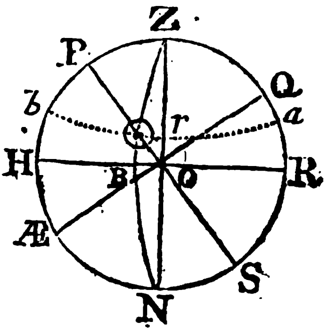
This is an Extreme disjunct, and the Angle at O is one of the Extremes; whence, by Lord *Nepier*'s fourth Rule, and its *Note*, Page 266,

As the Sine of O *d*, 22° . 36 , —— Co. Ar. 0.415335
Is to the Radius, —— —— 10.
So is the Sine of *f d*, 16° . 32′, —— 9.454194

To the Sine of *f* O *d*·47°.45′, South Latitude, — 9.869529

The Angle *f* O *d* may be meaſured by Prob. 53.

Prob. 103. *The Latitude of the Place 51° . 32′ North, and the Altitude of the Sun at Six o'Clock 18° . 9′, given, to find the Sun's Declination and Azimuth.*



Conſtruction. With the Chord of 60°, draw the Meridian H Z R N; draw the Horizon H O R, and the Prime Vertical Z O N at right Angles to it; lay 51° . 32′, the Latitude, from H to P, and from R to S; draw the Axis P O S, and at right Angles to it draw Æ O Q the Æquinoctial; lay 18° . 9′ from R to *a*, and from H to *b*; then lay the Half-Tangent of 18° . 9′ from O to *r*, and, by Prob. 17, draw *b r a*, the Parallel of Altitude, interſecting the Axis in ⊙, the Place of the Sun at Six o'Clock; then, by Prob. 17, draw the Azimuth-Circle Z ⊙ N, interſecting the Horizon in B, and forming the right-angled Triangle ⊙ O B, right-angled at B; in which is given B ⊙ 18° . 9′, and the Angle at O 51° . 32′, to find the Hypothenuſe O ⊙, the Sun's Declination; this is an Extreme disjunct, and the Hypothenuſe is one of the Extremes; wherefore, by Lord *Nepier*'s fourth-Rule, and its *Note*, Page 266,

As the Sine of ⊙ O B, 51° . 32′, — Co. Ar. 0.106255
Is to the Radius, —— —— 10.
So is the Sine of B ⊙, 18° . 9′, —— 9.493466

To the Sine of ⊙ O, 23° . 27′, the Declination, 9.599721

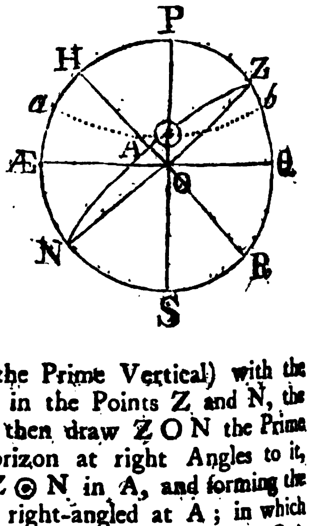
The Declination is North 23° . 27′; and the Hypothenuſe may be meaſured by Prob. 43, Caſe 1.

To find the Azimuth from the Eaſt or Weſt Point of the Horizon: We have the ſame Things given, to find the Baſe B O, which is an Extreme conjunct, and B O is the middle Part; wherefore, by Lord *Nepier*'s firſt Rule, and its *Note*, Page 265,

As the Radius ———— ————	10.
Is to the Tangent of B ⊙, 18°. 9′, ————	9.515631
So is the Co-tangent of ⊙ O B, 51°. 32′, ———	9.900087
To the Sine of B O, 15°. 6′, the Azimuth, —	9.415718

The Azimuth is 15°. 6′ from the Eaſt or Weſt, Northward; and the Side B O may be meaſured by Prob. 43, as the former.

Prob. 104. *The Sun's Declination* 22°. 58′ *North, and his Azimuth at Six o'Clock* 108°. 46′ *from the Meridian, or* 18°. 46′ *from the Eaſt Point of the Horizon Northward, given, to find the Latitude of the Place.*



Conſtruction. With the Chord of 60°, draw the Meridian Æ P Q S; draw Æ O Q the Æquinoctial, and P O S the Axis at right Angles to it; by Prob. 61, draw *a* ⊙ *b*, the Parallel of Declination, 22°. 58′ from the Æquinoctial, interſecting the Axis in ⊙; by Prob. 59, draw an oblique Circle thro' the Point ⊙, to make an Angle of 71°. 14′ (the Complement of 18°. 46′, the Azimuth from the Prime Vertical) with the primitive Circle, interſecting it in the Points Z and N, the Zenith and Nadir of the Place; then draw Z O N the Prime Vertical, and H O R the Horizon at right Angles to it, interſecting the oblique Circle Z ⊙ N in A, and forming the right-angled Triangle O ⊙ A, right-angled at A; in which is given the Hypothenuſe O ⊙ 22°. 58′, and the Baſe O A 18°. 46′, to find the Angle H O P, the Latitude of the Place; which is an Extreme conjunct, and the Angle at O is the middle Part; wherefore, by Lord *Nepier*'s firſt Rule, and its *Note*, Page 265,

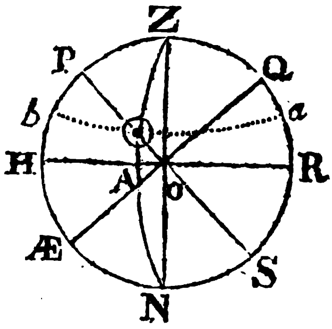
As the Radius, —— —— 10.

Is to the Co-tangent of O ☉, 22°.58′, — 10.372851
So is the Tangent of O A, 18°.46′, —— 9.531196

To the Co-ſine of ☉ O A, 36°.42′ North Lat., 9.904047

The Angle at O may be meaſured by Prob. 53.

Prob. 105. *The Sun's Azimuth at Six o'Clock 14°.55′ North of the Prime Vertical, and his Altitude 17°.57′, given, to find the Latitude of the Place.*



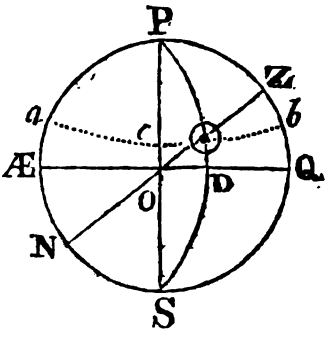
Conſtruction. With the Chord of 60°, draw the Meridian H Z R N; draw H O R the Horizon, and Z O N the Prime Vertical at right Angles to it; lay the Altitude, 17°.57′, from R to *a*, and from H to *b*, and draw the Parallel of Altitude, *a* ☉ *b*, by Prob. 61, at 17°.57′ Diſtance from the Horizon, interſecting the Prime Vertical in ☉; through ☉, and the Center O, draw the right Circle P ☉ O S, the Axis or Hour-Circle of Six, which forms the right-angled Triangle ☉ O A, right-angled at A; in which is given the Baſe A O 14°.55′, and the Side A ☉ 17°.57′, to find the Angle A O ☉, the Latitude of the Place; which is an Extreme conjunct, and the Angle at O is one of the Extremes; wherefore, by Lord *Nepier*'s ſecond Rule, and its *Note*, Page 266,

As the Tangent of ☉ A, 17°.57′, — Co. Ar. 0.489515
Is to the Radius, —— —— 10.
So is the Sine of A O, 14°.55′, —— 9.410632

To the Co-tang. of A O ☉, 51°.32′, North Lat. 9.900147

The given Declination being North, and the Sun being above the Horizon at Six o'Clock, the Latitude is 51°.32′ North. The Angle at O may be meaſured by Prob. 53.

Prob. 106. *The Sun being due Eaſt at ſeventeen Minutes paſt Eight in the Morning, and his Declination 19° . 57′ North, given, to find the Latitude of the Place.*



Conſtruction. With the Chord of 60°, draw the Meridian Æ P Q S; draw Æ O Q the Æquinoctial, and P O S the Axis at right Angles to it; lay 19° . 57′ from Æ to *a* and from Q to *b*; alſo lay the Half-Tangent of 19° . 57′ on the Axis from O to *c*, and, by Prob. 17, draw the Parallel of Declination *a c b*; ſubtracting eight Hours and ſeventeen Minutes from Twelve o'Clock, or Noon, there remains three Hours and forty-three Minutes for the Time the Sun wants of coming to the Meridian Æ P Q S; convert this three Hours and forty-three Minutes into Degrees and Minutes, by allowing 15° to an Hour, and 15′ of the Æquinox for one Minute of Time, and it is 55° . 45′; lay, by Prob. 42, Caſe 2, this 55° . 45′ on the Æquinox from Q to D, and, by Prob 17, draw the Meridian P D S, interſecting the Parallel of Declination *a c b* in ⊙, which is the Sun's Place at the given Time; then thro' the Point ⊙ draw the right Circle Z ⊙ O N, which is the Prime Vertical of the Place, and the two Points Z and N are the Zenith and Nadir. In the Triangle O ⊙ D, right-angled at D, given the Baſe O D 34° . 15′ the Complement of 55° . 45′ to 90°, and the Perpendicnlar D ⊙ 19° . 57′, to find the Angle at O, being the Diſtance of the Zenith from the Æquinox, which is always equal to the Latitude.

This is an Extreme conjunct, and the Angle at O, being one of the Extremes, may be found by Lord *Nepier*'s ſecond Rule, and its *Note*, Page 266,

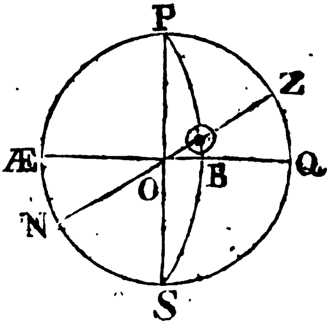
As the Tangent of D ⊙, 19° . 57′,	—	Co. Ar. 0.440115
Is to the Radius,	——— ———	10.
So is the Sine of O D, 34° . 15′,	———	9.750358
To the Co-tangent of ⊙ O D, 32° . 49′;	—	10.190473

The Declination being North, and the Sun being due Eaſt after he was above the Horizon, the Latitude is 32° . 49′ North.

The Angle at O, being made at the Center by the right Circles Æ O Q and Z O N, may be meaſured by Prob. 53.

If a right Circle were drawn at right Angles to the right Circle Z O N, it would be the Horizon of the Place, above which P, the North Pole, would be elevated 32° . 49′.

Prob. 107. *The Sun due Eaſt at forty-ſix Minutes after Seven in the Morning, and his Altitude* 28° . 53′ *at the ſame Time, given, to find the Latitude of the Place.*



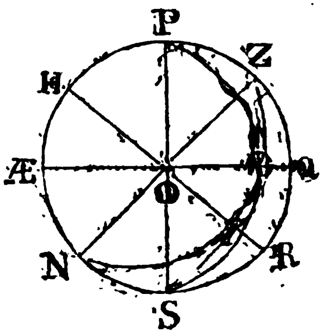
Conſtruction. With the Chord of 60°, draw the Meridian Æ P Q S; draw Æ O Q the Æquinoctial, and P O S the Axis at right Angles to it; then converting the Time from Six o'Clock, one Hour and forty-ſix Minutes, into Degrees and Minutes, it makes 26° . 15′, which lay from O to B, by Prob. 42, Caſe 1, and draw the Meridian P B S, by Prob. 17; then take the Half-Tangent of the given Altitude 28° . 53′ from the Scale, ſet one Foot of the Compaſſes in the Center O, and with the other interſect the Meridian P B S in the Point ☉; thro' the Point ☉ draw the right Circle Z ☉ O N, the Prime Vertical of the Place, which completes the Triangle O ☉ B, right-angled at B; where we have given the Hypothenuſe O ☉ 28° . 53′, and the Baſe O B 26° . 30′, to find the Angle at O, the Latitude of the Place.

This is an Extreme conjunct, and the Angle at O, being the middle Part, may be found by Lord *Nepier*'s firſt Rule, and its *Note*, Page 265:

As the Radius	10.
Is to the Co-tangent of O ☉, 28° . 53′,	10.258336
So is the Tangent of O B, 26° . 30′,	9.697736
To the Co-ſine of ☉ O B, 25° . 20′,	9.956072

Therefore the Latitude is 25° . 20′, and may be either North or South.

Prob. 108. *The Sun being in the Æquinoctial, his Azimuth 24° . 2′ at ſeventeen Minutes after One o'Clock in the Afternoon given, to find the Latitude of the Place.*



Conſtruction. With the Chord of 60, draw the Meridian Æ P Q S; draw the Æquinoctial Æ O Q, and P O S the Axis at right Angles to it; by Prob. 42, Caſe 2, lay 19° . 15′ (that is, one Hour and ſeventeen Minutes, the Time from Noon) on the Æquinoctial from Q to ⊙, and, by Prob. 17, draw the oblique Circle P ⊙ S; then, by Prob. 59, draw an oblique Circle thro' the Point ⊙, to make an Angle of 24° . 2′, the given Azimuth, with the Meridian, interſecting it in the two Points Z and N, the Zenith and Nadir of the Place; draw Z O N the Prime Vertical, and the Horizon H O R at right Angles to it, interſecting the oblique Circle Z ⊙ N in *a*, and forming the right-angled ſpherical Triangle O ⊙ *a*, right-angled at *a*; in which is given the Hypothenuſe O ⊙ 70° . 45′, the Complement of ⊙ Q, the Time from Noon, and the Baſe O *a* 65° . 58′, the Complement of the given Azimuth, to find the Angle ⊙ O *a*, the Complement of Z O Q, the Latitude of the Place ſought.

This is an Extreme conjunct, and the Angle at O, being the middle Part, may be found by Lord *Napier*'s firſt Rule, and its *Note*, Page 265:

As the Radius ——— ———	10.
Is to the Co-tangent of O ⊙, 70° . 45′, —	9.543094
So is the Tangent of O *a*, 65° . 58′, —	10.350737
To the Co-ſine of ⊙ O *a*, 38° . 27′, the Complement of Latitude, — —	9.893831

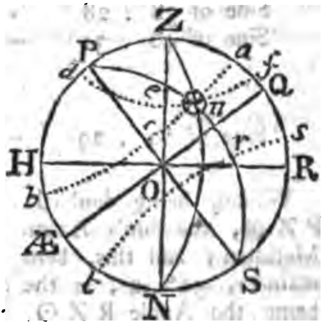
Therefore the Latitude is 51° . 33′ either North or South, as the Obſervation is made either on the North or South

Side of the Æquinox; but the Figure is here made for North Latitude.

The Angle at O, being made at the Pole of the primitive Circle, may be meaſured by Prob. 47. If it were required to find the Altitude, the ſame Parts being given, the Perpendicular *a* ⊙ may be found by Lord *Nepier*'s fourth Rule, and its *Note*, Page 266, as the required Perpendicular is one of the Extremes disjunct.

Having gone thro' the principal Aſtronomical Problems ſolvable by right-angled ſpherical Trigonometry, I ſhall now add ſome uſeful Problems in oblique Trigonometry.

Prob. 109. *The Latitude of the Place* 51°. 32′ *North, the Sun's Declination* 22°. 47′ *North, and his Altitude* 49°. 29′, *given, to find his Azimuth.*



Conſtruction. With the Chord of 60°, draw the Meridian H Z R N; draw H O R the Horizon, and at right Angles to it Z O N the Prime Vertical; lay 51°. 32′, the Latitude, from N to Æ, and from Z to Q; then draw Æ O Q the Æquinoctial, and at right Angles to it P O S the Axis; by Prob. 61, draw the Parallel of Declination *a c b*, at 22°. 47′ Diſtance from the Æquinoctial; and the Parallel of Altitude *d v f*, at 49°. 29′ Diſtance from the Horizon, interſecting the Parallel of Declination *a c b* in ⊙, the Sun's true Place; through which, by Prob. 17, draw the Meridian P ⊙ S, and the Azimuth-Circle Z ⊙ N at right Angles to it, which will complete the oblique-angled Triangle Z ⊙ P; in which is given the Side Z ⊙ 40°. 31′, the Complement of the given Altitude, the Side P Z 38°. 28′, the Complement of the given Latitude, and the Side P ⊙ 67°. 13′, the Diſtance of the Sun from the elevated Pole, which, in this Caſe, is the Complement of the given Declination, to find the Angle P Z ⊙, the Sun's Azimuth from the North Part of the Meridian.

Here are the three Sides given to find an Angle, which may be done by Prob. 90, where we have ſhewn the Solution of this Problem, both by letting fall a Perpendicular, and alſo by a Method without a Perpendicular, as follows:

Three Sides	38°. 28′	Z P.
	40 . 31	Z ☉.
	67 . 13	☉ P.
	146 . 12	Sum.
	73 . 6	Half-Sum.

73°. 6′	Half-Sum.
38 . 28	Z P ſubtract.
34 . 38	Remainder.
73°. 6′	Half-Sum.
40 . 31	Z ☉ ſubtract.
32 . 35	Remainder.

Sine of 34°. 38′	9.754595
Sine of 32 . 35	9.731206
Sine of 38 . 28	Co. Ar. 0.206168
Sine of 40 . 31	Co. Ar. 0.187308
	19.879277
Sine of 60 . 29	9.939638

Which, being doubled, gives 120°. 58′ for the Angle P Z ☉, the Sun's Azimuth from the North Part of the Meridian; and this, being ſubtracted from 180°, the Remainder, 59°. 2′, is the Sun's Azimuth from the South, being the Angle R Z ☉, which is made at the Periphery, and may be meaſured by Prob. 54.

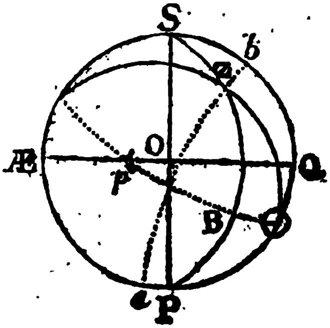
In the ſame Manner may the Angle at P be found, which is the Time from Noon. The two containing Sides in that Caſe will be P Z and P ☉, the Arithmetical Complements of their Sines added to the Sines of the Half-Sum and Remainder, as before: The Half of that Sum will be the Sine of 18°. 35′; whoſe Double, 37°. 10′, is the Angle Z P ☉ or Q P ☉, the Time from Noon; which, being converted into Time, is two Hours, twenty-eight Minutes, and forty Seconds; therefore, if it were in the Forenoon, it is thirty-one Minutes and twenty Seconds after Nine o'Clock; but, if in the Afternoon, twenty-eight Minutes and forty Seconds after Two.

Thus this Triangle contains the Complement of the Latitude in the Side P Z, and the Complement of the Sun's Declination in the Side P ☉, when the Declination and Latitude are both the ſame, as in the preſent Caſe; but, if the Declination and Latitude are, one North, the other South, then the Side P ⊙ will be the Sun's Declination added to 90°, as may be ſeen by drawing a Parallel of Declination on the contrary Side of the Æquinox, as *s r t*; for, in that Caſe, the Sun's Place will be at *n*, and the Side P *r* is the whole Quadrant P *u* more than the Declination *u r*. And, as the Angles at Z and P are the Sun's Azimuth and the Hour of the Day, this one Triangle is ſufficient to ſolve all the ſeveral Problems wherein three of theſe five Parts are given to find the other two. The Method of projecting the Triangle and finding the required Parts, in all the different Caſes that can be put, may be found in the Problems of oblique-angled ſpherical Triangles, from Prob. 80 to Prob. 91 incluſive.

But, as ſome of thoſe Caſes require the primitive Circle to be the Azimuth or Meridian-Circle, on which the Sun is, and not the Meridian of the Place, it will be proper to give an Example in each of thoſe Caſes; for, *when a given Side adjoins to a given Angle, that Side muſt always be laid on the primitive Circle.*

Prob. 110. *The Sun's Declination 22°. 54′ North, and his Altitude 14°. 36′ at three Minutes paſt Two o'Clock in the Afternoon, given, to find the Latitude of the Place: (Suppoſe South Latitude).*

Here are two Sides, *viz.* the Declination added to 90°, (the Latitude and Declination being contrary Ways) 112°. 54′; and the Complement of Altitude, 75°. 24′, with the Time from Noon, two Hours and three Minutes; which, being converted into Degrees, is 30°. 45′, the Angle oppoſite one of the Sides given, to find the third Side, being the Complement of the Latitude.



Conſtruction. With the Chord of 60°, draw the Meridian Æ S P Q; draw Æ O Q the Æquinox, and S O P the Axis at right Angles to it; as the Latitude is South and Declination North, lay 22°. 54′ from Q to ⊙, which will be the Sun's Place, and the Side S ⊙ will be the Sun's Diſtance from the viſible Pole, or his Declination added to 90°; by Prob. 48, draw an oblique Circle S Z B P, to make an Angle with the primitive Circle of 30°.45′ at the Point S; and S Z B P will be the Meridian of the Place; lay 75°.24′, the Complement of Altitude, from ⊙ to *a*, and from ⊙ to *b*; then draw the Parallel of the Sun's Altitude *a z b*, at 75°.24′ from the Point ⊙, interſecting the Meridian in *z*, the Zenith of the Place; and an oblique Circle, being drawn from the Point ⊙ thro' the Point *z*, will be the Azimuth-Circle on which the Sun is, and completes the oblique-angled Triangle S ⊙ *z*; in which we have given the Side S ⊙ 112°.54′, the Sun's Declination added to 90°, or his Diſtance from the elevated Pole, the Side ⊙ *z* 75°.24′, the Complement of the given Altitude, and the Angle at S 30°.45′, the Time from Noon converted into Degrees, &c. to find the Side S *z*, the Diſtance of the Zenith from the viſible Pole, being the Complement of the Latitude of the Place.

Here being two Sides, with an Angle oppoſite one of them, given, to find the third Side, it may be done by Prob. 87: For, if a Perpendicular be let fall from the End of the given Side S ⊙ on the Side S *z*, by drawing an oblique Circle from the Angle at ⊙ thro' *p* the Pole of the Meridian S *z p*, it will interſect the Circle S *z* P at right Angles in B, and thereby form the two right-angled Triangles S B ⊙ and *z* B ⊙, both right-angled at B; in the former of which are given the Side S ⊙ 112°.54′, and the Angle at S 30°.45′, to find the Perpendicular B ⊙; which is an Extreme disjunct, and the required Perpendicular is the middle Part; therefore it may be found by Lord *Nepier*'s third Rule, and its *Note*, Page 266: But, as the Side S ⊙ is more than a Quadrant, we muſt take the Supplement of it to 180°, which is 67°.6′, and the Proportion will be,

As the Radius ——— ———	10.
Is to the Sine of the Supplement of S ⊙, 67°.6′,	9.964347
So is the Sine of B S ⊙, 30°.45′, ———	9.708670
To the Sine of B ⊙, 28°.6′, ———	9.673017

By the ſame Parts given, to find the Side S B, is an Extreme conjunct, and one of the Extremes required; which may be found by Lord *Nepier*'s ſecond Rule, and its *Note*, Page 266: Only obſerve, that, in this Caſe, it will be the Supplement of the Side S B, which will be found by Calculation:

As

As the Co-tangent of the Supplement of S⊙, 67°.6′,	Co. Ar. 0.374259
Is to the Radius,	10.
So is the Co-ſine of B S ⊙, 30°.45′,	9.934199
To the Tang. of the Supplement of S B, 63°.49′,	10.308458

Which, being ſubtracted from 180°, gives S B 116°.11′. Then, in order to find S z, the Complement of Latitude, we muſt find the Side B z of the right-angled Triangle z B ⊙; in which we have given the Hypothenuſe ⊙ z 75°.24′, and the Perpendicular B ⊙ 28°.6′, by which either of the oblique Angles may be found, as the Angle at z; which is an Extreme disjunct, and one of the Extremes required; therefore, by Lord *Nepier's* fourth Rule, and its *Note*, Page 266,

As the Sine of ⊙ z, 75°.24′,	Co. Ar. 0.014255
Is to the Radius,	10.
So is the Sine of B ⊙, 28°.6′,	9.673032
To the Sine of B z ⊙, 29°.8′,	9.687287

Having found the Angle at z, with that, and the given Hypothenuſe ⊙ z, the Side B z may be found by Lord *Nepier's* ſecond Rule, and its *Note*, Page 266; being an Extreme conjunct, and one of the Extremes required:

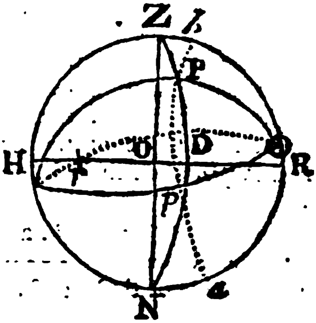
As the Co-tangent of ⊙ z, 75°.24′,	Co. Ar. 0.584225
Is to the Radius,	10.
So is the Co-ſine of B z ⊙, 29°.8′,	9.941258
To the Tangent of B z, 73°.24′,	10.525483

Which, being ſubtracted from the before-found S B 116°.11′, leaves S z 42°.47′, the Complement of Latitude; and that, being ſubtracted from 90°, leaves 17°.13′ South Latitude, ſought.

If the Sun's Azimuth had been required, the Angle at z is the Azimuth from the South, which we have found to be 29°.8′; whoſe Supplement to 180° is 151°.52′, the Sun's Azimuth from the North Part of the Meridian.

Prob. 111. *The Altitude of the Sun* 11°. 45′, *his Declination* 23°. 26′ *North, and his Azimuth from the South* 62°. 16′, *given, to find the Latitude of the Place.*

Here the Side repreſenting the Complement of the Altitude is adjoining to the Angle of the Azimuth; therefore the primitive Circle will be the Azimuth, on which the Sun is at that Time.



Conſtruction. With the Chord of 60°, draw the Azimuth-Circle H Z R N; thro' which draw H O R and Z O N at right Angles, for the Horizon and Prime Vertical; by Prob. 48, draw an oblique Circle, Z D N, to make an Angle of 62°. 16′, equal to the given Azimuth, with the primitive Circle in the Point Z, and Z D N is the Meridian of the Place; lay 78°. 15′, the Complement of the given Altitude, from Z to ⊙, which will be the Sun's Place at that Time; then lay 66°. 34′, the Complement of the Sun's Declination, both Ways, from ⊙ to *a* and *b*, and draw the Parallel of the Sun's Declination *a* P *b*, interſecting the Meridian in P and *p*; thro' either of theſe Points and Z N, draw the oblique Circle H P ⊙, (or H *p* ⊙) which will complete the Triangle Z ⊙ P, (or Z ⊙ *p*); in either of theſe are given the Side Z ⊙ 78°. 15′, the Complement of Altitude, the Side P ⊙ (or *p* ⊙) 66°. 34′, the Complement of Declination, and the Angle at Z 62°. 16′, the Sun's Azimuth from the South, to find the Side Z P, (or Z *p*) the Complement of the Latitude of the Place.

This may alſo be done by Prob. 87: A Perpendicular being let fall from the End of the Side Z ⊙ on the Side Z P, (or Z *p*) by drawing an oblique Circle from ⊙, thro' *r*, the Pole of the Circle Z P *p* N, will interſect it at right Angles in the Point D, and form the right-angled Triangles Z ⊙ D and P ⊙ D, (as alſo *p* ⊙ D) all right-angled at D; in the firſt of which we have the Side Z ⊙ 78°. 15′, and the Angle at Z 62°. 16′, to find the Perpendicular ⊙ D; which is an Extreme disjunct, and the middle Part required; whence, by Lord *Nepier*'s third Rule, and its *Note*, Page 266, As the Radius ——— ——— 10.

Is to the Sine of Z ⊙, 78°.15′, ——— 9.990803
So is the Sine of D Z ⊙, 62°.16′, ——— 9.947004

To the Sine of ⊙ D, 60°.4′, ——— 9.937807

Then, by the ſame Parts given, to find the Baſe Z D, is an Extreme conjunct, and one of the Extremes required; whence, by Lord *Nepier*'s ſecond Rule, and its *Note*, Page 266,

As the Co-tangent of Z ⊙, 78°.15′, Co. Ar. 0.681936
Is to the Radius, ——— ——— 10.
So is the Co-ſine of D Z ⊙, 62°.16′, ——— 9.667786

To the Tangent of D Z, 65°.55′, ——— 10.349722

Then, in order to find the Side Z P, the Complement of Latitude, we muſt firſt find the Side P D, the Baſe of the right-angled Triangle P ⊙ D; in which we have the Hypothenuſe P ⊙ 66°.34′, and the Perpendicular ⊙ D 64°.4′, by which either of the oblique Angles may be found, as the Angle at P; which is an Extreme disjunct, and the required Angle is one of the Extremes; therefore, by Lord *Nepier*'s fourth Rule, and its *Note*, Page 266,

As the Sine of P ⊙, 66°.34′, — Co. Ar. 0.037383
Is to the Radius, ——— ——— 10.
So is the Sine of ⊙ D, 60°.4′, ——— 9.937822

To the Sine of ⊙ P D, 70°.49′, ——— 9.975205

Now, by the Angle at P, and the Hypothenuſe P ⊙, given, to find the Baſe P D, is an Extreme conjunct, and the required Baſe is one of the Extremes; therefore, by Lord *Nepier*'s ſecond Rule, and its *Note*, Page 266,

As the Co-tangent of P ⊙, 66°.34′, Co. Ar. 0.363081
Is to the Radius, ——— ——— 10.
So is the Co-ſine of ⊙ P D, 70°.49′, ——— 9.516657

To the Tangent of P D, 37°.18′, ——— 9.879738

Which, being ſubtracted from Z D $65^\circ.55'$, before found, leaves Z P $28^\circ.9'$, the Complement of Latitude; and that, being ſubtracted from 90°, gives $61^\circ.51'$ North Latitude.

If the other Triangle, Z ☉ *p*, were taken, then Z *p* would be the Complement of Latitude; but, as Z *p* is more than a Quadrant, the Latitude would be South.

If the Hour of the Day were required, the Angle ☉ P D is the Time from Noon, which we have found to be $75^\circ.57'$; this, being converted into Time, gives five Hours, three Minutes, and forty-eight Seconds; ſo that, if it were in the Afternoon, it would be three Minutes and forty-eight Seconds paſt Five; but, if in the Forenoon, fifty-ſix Minutes and twelve Seconds paſt Six in the Morning.

Prob. 112. *To explain the* SOLAR SYSTEM, *commonly called the* Syſtem of the World.

Two different Syſtems of the Sun and Planets have obtained: One called the *Ptolomaic*, from its ſuppoſed Inventor CLAUDIUS PTOLOMÆUS, who flouriſhed in the ſecond Century; but was aſſerted by *Ariſtotle* *, *Hipparchus* †, and moſt of the antient Philoſophers, and followed by almoſt the whole World for many Ages: The other called the *Copernican*, from NICHOLAS COPERNICUS, born at *Thorn*, in *Pruſſia*, in 1473, the Reviver of this Syſtem, after it had been neglected for many Ages; tho' this was antiently aſſerted by ſeveral of the moſt eminent Philoſophers, particularly by PYTHAGORAS §, and therefore ſometimes called the *Pythagoric Syſtem*.

There has been alſo a third Syſtem, called the *Tychonic Syſtem*, from TYCHO BRAHE the Inventor, a noble *Dane*, born in 1546; but this has ſcarce been followed by any Aſtronomer of Note.

Of the Ptolomaic *Syſtem*.

This Syſtem is founded on the Suppoſition of the Earth's being at Reſt in the Center of the Univerſe, and the Sun and Planets moving round it in their ſeveral Orbs: But then, the whole Heavens are ſuppoſed to revolve round the Earth in twenty-four Hours; which, conſidering the immenſe Diſtance of the fixed Stars, is a Motion extremely rapid, and ſeems unneceſſary, ſince the ſame Appearances are produced by the Rotation of the Earth on its Axis in twenty-four Hours.

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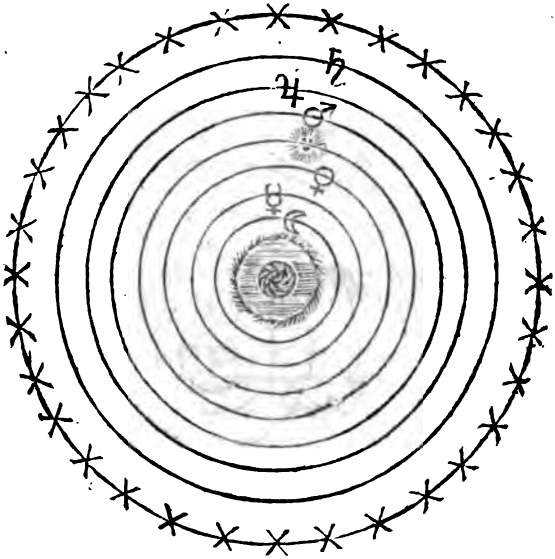
* *Ariſtotle* was a Diſciple of *Plato*, who was born 426 Years before Chriſt.

† *Hipparchus* was a *Greek* Aſtronomer, and lived about 120 Years before Chriſt.

§ *Pythagoras* was a *Greek* Philoſopher, and flouriſhed in the ſixtieth Olympiad that is, about 540 Years before Chriſt.

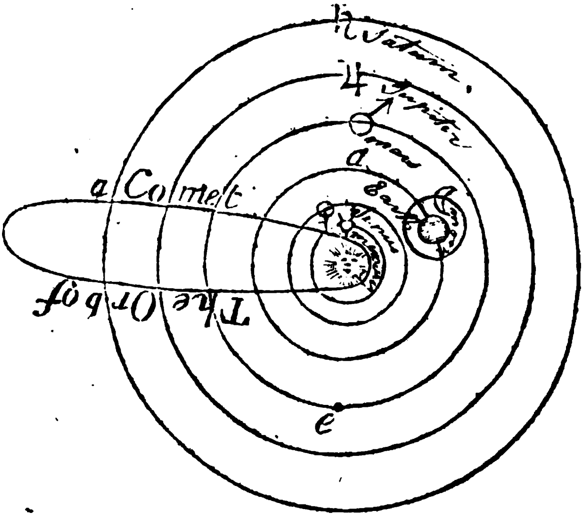


In this Syſtem, the *Earth* is placed in the Center, and ſuppoſed immoveable; next is the Orb of the *Moon*, then the Orb of *Mercury*, the Orb of *Venus* next, and then the Orb of the *Sun*; next is the Orb of *Mars*, then of *Jupiter*, then *Saturn*, and the higheſt is the Orb of the *fixed Stars*; all which are ſuppoſed to revolve round the Earth, from Eaſt to Weſt, in twenty-four Hours: Beſides which diurnal Motion, the Planets, in their ſeveral Orbs, have a Motion from Weſt to Eaſt, carrying them thro' the Heavens in different Periods. But, as the Phænomena of the Planets, according to aſtronomical Obſervations, cannot be ſolved by this Syſtem, it is now almoſt univerſally given up by learned Men.

Of the Copernican *Syſtem.*

In this Syſtem the Sun is placed in the Center, round which move the ſeveral Planets, *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn*; theſe ſix are called the primary Planets, the *Sun* being the Center about which their Motions are performed.

The *Earth* being the Center about which the Motion of the *Moon* is performed, the *Moon* is called a ſecondary Planet. *Jupiter* has four ſuch Moons moving round him, and *Saturn* five; theſe ſecondary Planets, or Moons, the Aſtronomers call the *Satellites* of *Jupiter*, or *Saturn*.



Copernicus, finding the *Ptolomaic* Syſtem to be void of Beauty, Order, and Proportion, peruſed the Works of the antient Philoſophers, to find if any of them had given a more rational Account of the celeſtial Motions. The firſt Hint he had, according to the learned Mr. *Maclaurin*, was from *Cicero*, (a *Roman* famous for his Eloquence); who ſays, that *Nicetus*, a *Syracuſian*, had taught, that the *Earth* turned round on its Axis, which occaſioned the whole Heavens to appear as if they had a diurnal Motion round the *Earth*. Afterwards, *Copernicus* learned from *Plutarch*, (a famous *Grecian*, born in the Reign of *Claudius*, Emperor of *Rome*) that *Philolaus*, the *Pythagorean*, had taught, that the *Earth* moved annually round the *Sun*. This great Man immediately perceived, that, by allowing theſe two Motions of the *Earth*, one round its Axis in twenty-four Hours, and the other thro' the Heavens in a Year, all that Confuſion, Irregularity, and Diſorder in the Motions of the Heavens, inexplicable by the *Ptolomaic* Syſtem, immediately vaniſhed and diſappeared: In the Room of which, a regular Diſpoſition of the planetary Orbs, and a beautiful Harmony in their Motions, appeared obviouſly reſulting from this Hypotheſis.

GALILEO, an illuſtrious *Italian* Philoſopher, towards the latter End of the ſixteenth Century, diſtinguiſhed himſelf in Defence of the *Copernican* Syſtem. He firſt applied a Teleſcope to the Heavens, and, by the Diſcoveries he made there with that noble Inſtrument, enlarged and confirmed the *Copernican* Syſtem. It had been objected to *Copernicus*, that, if his Syſtem of the World were true, *Venus* would undergo the ſame Changes and Phaſes as the Moon: To which, for Want of the Invention of Teleſcopes, he could only anſwer, *That, perhaps the Aſtronomers would, in future Times, diſcover* Venus *did really undergo all thoſe Changes.* This prophetic Conjecture was firſt fulfilled, when *Galileo*, by the Uſe of the Teleſcope, diſcovered *Venus* under thoſe ſeveral Changes. Beſides which, he diſcovered the mountainous and uneven Surface of the Moon, the Spots in the *Sun*, and his Revolution upon his own Axis. He alſo diſcovered that *Jupiter* had four Satellites, or Moons *, moving round him at different Diſtances, as the *Moon* moves round us: But his Teleſcope was not perfect enough to diſcover the Moons of *Saturn*. *Huygens*, in the Year 1655, on the twenty-fifth of *March*, with a Teleſcope twelve Feet long, firſt diſcovered one of *Saturn's* Moons: Since that Time, four others have been diſcovered by *Caſſini*, at different Times, by *Campani*'s Glaſſes, in Teleſcopes of one hundred and one hundred and thirty-ſix Feet long. At the ſame Time was diſcovered a Ring encompaſſing the Body of *Saturn*, which is ſuppoſed to reflect the Light and Heat of the *Sun* on the Body of *Saturn*, to ſupply the great Deficiency occaſioned by his Diſtance from that Fountain of Light and Heat.

The immortal Sir ISAAC NEWTON, the Glory of the *Britiſh* Nation, was the firſt that inveſtigated the true Cauſe of the planetary Motions: He applied the Principle of the univerſal Gravitation of Matter to Matter, to the Reſolution of the planetary Motions. The Accuracy and Agreement found betwixt the Concluſions drawn from the Theory of Gravity, with the Obſervations of Aſtronomers, is a rational and evident Proof of the Truth of his Hypotheſis. Thus, by one ſimple Principle, which obtains in all Parts of the material World, are the Motions of the ſeveral amazing Bodies in the Solar Syſtem performed.

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* Which he called the *Medicean* Stars, in Honour to the Name of *Medicis*, the Family of the Great Duke of *Tuſcany*, to whom he was chief Mathematician.

The Diameter of the *Sun* is computed at 763460 *Engliſh* Miles, whence the Circumference is 2398485 *Engliſh* Miles: It turns upon its Axis once in about twenty-ſeven Days, which is computed from the Motion of ſome Spots that appear near the Edge of the Diſk or Face of the *Sun*, and go over it, and diſappear for about the ſame Time they took to paſs over the Diſk. If the Magnitude of the Planets, *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn*, are added into one Sum, their Magnitude would be to the Magnitude of the *Sun* as about 1 to 523.

The Planet next to the *Sun* is *Mercury*, whoſe Diameter is 2460 *Engliſh* Miles, whence its Circumference is 7724 Miles. *Mercury* is diſtant from the *Sun* 32000000 Miles; the Circumference of the Orb is 201024000 Miles, which it moves through, or the periodical Time is eighty-ſeven Days, twenty-three Hours, and ſixteen Minutes, which is at the Rate of 95126 Miles in one Hour. The Heat *Mercury* receives from the *Sun* is ſeven times more than the Heat we have from the *Sun*. *Mercury* is never ſeen thirty Degrees from the *Sun*.

Next to *Mercury* is *Venus*, whoſe Diameter is 7906 *Engliſh* Miles, whence her Circumference is 24823 *Engliſh* Miles. The mean Diſtance of *Venus* from the *Sun* is 59000000 Miles; the Circumference of her Orb is 370636000 Miles, which ſhe paſſes over, or the periodic Time is two hundred and twenty-four Days, ſixteen Hours, and forty-nine Minutes, which is at the Rate of 68725 Miles in an Hour. *Venus* has alſo a Rotation on her Axis in twenty-three Hours; and receives from the *Sun* twice as much Light and Heat as we. *Venus* is never ſeen above forty-ſix Degrees from the *Sun*.

The *Earth* moves in the next Orb, accompanied with the *Moon*. The *Earth's* Diameter is 7964 *Engliſh* Miles, and its Circumference 25020 Miles, tho' ſome make it ſomewhat more. The *Earth*'s mean Diſtance from the *Sun* is 81000000 Miles; and the Circumference of the Orb is 508939200 Miles, which the *Earth* paſſes over, or the periodic Time is three hundred and ſixty-five Days, five Hours, and forty-nine Minutes, which is at the Rate of 58064 Miles in an

Hour. The *Earth* has a Rotation about its Axis in twenty-four Hours.

Round the *Earth* moves the *Moon*, whoſe Diameter is 2175 Miles, and her Circumference is 6834 Miles: Her mean Diſtance from the *Earth* is 240000 Miles; and the Circumference of the Orb is 1507963 Miles, which ſhe encompaſſes in twenty-ſeven Days, ſeven Hours, and forty-three Minutes, which is at the Rate of 2309 Miles in an Hour. The *Moon* revolves about her own Axis in the ſame Time, *viz.* twenty-ſeven Days, ſeven Hours, and forty-three Minutes. Beſides this Motion of the *Moon* in her Orb, ſhe is carried along with the *Earth* round the *Sun* once in a Year.

Mars moves round the *Sun* next above the *Earth*. The Diameter of *Mars* is 4442 Miles, and his Circumference is 13960 Miles: His mean Diſtance from the *Sun* is 123000000 Miles; and the Circumference of the Orb is 773686000 Miles, which he encompaſſes, or his periodic Time is ſix hundred and eighty-ſix Days, twenty-ſeven Hours, and twenty-ſeven Minutes, or one Year, three hundred and twenty-one Days, ſeventeen Hours, and thirty-eight Minutes, which is about 46925 Miles in an Hour. *Mars* performs his Revolution upon his Axis in twenty-four Hours and forty Minutes. The Light and Heat this Planet receives from the *Sun* is leſs by one Third than what we enjoy.

This Planet, moving in the next Orb to the *Earth*, is ſometimes very near the *Earth*, and at others at a far greater Diſtance: Thus, when the *Earth* is at *a*, if *Mars* be at ♂, the Diſtance will be but 42000000 Miles; but if, when the *Earth* is in *a*, *Mars* is at *e*, the Diſtance will then be about 20400000 Miles; which is the Reaſon that the apparent Diameter of *Mars* is five times greater when the *Earth* is at *a* and *Mars* at ♂, or in Oppoſition to the *Sun*, than it is when the *Earth* is at *a* and *Mars* in *e*, or in Conjunction with the *Sun*.

The next is *Jupiter*, whoſe Diameter is 81155 Miles, and his Circumference is 254908 Miles: His mean Diſtance from the *Sun* is 424000000 Miles; and the Circumference of his Orb is 2662280000 Miles, which he paſſes over in four thouſand, three hundred and thirty-two Days, twelve Hours, and twenty Minutes, his periodical Time, which is near twelve Years; whence he moves about 25603 Miles in an Hour. The Proportion of Heat this Planet receives from the *Sun* is but one thirtieth Part of what we receive. *Jupiter* performs his diurnal Motion on his Axis in nine Hours and fifty-ſix Minutes.

Jupiter has four Satellites, or Moons, attending him, to ſupply the Defect of the *Sun*'s Light. The firſt Satellite of *Jupiter*, or that which is neareſt to him, is diſtant from *Jupiter* about 227234 *Engliſh* Miles, and moves round him in one Day, eighteen Hours, twenty-eight Minutes, and thirty-ſix Seconds: The ſecond is diſtant 365197 Miles, and moves 'round him in three Days, thirteen Hours, eighteen Minutes, and fifty-two Seconds: The third, at 568085 Miles Diſtance, moves round him in ſeven Days, three Hours, fifty-nine Minutes, and forty Seconds: The fourth and laſt is diſtant from *Jupiter* 1027949 Miles, and moves round him in ſixteen Days, eighteen Hours, five Minutes, and ſix Seconds.

Another Phænomenon peculiar to *Jupiter*, is, that he has three lucid Zones, or Belts, encompaſſing his Body, which Sir *Iſaac Newton* ſuppoſes to be made in his Atmoſphere. In theſe Belts are ſeveral Spots, by which, it is ſaid, *Jupiter*'s Motion on his Axis was firſt diſcovered.

The higheſt Planet in our Syſtem is *Saturn*, whoſe Diameter is 67870 Miles, his Circumference 254908 Miles. His mean Diſtance from the Sun is 777000000 Miles, and the Circumference of his Orb is 4881891000 Miles. His periodic Time being ten thouſand ſeven hundred fifty-nine Days, ſix Hours, and thirty-ſix Minutes, which is near thirty Years, his hourly Motion is about 18905 Miles. The Proportion of Heat this Planet receives from the *Sun* is about one ſixtieth Part of what we enjoy on the Earth.

Saturn is attended by five Satellites, or Moons: The neareſt is diſtant from *Saturn* 148465 *Engliſh* Miles, and moves round him in one Day, twenty-one Hours, eighteen Minutes, and thirty-one Seconds: The ſecond is diſtant 190036 Miles, and moves round him in two Days, ſeventeen Hours, forty-one Minutes, and twenty-ſeven Seconds: The third is diſtant 271480 Miles, and moves round him in four Days, thirteen Hours, forty-ſeven Minutes, and ſixteen Seconds: The fourth is diſtant 610830 Miles, and performs his Revolution in fifteen Days, twenty-two Hours, forty-one Minutes, and eleven Seconds: The fifth and laſt is diſtant 1832490 Miles, and moves round him in ſeventy-four Days, ſeven Hours, fifty-three Minutes, and fifty-ſeven Seconds.

The great Diſtance of the fourth and fifth Satellites of *Saturn* has given ſome Suſpicion that there was one betwixt them, but it has not been diſcovered, notwithſtanding the Improvements that have been made in Teleſcopes.

Theſe Secondaries, or Moons, beſides the great Uſe to give Light to their primary Planets, have been of ſingular Uſe in ſolving two noble Problems in Philoſophy; to determine the Longitude of Places on the Earth, and the Velocity of Light. *Roemer* was the firſt that made any Obſervations of this Kind; and he determined the Velocity of Light to be ſuch, that it came from the *Sun* to us in about eleven Minutes; but the immortal *Newton* has ſince ſhewn that Light does not take up ſeven Minutes in coming from the *Sun* to us.

Beſides theſe Bodies continually moving round the *Sun*, and called the primary and ſecondary Planets, there are other Bodies that make their Appearance for ſome Time, and then diſappear, called *Comets*. It is now well known that the Comets obſerve the ſame Law in their Revolutions round the *Sun* as the Planets; but then, as the Planets move through the Heavens all the ſame Way, the Comets move through the Heavens in all Manner of Directions.

The antient *Ægyptians* and *Chaldeans*, by a long Courſe of Obſervations, were ſaid to be able to predict the Apparition of Comets; but as, by the ſame Arts, it is ſaid they have foretold Earthquakes and Tempeſts, their Knowledge was rather the Reſult of Aſtrological Calculations than of Aſtronomical Theories. *Ariſtotle*'s Opinion, that Comets were nothing but Vapours or airy Meteors, prevailing among the *Greeks*, brought this Part of the celeſtial Science into neglect.

But *Seneca* the Philoſopher, having conſidered two remarkable Comets that appeared in his Time, placed them amongſt the celeſtial Bodies, believing them to be Stars of equal Duration with the World; and, tho' he was ignorant of the Laws that governed their Motions, yet foretold that there ſhould be Ages hereafter to whom Time and Diligence ſhould unfold all theſe Myſteries. Yet almoſt all the Aſtronomers differed from *Seneca*; and, by the learned Dr. *Halley*'s Account, he found nothing of Service in the Aſtronomy of Comets 'till 1337, when the Path of a Comet amongſt the fixed Stars was deſcribed by *Gregorius*, an Hiſtorian and Aſtronomer of *Conſtantinople*. After this another Comet was obſerved by *Regiomontanus*, 1472. And the Aſtronomers ſtill conſidered the Comets to be below the Moon: But, in the Year 1577, a remarkable Comet appeared, whoſe Motion the noble *Tycho* obſerved with great Application, and found that it was much higher than the Orb of the *Moon*, and might be placed amongſt the Orbs of the Planets. The ſagacious *Kepler* had the Opportunity of obſerving two Comets, one of which was very remarkable: This great Aſtronomer concluded, that *the Comets moved freely through the planetary Orbs, with a Motion not much different from a rectilinear one, but of what Kind he could not preciſely determine.* Then *Hevelius* obſerved the Comets, and, tho' he embraced *Kepler*'s Hypotheſis, yet complained his Calculations did not agree with his Obſervations; and was aware that *the Path of a Comet was bent into a curve Line, concave towards the* Sun. But the prodigious Comet of 1680 appearing for four Months, its Motion was obſerved, with the utmoſt Accuracy, by thoſe celebrated Aſtronomers, Mr. *Caſſini* at *Paris*, and Mr. *Flamſtead* at *Greenwich*. Not long after this the incomparable *Newton*, writing his admired *Mathematical Principles of Natural Philoſophy*, ſhewed how to deſcribe the Orbits of Comets geometrically; a Problem, in the Judgment of that great Aſtronomer Dr. *Halley*, ſcarce acceſſible to any but himſelf: And Dr. *Halley*, following his Principles, has calculated the Orbs of twenty-four Comets.

The Orbits of Comets are very eccentric Ellipſes; I have added one in the *Copernican* Syſtem, to give the Reader ſome Idea of the Curve in which they move. Mr. *Whiſton* has given a neat Conſtruction of the Solar Syſtem, with the cometary Orbs.

The famous Comet which appeared in 1680 came ſo near the *Sun*, as to be within a ſixth Part of the *Sun*'s Diameter from its Surface; when it ſuſtained a Degree of Heat 28000 times greater than the Heat of the *Sun* upon our Earth, and might have contracted a Heat 2000 times greater than the Heat of red-hot Iron. A Globe of red-hot Iron equal to our Earth, Sir *Iſaac Newton* has computed, would not cool in leſs than 50000 Years *. The longeſt Diameter of the Orb of this Comet Dr. *Halley* has computed to be about 11200000000 Miles; its periodic Time is computed to be about 575 Years.

The Tails of Comets has employed the Curious to diſcover their Cauſe and Origin. The illuſtrious Sir *Iſaac Newton* proves that it conſiſts of a Vapour ariſing continually from the Body of the Comet. The Tail of the Comet of 1680 was of ſuch a prodigious Size, as to be extended from its Head to a Diſtance ſcarcely inferior to that of the *Sun* from the *Earth*.

The Planets are ſuppoſed, and very juſtly, to be Worlds; but the great Extremes of Heat and Cold, which the Comets ſuſtain in their Approach to and Receſs from the *Sun*, make ſuch a Suppoſition, with Regard to Comets, not ſo probable; tho' it is by no Means impoſſible for God to create Beings ſubject to ſuch Laws as will enable them to bear ſuch a prodigious Change of Circumſtances. But the Comets are ſo far from being the Meſſengers of Harm to Mankind, Expectations which naturally ariſe out of Superſtition, and Ignorance of the Cauſe of any uncommon Phænomena, that the great *Newton* has aſſigned two very probable and important Uſes of Comets: The firſt is, that, by their nearer and nearer Approach to the *Sun* at every Revolution, they will at laſt fall into him, in order to reſtore the Waſte that great Body muſt ſuffer in ſupplying the Syſtem with Light and Heat: The other important Uſe is, that, from the great Heat they ſuſtain in their Approach to the *Sun*, the Parts that fly off from them are ſo rarefied as to ſupply the Planets with an exceeding fine elaſtic Medium, neceſſary for the Support of Animals and Vegetables.

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* *Newton*'s Mathematical Principles, tranſlated by Mr. *Motte*, Vol. ii. P. 361.

Beyond the Orbs of the Planets and Comets, at an immenſe Diſtance, are the fixed Stars, ſo called from their keeping the ſame Diſtance one from another. Theſe are at ſo great a Diſtance from us as to have no ſenſible Parallax, that is, the Diameter of the *Earth*'s Orb bears no ſenſible Proportion to their Diſtance from the *Earth*, whence their Diſtance cannot be known; but, that they are farther from us than the remoteſt Planet, is hence evident, ſince *Saturn*, the higheſt of the Planets, has a conſiderable Parallax.

It has been a popular Opinion that the fixed Stars were numberleſs, but this, like many other vulgar Notions, is proved to be a Miſtake, that is, with Regard to thoſe viſible to the naked Eye; for, tho' the Number of Stars in a Winter's Night appears ſo vaſtly great, yet the Stars ſeen by a good Eye are much fewer in Number than can be imagined: The Twinkling of the Stars deceives our Sight, and makes us think them more than they are, as is evident from the Catalogues that have been made of them. *Hipparchus* was the firſt that attempted a Catalogue, on Occaſion of a new Star that appeared in his Time: His Catalogue conſiſted of 1022 viſible Stars, to which *Ptolomy* added four more. The noble *Tycho* publiſhed a Catalogue of 777 from his own Obſervations. But this Number has been vaſtly increaſed by the Induſtry of modern Aſtronomers: Dr. *Flamſtead*'s Catalogue amounts to 3000, but then many of theſe are not viſible without a Teleſcope.

Yet, that the Stars are really innumerable, is evidently proved by the Uſe of the Teleſcope, which, being directed to any Part of the Heavens, diſcovers great Numbers which are inviſible to the naked Eye. In the *Pleiades*, inſtead of ſix or ſeven ſeen by the Eye, Dr. *Hook*, with a Teleſcope twelve Feet long, told ſeventy-eight; and, with larger Glaſſes, diſcovered many more of different Magnitudes. *Huygens*, looking at the Star in the Middle of *Orion*'s Sword, found it to be a Concourſe of twelve Stars. To this we may add, that the *Galaxy*, or *Milky-Way*, is now known to be an Aſſemblage of innumerable Stars too ſmall for the Sight.

If theſe Stars are all Suns, with Choruſes of Worlds moving about them, as is generally ſuppoſed, what an exalted Idea muſt it give us of the great Author of Nature? Without ſome ſuch Suppoſition, we can ſcarce form any Idea of the Uſe ſuch a prodigious Number of vaſt Bodies can be of, as far the greater Part are, by their immenſe Diſtance, hid from our Sight, and had not been known to exiſt 'till ſince the Invention of Teleſcopes in the laſt Age.

Of DIALLING.

DIALLING is the Art of drawing Lines on a Plane, in ſuch Manner as that the Shadow of a Style may point out the Hours of the Day.

Of Dials there are various Sorts, according to the Situation of the Planes on which they are deſcribed: The moſt uſual are the Horizontal, Vertical, Æquinoctial, Polar, &c.

An Horizontal Dial is deſcribed on an Horizontal Plane: This is the moſt uſeful of all Dials, as it gives the Hour of the Day at all Times, while the Sun is above the Horizon, both in Summer and Winter.

To calculate an Horizontal Dial for the Latitude of 51°. 32′.

For the Angle the Hour-Lines of Eleven and One make with the Meridian, the Proportion is,

As the Radius ——— ———	10.
Is to the Sine of the Latitude, 51°. 32′, —	9.893745
So is the Tangent of 15°, or one Hour, —	9.428052
To the Tangent of 11°. 51′, the Angle the Hour-Lines of 11 and 1 make with the Meridian,	9.321797

To find the Angle the Hour-Lines of Ten and Two make with the Meridian:

As the Radius	10.
Is to the Sine of the Latitude,	9.893745
So is the Tangent of 30°, or two Hours,	9.761439
To the Tangent of 24°. 19′,	9.655184

In the ſame manner may the Angles of the Hour-Lines for Nine and Three be found, by putting the Tangent of 45°, or three Hours, in the third Place of the Proportion: And ſo of the reſt. Then, for the Halves and Quarters, obſerve, that each Half-Hour is 7°. 30′, and each Quarter 3°. 45′: Thus, to find the Angle for Half an Hour after Three, or Half an Hour before Nine, the Proportion is,

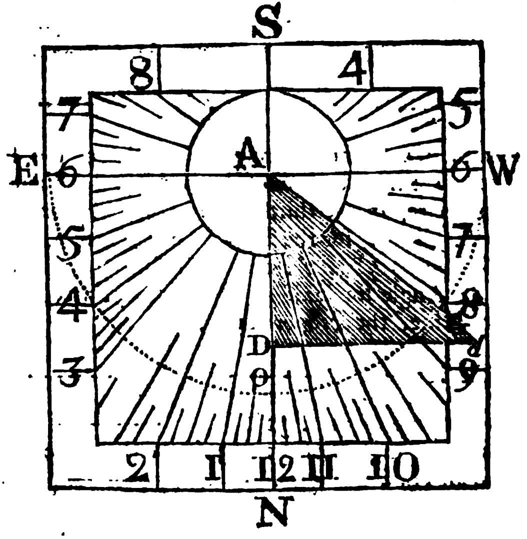
As the Radius	10.
Is to the Sine of 51°. 32′,	9.893745
So is the Tangent of 52°. 30′, or three Hours and an Half,	10.117113
To the Tangent of 45°. 43′,	10.010858

By the ſame Proportion may all the Hours, Halves, and Quarters be found, as in the following Table:

A Table of the Angles of the Hour-Lines.

Hours.	Hours in Deg. & Min.	Diſt. from the Merid.	Hours.	Hours in Deg. & Min.	Diſt. from the Merid.
11 ¾ or 12 ¼	3°. 45′	2°. 56′	8 ¾ or 3 ¼	48°. 45′	41°. 45
11 ½ or 12 ½	7 . 30	5 . 53	8 ½ or 3 ½	52 . 30	45 . 34
11 ¼ or 12 ¾	11 . 15	8 . 51	8 ¼ or 3 ¾	56 . 15	49 . 31
11 or 1	15 . 00	11 . 51	8 or 4	60 . 00	53 . 36
10 ¾ or 1 ¼	18 . 45	14 . 53	7 ¾ or 4 ¼	63 . 45	57 . 48
10 ½ or [illegible]	22 . 30	17 . 58	7 ½ or 4 ½	67 . 30	62 . 7
10 ¼ or [illegible]	26 . 15	21 . 7	7 ¼ or 4 ¾	71 . 15	66 . 34
10 or 2	30 . 00	24 . 19	7 or 5	75 . 00	71 . 6
9 ¾ or 2 ¼	33 . 45	27 . 37	6 ¾ or 5 ¼	78 . 45	75 . 45
9 ½ or 2 ½	37 . 30	30 . 59	6 ½ or 5 ½	82 . 30	80 . 27
9 ¼ or 2 ¾	41 . 15	34 . 28	6 ¼ or 5 ¾	86 . 15	85 . 13
9 or 3	45 . 00	38 . 3	6 6	90 . 00	90 . 00

To draw the Dial.



Draw two Squares, one within the other, ſo as to leave Room for the Figures of the Hours; thro' the Middle of the Squares draw a Meridian Line, S N, and on a Point aſſumed, as A, draw E W perpendicular to it; on A, with the Chord of 60°, draw a Semicircle, E O W, on which, by the Line of Chords, ſet off the ſeveral Hours, Halves, and Quarters in the Table; draw a ſmall Circle round A, then lay a Ruler from the Center A to each of the Diviſions, and draw Lines from the ſmall Circle to the inner Square for each Hour, and ſhorter for the Halves and Quarters: The Hours before and after Six are made by drawing the Hours of Four and Five in the Afternoon thro' the Center, for Four and Five in the Morning; and Seven and Eight in the Morning being drawn thro' the Center, you have the Hour-Lines for Seven and Eight at Night.

Then, for the Style, lay 51°. 32', the Latitude of the Place, from O to G, and draw A G; the Angle O A G will be the Height of the Style. Or, the Dial being made, you may aſſume any Part of the Line A O N [illegible] which to erect the Style; and meaſure the Diſtance from A to that Point, as D, which let be eighteen Inches, to find the perpendicular Height of D *d*:

As the Sine of the Angle A G D, 38°. 28′, Co. Ar. 0.206168
Is to A D, 18 Inches, ——— 1.255272
So is the Sine of the Angle O A G, 51°. 32′, — 9.893745

To D *d*, 22.67 Inches, ——— 1.355185

And ſo high muſt the upper Surface of the Style be from the Plane at D.

To make an erect direct South Dial for the Lat. of 51°. 32′.

To calculate the Hour-Lines for this Dial the ſame Proportion is uſed as in the former, only the ſecond Number in the Proportion is the Co-ſine of the Latitude, which there was the Sine. And it is to be obſerved, that the Sun never ſhines on this Dial but from Six in the Morning to Six at Night. For the Hours of Eleven and One ſay,

As the Radius ——— ——— 10.

Is to the Co-ſine of the Latitude, 51°. 32′, — 9.793832
So is the Tangent of 15°, or one Hour, —— 9.428052

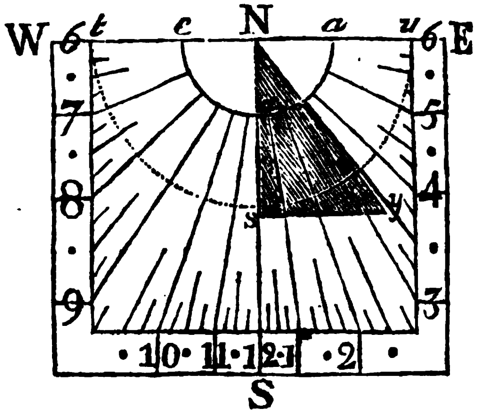
To the Tangent of 9°. 28′, —— —— 9.221884

The other Hours as in the following Table:

A Table of the Angles of the Hour-Lines with the Meridian.

Hours.	Hours in Deg. & Min.	Diſt. from the Merid.	Hours.	Hours in Deg. & Min.	Diſt. from the Merid.
11 ¾ or 12 ¼	3°. 45′	2°. 20′	8 ¾ or 3 ¼	48°. 45′	35°. 21′
11 ½ or 12 ½	7 . 30	4 . 41	8 ½ or 3 ½	52 . 30	39 . 2
11 ¼ or 12 ¾	11 . 15	7 . 3	8 ¼ or 3 ¾	56 . 15	42 . 57
11 or 1	15 . 00	9 . 28	8 or 4	60 . 00	47 . 8
10 ¾ or 1 ¼	18 . 45	11 . 55	7 ¾ or 4 ¼	63 . 45	51 . 36
10 ½ or 1 ½	22 . 30	14 . 27	7 ½ or 4 ½	67 . 30	56 . 21
10 ¼ or 1 ¾	26 . 15	17 . 3	7 ¼ or 4 ¾	71 . 15	61 . 22
10 or 2	30 . 00	19 . 45	7 or 5	75 . 00	66 . 42
9 ¾ or 2 ¼	33 . 45	22 . 34	6 ¾ or 5 ¼	78 . 45	72 . 16
9 ½ or 2 ½	37 . 30	25 . 31	6 ½ or 5 ½	82 . 30	78 . 3
9 ¼ or 2 ¾	41 . 15	28 . 37	6 ¼ or 5 ¾	86 . 15	83 . 59
9 or 3	45 . 00	31 . 53	6 6	90 . 00	90 . 00

To draw the Dial.



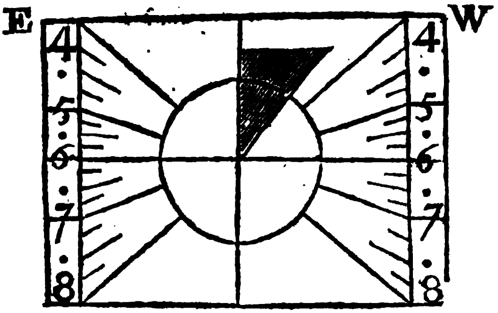
Draw an horizontal Line, W E; on a Point aſſumed, N, erect a Perpendicular, NS, for the Meridian, or Twelve o'Clock Line; extend each of theſe to a convenient Length, as in the Cut, and draw the internal Square; then draw three Lines to encompaſs it, leaving Room for the Figures: With any ſmall Extent of the Compaſſes, ſet one Foot in N, and draw the ſmall Semicircle *a x e*; and, with the Chord of 60°, draw the Semicircle *u r s t*, on which lay the Hours, as in the Table, from the Meridian N S towards E and W; then lay a Ruler from N to each of thoſe Diviſions, and draw the Hour-Lines from the inner Circle to the inner Square: In the ſame Manner may the Half-Hours and Quarters be drawn.

For the Style, take 38°.28′, the Complement of the Latitude, from the Line of Chords, and lay it from *s* to *r*, and draw the Line N *r*; thus will the Angle *s* N *r* be the Height of the Style, which is to ſtand on the Meridian, or Twelve o'Clock Line. Or, the Diſtance N *s* being meaſured on the Plane, which let be fourteen Inches, to find the perpendicular Height of the Style *s y*,

As the Sine of the Angle *s y* N, 51°.32′, Co. Ar.	0.106255
Is to *s* N, 14 Inches, ——— ———	1.146128
So is the Sine of the Angle *s r* N, 38°.28′, —	9.793832
To the perpendicular Height of *s r*, 11.12 Inches,	1.046215

And this is the Height of the upper Surface of the Style at the Point *y*.

The



The North Dial requires the Hour-Lines from Four to Eight in the Morning, and from Four to Eight at Night. If you take the South Dial, and draw the two Hours after Six, and before Six o'Clock, calling them Six, Seven, and Eight, as in the Figure, and continue them thro' the Center for the Hours of Four and Five, it is an erect direct North Dial; in which the Style muſt be fixed on the Twelve o'Clock Line, pointing upwards, as in the Scheme.

Of Eaſt and Weſt Dials.

The Planes of all direct Eaſt and Weſt Dials are in the Meridians, which paſſing thro' the Poles of the World, Eaſt and Weſt Dials have no Latitude, and therefore no Center; and the Hour-Lines are all parallel to one another. The Style in theſe Dials is either an upright Pin, or a Plate fixed on the Hour-Line of Six o'Clock, the Height being equal to the Diſtance of the Hour-Lines of Six and Nine in Eaſt Dials, or Three and Six in Weſt Dials.

To calculate the Diſtances of the Hour-Lines from the Six o'Clock Line for direct Eaſt or Weſt Dials.

To do this, the Height of the Style muſt be given, which ſuppoſe ten Inches; then, for two Hours either before or after Six, the Proportion is,

As the Co-ſine of 30°, or two Hours, — Co. Ar.	0.062469
Is to 10 Inches, the Height of the Style, —	1.000000
So is the Sine of 30°, — —	9.698970
To 5.77 Inches, — —	c.761439

In the ſame Manner may any other Hour, Half, or Quarter be found; as for four-Hours and an Half, which is 67° . 30′, ſay,

As the Co-ſine of 67° . 30′, —— ——	Co. Ar. 0.417160
Is to 10 Inches, —— ——	1.000000
So is the Sine of 67° . 30′, —— ——	9.965615
To 24.14 Inches, —— ——	1.382775

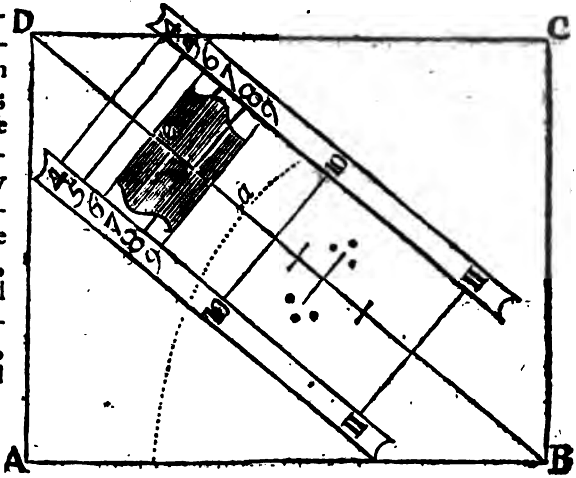
By the ſame Proportion may all the others be found, as in the annexed Table; where it is to be obſerved, that the Hour-Diſtances are here reckoned from Six, the Subſtyle of theſe Dials, as they are from Twelve in Horizontal and South Dials.

A Table of the Hour-Diſtances for an Eaſt Dial.

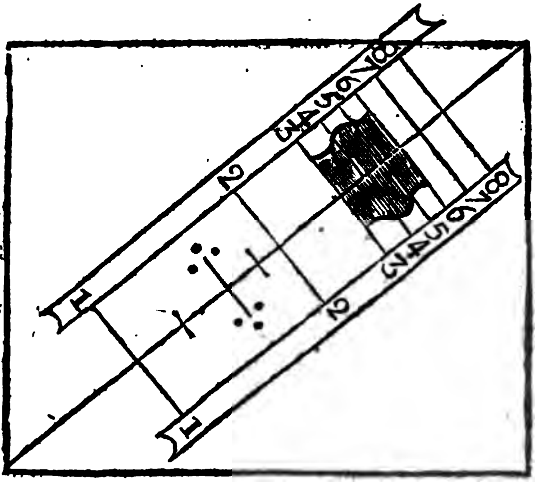
Hours, &c.	Hours in Deg. & Min.	Inch.	Hours.	Hours in Deg. & Min.	Inches.
6 ¼ or 5 ¾	3° . 45′	.66	8 ¾	41° . 15′	8.77
6 ½ or 5 ½	7 . 30	1.32	9	45 . 00	10
6 ¾ or 5 ¼	11 . 15	1.99	9 ¼	48 . 45	11.41
7 or 5	15 . 00	2.68	9 ½	52 . 30	13.03
7 ¼ or 4 ¾	18 . 45	3.39	9 ¾	56 . 15	14.96
7 ½ or 4 ½	22 . 30	4.14	10	60 . 00	17.33
7 ¾ or 4 ¼	26 . 15	4.93	10 ¼	63 . 45	20.27
8 or 4	30 . 00	5.77	10 ½	67 . 30	24.14
8 ¼	33 . 45	6.68	10 ¾	71 . 15	29.46
8 ½	37 . 30	7.67	11	75 . 00	37.32

To draw an Eaſt Dial for the Latitude of 51° . 32′.

Let A B C D be the Dial-Plane; take 60° from the Line of Chords, ſet one Foot of the Compaſſes in B, and draw the Arch *b a*; take 38° . 28′, the Complement of the Latitude, and ſet it from *b* to *a*, and draw B *a* thro' the whole Plane to D; and obſerve, that the Diagonal B D muſt be above ſix times the Length of the Style's Height; at about Once and an Half of the Style's Height from D, as at *e*, draw 6 *e* 6 for the Six o'Clock Line; from *e*, by the *general Rule*, Page 216, ay the ſeveral Hour-Diſtances in the Table; thro' thoſe ſeveral Diviſions draw Perpendiculars to the Diag. B D, which will be the ſeveral Hours, Halves, and Quarters.



In making an Eaſt Dial we have a Weſt Dial, with only this Difference, that the Diagonal is drawn from the left-hand Corner to the right-hand, and, inſtead of the Morning Hours from Four to Eleven, here we have the Evening Hours from Eight to One, at the ſame Diſtances as in the Table; ſo that the Table will ſerve for the Diſtances of any direct Eaſt or Weſt Dial, in any Latitude; as alſo for Polar Dials, as ſhall be ſhewn hereafter.



The Smallneſs of the Schemes not admitting the Diviſions into Quarters, I have done it only betwixt Ten and Eleven in the Eaſt, and One and Two in the Weſt Dial.

In making an Eaſt or Weſt Dial, the Height of the intended Style being given, the Hour-Diſtances may be found by the *Rule of Three* from this Table: As ſuppoſe your Style were ſix Inches, to find the Diſtance of the firſt Hour from Six, ſay,

As 10 Inches is to 6 Inches, ſo is 2.68 Inches.

$$\begin{array}{r} 6 \\ \hline 10)\,16.08\,(1.61 \text{ very near.} \end{array}$$

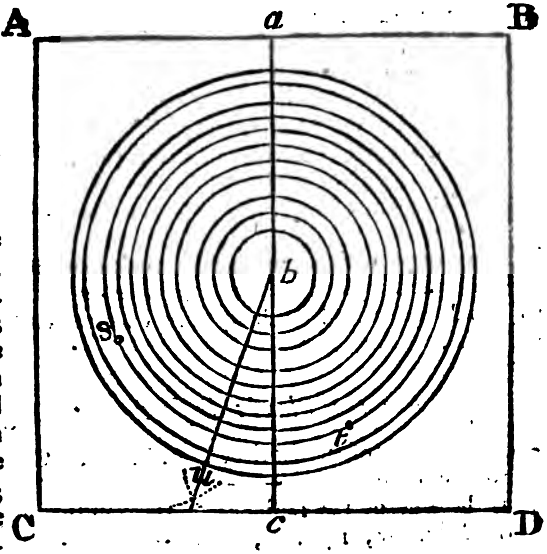
These five Dials may be all fixed on a Poſt, the Horizontal Dial on the Top, and the other four on the four Sides; Care being taken that the Sides are ſet ſo as to face the four Cardinal Points of Eaſt, Weſt, North, and South directly.

Of Declining Dials.

Dials being uſually fixed againſt the Walls of Houſes, which do not always ſtand directly facing the Cardinal Points, but more commonly declining, the moſt common Dials are what are uſually called Declining Dials, that is, South or North Dials, declining towards the Eaſt or Weſt: In this Caſe, the Declination of the Plane from the North or South Points, towards the Eaſt or Weſt, muſt be firſt known, before the Dial can be made.

To find the Declination of any Plane.

There are a Variety of Methods to find the Declination of Planes, of which the following, as it may be performed without Mathematical Inſtruments, will be of more univerſal Uſe: Take an even, well-planed Board, of ſix, eight, or ten Inches ſquare, and draw a Line, *a b c*, thro' the Middle, perpendicular to the Sides A B and C D; on the Center *b* draw ſeveral eccentric Circles at ſmall Diſtances, at Pleaſure, it not being neceſſary to make them at equal Diſtances one from another; and ſtick a Pin in the Center, which muſt ſtand directly upright: Place the Edge A B againſt the Wall whoſe Declination is required, between the Hours of Eight and Ten in the Forenoon; obſerve when the Shadow of the Top of the Pin juſt touches one of the Circles, and make a Mark at that Place in the Circle; then, in the Afternoon, watch 'till the Shadow comes to touch the ſame Circle again, and make another Mark,



Mark, as at *s* and *t*: Open your Compaſſes to above Half the Diſtance of thoſe two Marks, ſet one Foot in one of the Marks, as *s*, and draw a ſmall Arch, *u*; then ſet the Compaſſes in the other, and draw an Arch croſſing the former in *u*; draw a Line from the Center *b* thro' the Interſection of thoſe Arches, which will be a true Meridian-Line; and the Angle *u b c*, being meaſured on a Line of Chords, gives 18°. 25′, the Declination of the Plane Eaſt, as the Line *b c* is to the right Hand of the Meridian-Line *u b*.

Thus having the Declination 18°. 25′ Eaſt, and the Latitude 51°. 32′ North, there are three Things required to be found before we come to calculate the Hours, *viz.* the Height of the Style, or Elevation of the Pole above the Plane, the Angle the Subſtyle makes with the Meridian on the Dial, and the Plane's Difference of Longitude.

To find the Height of the Style, or Elevation of the Pole above the Plane:

As the Radius ——— ——— 10.

Is to the Co-ſine of the Latitude, 51°. 32′, — 9.793832
So is the Co-ſine of the Declination, 18°. 25′, — 9.977167

To the Sine of 36°. 10′, the Height of the Style, 9.770999

Which is alſo the Height of the Pole above the Plane, for the Styles of all Dials muſt point directly to the Poles of the World.

To find the Angle of the Subſtyle with the Meridian.

As the Radius	10.
Is to the Sine of the Declination, 18°. 25′,	9.499584
So is the Co-tangent of the Latitude, 51°. 32′,	9.900086
To the Tangent of 14°. 5′,	9.399670

The Angle which the Subſtyle makes with the Meridian; that is, the Angle made by the Line on which the Style is to ſtand and the Meridian, or Twelve o'Clock Line of the Dial.

To find the Plane's Difference of Longitude.

As the Co-ſine of the Latitude, 51°. 32′,	Co. Ar. 0.206168
Is to the Radius,	10.
So is the Sine of the Diſtance of the Subſtyle from the Meridian, 14°. 5′,	9.386201
To the Sine of 23°. 2′, the Plane's Difference of Longitude,	9.592369

The Plane's Difference of Longitude, being 23°. 2′, is more than one Hour, or 15°, and leſs than 30°, or two Hours; therefore the Subſtyle muſt ſtand between the firſt and ſecond Hours from Noon, that is, as the Plane declines Eaſt, between the Hours of Ten and Eleven o'Clock. If 15°, or one Hour, be taken from the Plane's Difference of Longitude, the Remainder is 8°. 2′ for the Æquinoctial Diſtance of the Eleven o'Clock Hour-Line from the Subſtyle; and 23°. 2′, being taken from 30°, or two Hours, leaves 6°. 58′, the Diſtance of the Ten o'Clock Line from the Subſtyle. In the ſame Manner, 23°. 2′, being ſubtracted from 45°, or three Hours, 60°, or four Hours, *&c.* the Degrees and Minutes of all the Forenoon-Hours may be obtained: But, for the Afternoon-Hours, adding 23°. 2′, the Plane's Difference of Longitude, to 15° for One o'Clock, 30° for Two o'Clock, *&c.* you have the Æquinoctial Diſtances of the Afternoon-Hours, as in the ſecond Column of the Table, where the Degrees and Minutes are ſet down to every Hour and Half-Hour, from Five in the Morning to Four in the Afternoon. Then, for finding the Angles the Hour-Lines make with the Subſtyle on the Dial, the Proportion is the ſame as in Horizontal Dials ; thus, to find the Angle of the Hour-Line for Eleven, the Height of the Pole above the Plane being 36°. 10′, the Proportion is,

As the Radius ——— ——— 10.

Is to the Sine of 36°. 10′, ——— 9.770952
So is the Tangent of 8°. 2′, ——— 9.149632

To the Tangent of 4°. 46′, the Angle, ——— 9.920584

For the Angle of the Two o'Clock Hour-Line ſay,

As the Radius ——— ——— 10.

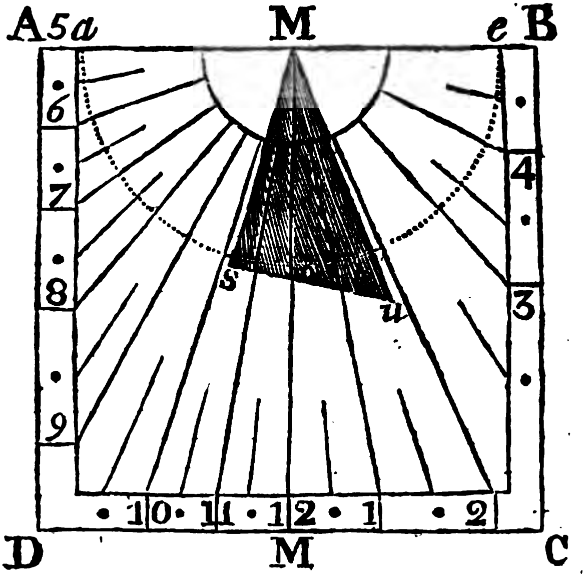
Is to the Sine of 36°. 10′, ——— 9.770952
So is the Tangent of 53°. 2′, ——— 10.123411

To the Tangent of 38°. 6′, the Angle, — 9.894363

In the ſame Manner may the Angle of any Hour or Half-Hour be found, by the Tangent of the Æquinoctial Diſtance being made the third Article in the Proportion ; by which the Angles will be as in the third Column of the Table.

A Table of the Æquinoctial Degrees, and Angles of the Hours.

Hours.	Hours in Deg. & Min.	Angles.	Hours.	Hours in Deg. & Min.	Angles.
5	81°. 58′	76°. 43′	11	8°. 2′	4°. 46′
5 ½	74 . 28	64 . 47	11 ½	15 . 32	9 . 19
6	66 . 58	54 . 14	12	23 . 2	14 . 5
6 ½	59 . 28	45 . 1	12 ½	30 . 32	19 . 12
7	51 . 58	37 . 2	1	38 . 2	24 . 47
7 ½	44 . 28	30 . 5	1 ½	45 . 32	31 . 1
8	36 . 58	23 . 57	2	53 . 2	38 . 6
8 ½	29 . 28	18 . 27	2 ½	60 . 32	46 . 15
9	21 . 58	13 . 23	3	68 . 2	55 . 39
9 ½	14 . 28	8 . 39	3 ½	75 . 32	66 . 23
10	6 . 58	4 . 7	4	83 . 2	78 . 18
10 ½	00 . 32	00 . 19			



To draw the Dial.

Let A B C D be the Dial-Plane; thro' the Middle draw M M perpendicular to A B, for the Meridian or Twelve o'Clock Line; draw a ſmall Semicircle, and then, with the Chord of 60°, draw *a o e*; lay 14° . 5', the Angle of the Subſtyle with the Meridian, from *o* to *s*, and draw M *s* for the Subſtyle: Then, with your Line of Chords, lay down the Hour-Angles according to the Table, from the Subſtyle, both Ways, as in the Scheme; but obſerve, that the Hour of Five in the Morning is not exactly the Line A B, for the Line A B makes an Angle of 75° . 55' with the Subſtyle, which is leſs by 48' than the Angle the Five o'Clock Line makes, as in the Table. For the Height of the Style, lay 36° . 10' from *s* to *t*, and draw the Line M *t* for the Style: Otherwiſe, meaſure the Line M *s*, which ſuppoſe twelve Inches, and the Angle *s* M *t* being 36° . 10', a Perpendicular, *t u*, being raiſed on the Subſtyle, the Angle *s u* M will be 53° . 50', the Complement of *s* M *u*; whence the Length of *s u* may be found by the following Proportion:

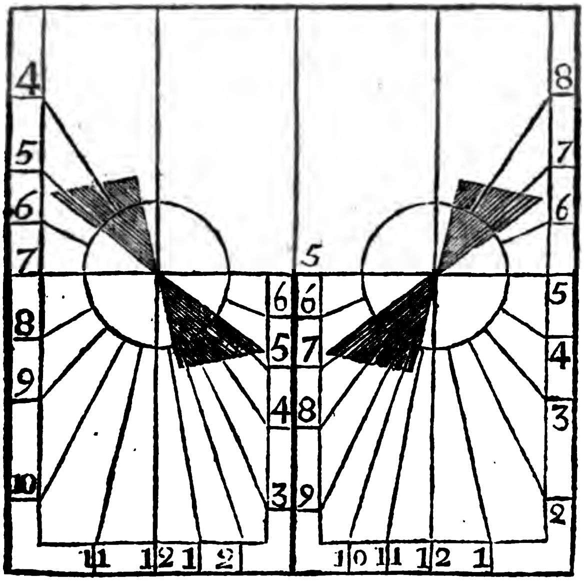
As the Sine of M *s*, 53° . 50',	——	Co. Ar. 0.092963
Is to 12 Inches,	———— ————	1.079181
So is the Sine of 36° . 10'	————	9.770260
8.75 Inches, the Height of *s u*,	——	0.942404

Then from the Diagonal Scale, by the *general Rule*, P. 216, lay 8.75 from *s* to *u*, and draw the Hypothenuſe M *u*, which forms the Style.

In making this Dial you make four Dials, that is, beſides this South declining Eaſt, a South declining Weſt, a North declining Eaſt, and a North declining Weſt, 18°. 25′, the ſame with the Example, by ſetting off the Angles for the Hours and the Style different, according to the different Situation of each Plane: Thus, the Morning-Hours in this will be the Afternoon-Hours in the South declining Weſt, and the Afternoon-Hours will be the Morning-Hours: Then, for the North Dials, draw the Hour-Lines of the South Decliners thro' the Center, and they will become Hour-Lines for the North Decliners.

To make this more plain, I have here drawn all the four Dials together, where it appears that the North Dials have their Styles pointing upwards.

North declining Eaſt 18°. 25′. | North declining Weſt 18°. 25′.



South declining Weſt 18°. 25′. | South declining Eaſt 18°. 25′.

In far-declining Planes, the Hour-Lines become ſo near parallel that they cannot be drawn with Centers: In this Caſe, it is neceſſary to draw them geometrically; and, as ſuch Planes are pretty common, I ſhall give an Example.

To draw a Dial for the Latitude of 51°. 32′ on a Plane declining from the South Weſtward 72°. 26′.

Before we begin the geometrical Part, the Height of the Pole, or Style, above the Plane, the Diſtance of the Subſtyle from the Meridian, and the Plane's Difference of Longitude, muſt be firſt found, as in the laſt Example:

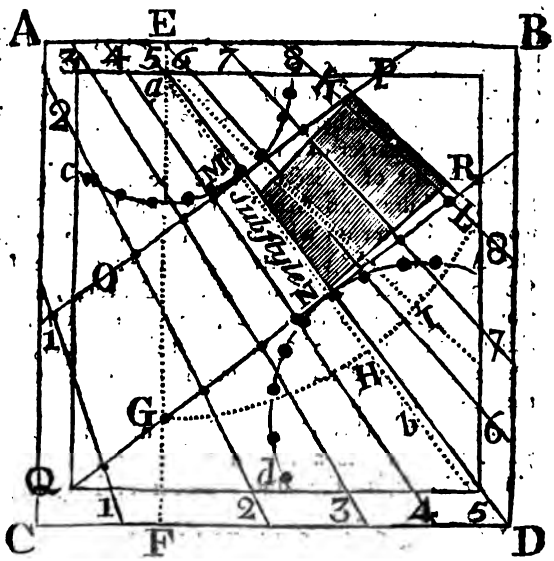
As the Radius ——— ———	10.
Is to the Co-ſine of the Latitude, 51°. 32′, —	9.793832
So is the Co-ſine of the Declination, 72°. 20′,	9.482128
To the Sine of 10°. 53′, the Height of the Pole, or Style, above the Plane. ———	9.275960

As the Radius ——— ———	10.
Is to the Sine of the Declination, 72°. 20′, —	9.979019
So is the Co-tangent of the Latitude, 51°. 32′,	9.900086
To the Tangent of 37°. 8′, the Diſtance of the Subſtyle from the Meridian, ——	9.879105

As the Co-ſine of the Latitude, 51°. 32′, Co. Ar.	0.206168
Is to the Radius, ——— ———	10.
So is the Sine of the Diſtance of the Subſtyle from the Meridian, 37°. 8′, ———	9.780801
To the Sine of the Plane's Difference of Longitude, 76°. 2′, —— ——	9.986969

To draw the Dial geometrically.

Draw A B C D for the Dial-Plane; from a Point aſſumed, as E, towards the left Hand of the Square, as the Declination of the Plane is Weſt, draw a Line, E F, perpendicular to A B and C D; with the Chord of 60°, ſet one Foot of the Compaſſes



Compaſſes where this Perpendicular croſſes the upper Line of the internal Square, and draw the Arch G H I; take 37°. 8′, the Diſtance of the Subſtyle from the Meridian, and lay it from G to H; then lay 10°. 53′, the Height of the Style above the Plane, from H to I, and draw E H for the Subſtyle, and E I for the Style; but, as the Diſtances of the Hour-Lines are proportional to the Height of the Style, it will be neceſſary to make the Style ſomewhat higher, which is done by drawing K L parallel to E I at a convenient Diſtance: Aſſume two Points, M and N, in the Subſtyle, thro' which draw Lines perpendicular to the Subſtyle, as O P and Q R; from the Point M take the neareſt Diſtance to the Top of the Style K L, ſet it on the Subſtyle from M to *a*, and on *a*, as a Center, with the Extent *q* M, draw an Arch of a Circle; alſo take the neareſt Diſtance from N to K L, ſet it from N to *b*, and on *b*, as a Center, with the Extent *b* N, draw another Arch; on each of theſe Arches ſet off 76°. 20′, the Plane's Difference of Longitude, from M to *c*, and from N to *d*: Then, ſetting a Sector to the Radius *b* N, take 15° in the Compaſſes, and, ſetting one Foot in *d*, turn them over on the Arch ſo as to make eight Marks from *d*; a Ruler, being laid from *b* to each of thoſe Marks, will cut the Line Q R in the ſeveral Points thro' which each of the eight Hour-Lines muſt paſs; then taking in the Compaſſes 15°, from the Sector ſet to the Radius *a* M, and, ſetting one Foot in *a*, turn them over on the Arch ſo as to make eight Marks, as on the former; a Ruler, being laid from *a* to each of theſe Marks, will cut the Line O P in each of the other Points thro' which the eight Hour-Lines muſt paſs; thus, having two Points for each Line, the Hour-Lines may be drawn, as in the Scheme. The ſame Method may be uſed, on a large Board or Braſs Plate, to draw Half-Hours and Quarters, remembering that 7°. 30′ is Half an Hour, and 3°. 45′ a Quarter of an Hour.

For Want of a Sector, take the common Radius from the Scale, and draw an Arch, on which lay 15°, and draw Lines from the Center on which the Arch was drawn to each End of the Arch of 15°; then take either of the Radius's in the Compaſſes, ſet one Foot in the Center, and draw an Arch from one Line to the other, and the Arch included between thoſe Lines will be 15°. In this Manner, any Number of Degrees to any Radius, either longer or ſhorter than that on the Scale, may be had.

In making this Dial we have made a South declining Eaſt 72°. 20′, as alſo a North declining Eaſt, and a North declining Weſt, to the ſame Number of Degrees, as was ſhewn in the Eaſt Dial.

When the Style of this Dial is adjuſted to its proper Height, and the two Perpendiculars O P and Q R are drawn, the Diſtances of the Hour-Lines on theſe Perpendiculars may be found by Trigonometrical Calculation: Thus, let the upper Part of the Style from M to where the Perpendicular O P cuts the Top of the Style be 8.5 Inches, and let the Height from N to where Q R cuts the Top of the Style be 10 Inches. To find the Diſtance of each Hour-Line from the Subſtyle, firſt find the Diſtance of each Hour from the Subſtyle in Degrees and Minutes of the Æquinoctial.

The Plane's Difference of Longitude being 76°. 2′, which is more than five Æquinoctial Hours, or 75°, and leſs than 90°, or ſix Hours; therefore the Subſtyle will fall between the Hour-Lines of Five and Six. If 75°, or five Hours, are ſubtracted from 76°. 2′, the Remainder, 1°. 2′, is the Æquinoctial Diſtance of the Subſtyle from the Five o'Clock Hour-Line: And in the ſame Manner may the Diſtances of One, Two, Three, and Four be found, by ſubtracting 15° for each Hour from 76°. 2′; but, for the Hours of Six, Seven, and Eight, ſubtract 76°. 2′ from 90°, 105°, and 120°, and the Remainders are the Diſtances, as in the Table. Then, for the Diſtances of the Hour-Lines on the Perpendiculars O P and Q R, the Proportion is the ſame as in direct Eaſt and Weſt Dials; only here, the Style being of an unequal

Height at the Perpendiculars, there muſt be two Calculations for each Hour: Thus, for the Hour-Line of Six,

As the Co-ſine of 13° . 58′, ——	Co. Ar. 0.013033
Is to 8.5 Inches, —— ——	0.929419
So is the Sine of 13° . 58′, ——	9.382660
To 2.11 Inches, the Diſtance of the Hour-Line of Six from M on the Perpendicular O P, —	0.325112

And,

As the Co-ſine of 13° . 58′, ——	Co. Ar. 0.013033
Is to 10 Inches, —— ——	1.000000
So is the Sine of 13° . 58′, ——	9.382660
To 2.49 Inches, the Diſtance on Q R, ——	0.395693

For any Hour before Five, as for the Three o'Clock Line,

As the Co-ſine of 31° . 2′, ——	Co. Ar. 0.067086
Is to 8.5 Inches, —— ——	0.929419
So is the Sine of 31° . 2′, ——	9.712260
To 5.11 Inches, the Diſtance on O P, —	0.708765

And,

As the Co-ſine of 31° . 2′, ——	Co. Ar. 0.067086
Is to 10 Inches, —— ——	1.000000
So is the Sine of 31° . 2′, —— ——	9.712260
To 6.02 Inches, the Diſtance on Q R, ——	0.779346

In the ſame Manner may all the other Hour-Diſtances be found, as in the Table.

A Table of the Diſtances of the Hour-Lines from the Subſtyle, on each Perpendicular.

Hours.	Hours in Deg. & Min.	Diſt. on O P in Inches.	Diſt. on Q R in Inches.
1	01° . 2′	15.35	18.1
2	46 . 2	8.81	10.37
3	31 . 2	5.11	6.02
4	16 . 2	2.44	2.87
5	1 . 2	.15	.15
6	13 . 58	2.11	2.49
7	28 . 58	4.7	5.53
8	43 . 58	8.2	9.96

Of Reclining and Inclining Dials.

Dials are ſometimes required to be made for Planes that are not erect, but which either recline from the Horizon, or incline towards it: Of theſe, as well as of erect Dials, there are two different Kinds, for the reclining Plane may either directly face one of the Cardinal Points, or decline from the North or South, either towards the Eaſt or Weſt: Hence ariſes the Diſtinction of *direct* reclining or inclining Dials, and *declining* reclining or inclining Dials, each of which ſhall be explained in their Order.

But, before we come to the making theſe Dials, it will be neceſſary to ſhew how to take the exact Reclination or Inclination of any Plane.

To take the Reclination of a Plane.

Lay a Ruler, or any ſtraight Rod, on the Plane, and let one End lie clear of the Plane; apply a Quadrant to that End of the Rod, and then, the Plummet hanging at Liberty, the Thread will cut the Degrees of the Plane's Reclination on the Limb. In the ſame Manner may the Inclination of a Plane be found, by laying the other Edge of the Quadrant to the Plane.

Having found the Reclination of the Plane, if it be a direct South Plane, conſider whether the Reclination be more or leſs than the Complement of the Latitude of the Place; if the Reclination of the Plane be leſs than the Complement of the Latitude, ſubtract the Reclination from the Complement of the Latitude, and the Remainder is the Height of the Pole above the Plane; but, if the Reclination is more than the Complement of the Latitude, ſubtract the Complement of Latitude from the Reclination, and the Remainder is the Height of the Pole, or Style, above the Plane.

To make a direct South Dial for a Plane reclining 10°. 56′ *in the Latitude of* 51°. 32′.

Subtract 10°. 56′ from 38°. 28′, the Complement of Latitude, and the Remainder, 27°. 32′, is the Height of the Pole above the Plane.

Having the Height of the Pole, the Angles the Hour-Lines make with the Meridian may be found by the ſame Proportion uſed in the Horizontal or erect direct South Dials: The Hour-Lines being found, the drawing the Dial is the ſame;

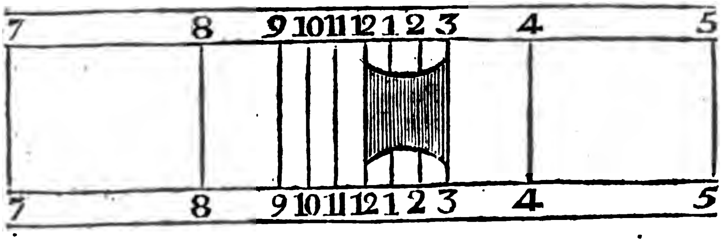
and the Style muſt be made to an Angle of 27° . 32′, pointing downwards, as in the erect South Dial.

To make a direct South Dial reclining 79°. 34′ for the Latitude of 51°. 32′.

Subtract the Complement of the Latitude from the Reclination, and the Remainder, 41° . 6′, is the Height of the Pole; by which the Angles of the Hour-Lines may be calculated, as in Horizontal Dials. The Style here muſt be made to the Angle of 41°. 6′, and ſet on the Twelve o'Clock Line, pointing upwards to the North Pole.

But, if the Reclination of the Plane is equal to the Complement of the Latitude, then is the Dial a Polar Dial, which is made by the Rules laid down for Eaſt and Weſt Dials: The Hour-Diſtances in this are proportioned to the Height of the Style; only obſerving, that the Style of a Polar Dial ſtands on the Twelve o'Clock Line, that being parallel to the Axis of the World, as the Six o'Clock Line is in Eaſt or Weſt Dials; therefore, making the Style ten Inches, as in that, the ſame Table will ſerve for this, taking the Diſtances, both Ways, from the Line of Six for the Diſtances from Twelve, as in the following Scheme.

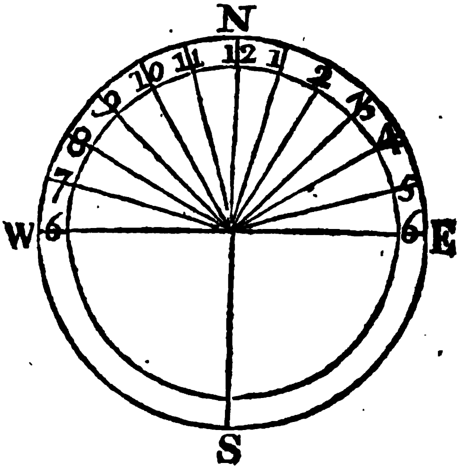
A Polar Dial for the Latitude of 51°. 32′.



In North reclining Planes, the Height of the Pole above the Plane is found by adding the Complement of the Latitude to the Reclination of the Plane, and the Sum, if leſs than 90°, is the Height of the Pole above the Plane; but, if the Sum exceeds 90°, ſubtract it from 180°, and the Remainder is the Height of the Pole. The Height of the Style muſt be the ſame as the Height of the Pole above the Plane, and muſt ſtand on the Meridian, pointing upwards towards the North Pole.

But, if the Sum of the Reclination and the Complement of Latitude be exactly 90°, then is the Plane parallel to the Æquinoctial, and is called an Æquinoctial Dial.

To draw an Æquinoctial Dial.



Draw a Circle, NESW; draw the Diameter N S for the Meridian, and E W for the Axis, or Hour-Line of Six, at right Angles to it; divide the Arches E N and N W into ſix equal Parts, and draw Diameters from each of thoſe Diviſions thro' the Center, which will be the Hour-Lines of the Dial; only, if it is made for any particular Latitude, the Lines may be omitted before Six in the Morning and after Six at Night, as they are here.

The Style for this Dial is a ſtraight Pin, ſet upright in the Center of the Dial, which may be of any Length at Pleaſure.

Of Eaſt and Weſt Reclining Dials.

In Eaſt and Weſt reclining Dials, the Latitude of the Place and Reclination of the Plane being given, the Height of the Pole above the Plane, the Plane's Difference of Longitude, and the Diſtance of the Subſtyle from the Meridian, muſt be found as in South declining Planes. To give an Example,

Of an Eaſt Plane in the Latitude of 51°. 32′ reclining 19°. 49′.

To find the *Height of the Pole above the Plane*, the Proportion is,

As the Radius ——— ———	10.
Is to the Sine of the Latitude, 51°. 32′, ——	9.893745
So is the Sine of the Reclination, 19°. 49′, —	9.530215
To the Sine of 15°. 24′, the Height of the Pole above the Plane, ——— ———	9.423960

For the *Diſtance of the Subſtyle from the Meridian*, the Proportion is,

As the Radius	10.
Is to the Tangent of the Latitude, 51° . 32′,	10.099913
So is the Co-ſine of the Reclination, 19° . 49′,	9.973489
To the Tangent of 49° . 50′, the Angle of the Subſtyle with the Meridian,	10.073402

Then, for the *Plane's Difference of Longitude*, the Proportion is,

As the Sine of the Latitude, 51° . 32′,	Co. Ar. 0.106254
Is to the Radius,	10.
So is the Sine of 49° . 50′, the Diſtance of the Subſtyle from the Meridian,	9.883191
To the Sine of 77° . 22′, the Plane's Difference of Longitude,	9.989445

The Plane's Difference of Longitude, 77° . 22′, being more than five Hours Diſtance from the Meridian, and leſs than ſix Hours, the Subſtyle will ſtand between the Hour-Lines of Six and Seven in the Forenoon, as it is an Eaſt Dial.

To find the Æquinoctial Diſtances of the Hours, ſubtract 15° for each Hour leſs than 75°, or five Hours, Diſtance from the Meridian, from the Plane's Difference of Longitude; but, for all the Hours more than five Hours, ſubtract the Plane's Difference of Longitude from the Hours, as has been ſhewn; and the Remainders are the Æquinoctial Diſtances of each reſpective Hour, as in the Table: By which, and the Height of the Pole above the Plane, the Angles the Hour-

A Table of the Angles of the Hour-Lines.

Hours. E.	Hours. W.	Hours in Deg. & Min.	Hour-Angl. with Subſt.
4	8	42° . 38′	13° . 33′
5	7	27 . 38	7 . 55
6	6	12 . 38	3 . 24
7	5	2 . 22	0 . 38
8	4	17 . 22	4 . 45
9	3	32 . 22	9 . 33
10	2	47 . 22	16 . 5
11	1	62 . 22	26 . 54
12	12	77 . 22	49 . 50

Lines make with the Subſtyle may be found, by the ſame Proportion already delivered, to be as in the Table. The Meridian-Line, or Hour of Twelve, is an Horizontal Line drawn thro' the Middle of the Dial.

The drawing the Hour-Lines has been already ſhewn: And the Style may be ſet off, by a Line of Chords, to the Angle of 15°. 24′, the Plane's Latitude, or calculated; both which Methods have been already taught, both in the Horizontal and erect direct South Dials.

A Weſt reclining Dial is made in the ſame Manner; only, as the Style here ſtands between the Hours of Six and Seven in the Morning, there it muſt ſtand between Five and Six in the Afternoon, for which Reaſon the Table is made for both.

Of Inclining Planes.

Dials for inclining Planes may be made from their oppoſite Recliners: Thus, the firſt Example of a South reclining Dial being drawn, if you look thro' the Paper, it becomes a North Dial for a Plane inclining 10°. 56′; only the Morning-Hours in that will be the Afternoon-Hours in this, and ſo *vice verſâ*; alſo the Style muſt be ſet pointing upwards to the North Pole: And the ſecond Example, in the ſame Manner, becomes a North Dial for a Plane inclining 79°. 34′, exchanging the Numbers of the Hours, and ſetting the Style pointing downwards. If a North Plane's Inclination be equal to the Complement of the Latitude of the Place, the Dial is an under Polar Dial, whoſe Style may be a thin Plate, fixed in the Middle, which here is the Twelve o'Clock Line for Midnight; and the Hours of Four and Five in the Morning, and Seven and Eight at Night, will be the only Hour-Lines that can be of Uſe.

In the ſame Manner may South inclining Dials be made from North Recliners: And, if a South Plane's Inclination be equal to the Latitude, the Dial is an under Æquinoctial Dial, and may be drawn as the upper Æquinoctial Dial on the North reclining Plane, to ſhew the Hours before Six in the Morning, and after Six at Night.

It being very ſeldom that Dials are made for declining Planes, that either recline from the Horizon, or incline towards it, and they being of difficult Calculation, I ſhall omit giving Examples of thoſe Dials.

I have ſhewn the Way of calculating the Hour-Lines by the Sines and Tangents, that being more exact than the geometrical Conſtruction. If, in large Dials with Centers, the Lines of Chords on the common Scales are not ſufficient for laying down the Hour-Lines, as they are only made to Degrees, ſo that the Minutes can be only taken by Gueſs, the Tangent-Line may be uſed, where the Diviſions are larger, except at the Beginning: Therefore, taking the largeſt Radius on the Scale, ſet it from the Center of the Dial, on the Subſtylar-Line; then croſs the Subſtyle in that Point with a Perpendicular, on which lay the Tangent of the ſeveral Angles of the Hour-Line with the Subſtyle, from the Tangent to the ſame Radius; and draw the Hour-Lines from the Center of the Dial to the Tangent of each reſpective Angle.

An Account of the Gregorian *or* New Style, *ſettled by Act of Parliament; the Method of finding the* Epact, Moon's Age, Tides, *&c.*

POPE *Gregory* the XIIIth made a Reformation of the Calendar. The *Julian* Calendar had before that Time been in general Uſe all over *Europe*. The Year, according to the *Julian* Calendar, conſiſts of three hundred and ſixty-five Days and ſix Hours; which ſix Hours being one fourth Part of a Day, the common Years conſiſted of three hundred and ſixty-five Days, and every fourth Year one Day was added to the Month of *February*, which made each of thoſe Years three hundred and ſixty-ſix Days, which have been uſually called Leap-Years.

This Computation, tho' near the Truth, is more than the Solar Year by eleven Minutes, which in one hundred and thirty-one Years amounts to a whole Day. By which the Vernal Æquinox was anticipated ten Days from the Time of the general Council of *Nice*, held in the Year 325 of the Chriſtian Æra, to the Time of Pope *Gregory*; who therefore cauſed ten Days to be taken out of the Month of *October* in 1582, to make the Æquinox fall on the twenty-firſt of *March*, as it did at the Time of that Council. And, to prevent the like Variation for the future, he ordered that three Days ſhould be abated in every four hundred Years, by reducing the Leap-Year at the Cloſe of each Century, for three ſucceſſive Centuries, to common Years, and retaining the Leap-Year at the Cloſe of each fourth Century only.

This was, at that Time, eſteemed as exactly conformable to the true Solar Year. But Dr. *Halley* makes the Solar Year to be three hundred and ſixty-five Days, five Hours, forty-eight Minutes, fifty-four Seconds, forty-one Thirds, twenty-ſeven Fourths, and thirty-one Fifths: According to which, in four hundred Years, the *Julian* Years of three hundred and ſixty-five Days and ſix Hours will exceed the Solar by three Days, one Hour, and fifty-five Minutes, which is near two Hours, ſo that in fifty Centuries it will amount to a Day.

The greateſt Part of *Europe* have long uſed the *Gregorian* Calendar, or Old Style; but *Great Britain* retained the *Julian* Calendar 'till the Act of Parliament paſſed in the twenty-fourth Year of his preſent Majeſty's Reign took Place, on the firſt of *January*, 1752; and, in *September* following, the eleven Days were adjuſted, by calling the third Day of that Month the fourteenth, and continuing the reſt in their Order. The Proviſion made by that Act, for preventing the Anticipation of Payments of Rents, Expiration of Leaſes, or any other Contracts or Cuſtoms, wherein the Property of the Subjects were concerned, has been fully explained by the common Almanacsk, therefore it will not be neceſſary to ſay any thing further of it here.

But, as there are three Leap-Years to be abated in every four Centuries, it will be proper to ſhew how to find on which Century the laſt Year is to be a Leap-Year, and in which it is not. This may be done thus; cut off the two Cyphers, and divide the remaining Figures by 4; if nothing remains, the Year is a Leap-Year; but, if any thing remains, it is a common Year.

Exam. 1. The Year 18|00.
4) 18 (4
16
2

Exam. 2. The Year 20|00.
4) 20 (5
20
0

Exam. 3. The Year 27|00.
4) 27 (6
24
3

Exam. 4. The Year 32|00.
4) 32 (8
32
0

The firſt and third Examples having Remainders, the Years are common Years of three hundred and ſixty-five Days; but the ſecond and fourth, having no Remainders, are Leap-Years of three hundred and ſixty-ſix Days.

With Regard to any other Years, the whole Date is to be divided by 4; and, if there's no Remainder, it is Leap-Year; but, if 1, 2, or 3 remains, it is the firſt, ſecond, or third after Leap-Year; according to the Diſtich publiſhed by Mr. *Street*, in his *Aſtronomia Carolina:* Where he has given Diſtichs for finding the Cycle of the Moon or Golden Number, Epact, Moon's Age, *&c.* which being of great Uſe to aſſiſt the Memory, I ſhall inſert ſuch as are agreeable to the *Gregorian* or New Style. That for finding the Leap-Year is,

Divide the Year by 4, what's left ſhall be
For Leap-Year 0, for paſt 1, 2, or 3.

Exam. For the Year 1754.

```
4) 1754 (438
   16
   --
    15
    12
    --
     34
     32
     --
      2 ſecond after Leap-Year.
```

To find the Dominical or Sunday Letter for any Year.

The Diſtich for this, made to anſwer to the *Gregorian* Calendar, is,

Divide the Year and Fourth of it by 7,
What's left ſubtract from 7, the Letter's given.

That is, to the Year of the Chriſtian Æra add the fourth Part, and divide this Sum by 7; then ſubtract what remains (even if it be a Cypher) from 7, and the Remainder is the Dominical Letter, accounting as follows:

A	B	C	D	E	F	G
[illegible]	2	3	4	5	6	7

Exam. For the Year 1754.
Its Fourth 438

7) 2192 (313
21
9
7
22
21
1

From 7
Subtract 1
6 F

Here it is to be obſerved, that every Leap-Year has two Dominical Letters; that found by this Rule is the Sunday Letter from the twenty-fifth Day of *February* to the End of the Year; and the next in the Order of the Alphabet ſerves from the firſt of *January* to the twenty-fourth of *February*.

Exam. For the Year 1756.
Its Fourth 439

7) 2195 (313
4

From 7
Subtract 4
3 C

Hence C is the Dominical Letter for the Year; but D, the next in the Order of the Alphabet, is the Dominical Letter for *January* and *February*. From this Interruption of the Dominical Letter every fourth Year, it is twenty-eight Years before the Dominical Letter returns to the ſame Order; which, were it not for the Leap-Years, would return to the ſame every ſeven Years. This Cycle of twenty-eight Years is called the Cycle of the Sun; tho' it has no Relation to the Sun's Motion, but is ſo called from its Uſe in finding the Sunday Letter, becauſe Sunday uſed to be called *Dies Solis*, the Sun's Day.

The Diſtich for finding the Golden Number, Cycle of the Sun, and *Roman* Indiction, is this,

When 1, 9, 3, to the Year hath added been,
Divide by 19, 28, 15.

That is, for the Golden Number, add 1 to the Year, then divide by 19, and the Remainder is the Golden Number; if nothing remains, 19 is the Golden Number. (The Uſe of the Golden Number will be ſhewn below.) In the ſame Manner, 9 being added to the Year, and the Sum divided by 28, the Remainder, if any, is the Cycle of the Sun; if nothing remains, the Cycle is 28.

Exam. For the Year 1754.
Add 9
28) 1763 (62
168
83
56
27 Cycle of the Sun.

The Uſe of this Cycle is for finding the Dominical Letter by the following Table.

A Table of the Dominical Letters for the New Style, according to the Cycle of the Sun.

Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.
1	D C	8	B	15	G	22	E
2	B	9	A G	16	F	23	D
3	A	10	F	17	E D	24	C
4	G	11	E	18	C	25	B A
5	F E	12	D	19	B	26	G
6	D	13	C B	20	A	27	F
7	C	14	A	21	G F	28	E

This Table, by the preſent Rule, will ſerve but to the End of this Century. The Leap-Year being to be omitted in the Year 1800, will make it neceſſary to add 25 to the Date of the Year, and then dividing by 28, it will give the Cycle right for the Table during the next Century. And this is a general Rule to be obſerved, that, when a Leap-Year has been abated, add 16 to the Number which was before added to the Year, and reject 28 when it exceeds it; then, this Number being added to the Year, and the Sum divided by 28, the Remainder will be the Cycle for finding the Dominical Letter. Thus, in the nineteenth Century it will be 25, and in the twentieth Century 13, which Number will ſerve two Centuries, for the Year 2000 is a Leap-Year.

Example. To find the Dominical Letter for the Year 1975, and for the Year 2056.

1975
13
28) 1988 (70
196
28 Cycle of the Sun,
(the Letter is E.

2056
13
28) 2069 (73
196
109
84
(Letters BA.
25 Cycle of the Sun,

Of the Golden Number, or Lunar Cycle.

The Golden Number is a Period of nineteen Years, invented by *Meton*, an *Athenian*, and from him called the *Metonic* Cycle. The Uſe of this Cycle is to find the Change of the Moon; becauſe, after nineteen Years, the Changes of the Moon fall on the ſame Days of the Month as in the former nineteen Years; tho' not at the ſame Time of the Day, there being an Anticipation of one Hour, twenty-ſeven Minutes, forty-one Seconds, and thirty-two Thirds; which, in three hundred and twelve Years, amounts to a whole Day: Hence, the Golden Number will not ſhew the true Change of the Moon for more than three hundred and twelve Years, without being varied. But the Golden Number is not ſo well adapted to the *Gregorian* as the *Julian* Calendar, the Epact being more certain in the New Style, to find which the Golden Number is of Uſe. The Way of finding the Golden Number has been already ſhewn above.

Thus for the Year 1754.
Add 1
19) 1755 (92
171
45
38
7 Golden Number.

The Epact is the Exceſs of the Solar above the Lunar Year, or twelve Synodical or Lunar Months; which being ſomewhat more than ten Days and twenty-one Hours, to avoid Fractions, the annual Epact is eleven Days.

By the Golden Number, called alſo the Prime, to find the Epact.

For the *Gregorian* or New Style, take 1 from the Golden Number or Prime; then

Divide by 3, for each one left add 10,
30 reject, the Prime makes Epact then.

That is, after 1 has been taken from the Golden Number, divide the Remainder by 3; if 1 remains, add 10 to the Dividend, and that will be the Epact; if 2 remains, add 20 to the Dividend; but, if nothing remains, the Dividend is the Epact.

Exam. 1.	*Exam.* 2.	*Exam.* 3.
Year 1753.	Year 1754.	Year 1755.
Gold. Num. 6	Gold. Num. 7	Gold. Num. 8
Subtract 1	Subtract 1	Subtract 1
3) 5 (1	3) 6 (2	3) 7 (2
3	6	6
2 Epact 25.	0 Epact 6.	1 Epact 17.

A Table of the nineteen Epacts for the Julian *and* Gregorian *Accounts, by the Gold. Num.*

G. N.	Julian Epact.	Greg. Epact.	G. N.	Julian Epact.	Greg. Epact.
1	11	29	11	1	20
2	22	11	12	12	1
3	3	22	13	23	12
4	14	3	14	4	23
5	25	14	15	15	4
6	6	25	16	26	15
7	17	6	17	7	26
8	28	17	18	18	7
9	9	28	19	29	18
10	20	9			

To find the Age or Change of the Moon by the Epact, the Diſtich is,

To *Janus* 0, 2, 1, 3, 4, 5, 6,
8, 8, 10, 10, theſe to the Epacts ſix;
The Sum, bate 30, to the Month-Day add,
Or take from 30, Age or Change is had.

The Senſe of which is, that, if to the Epact for the Year the Number of the Month, taken in the Order they ſtand in the Diſtich, be added, and to this add the Day of the Month, this Sum, rejecting 30 if it exceed that, is the Moon's Age; or, taking the Sum of the Epact and Number of the Month from 30, it gives the Day of the Moon's Change in that Month.

The Numbers for each Month are, for *January* 0, *February* 2, *March* 1, *April* 2, *May* 3, *June* 4, *July* 5, *Auguſt* 6, *September* 8, *October* 8, *November* 10, *December* 10.

Exam. 1. To find what Day of the Month of *July*, 1754, the Moon changes, and the Age of the Moon on the twelfth of that Month.

The *Gregorian* Epact for 1754 is	6	From	30
Number for *July* add —	5	Subtract	11
	11		19 Day of the Change.
Given Day of the Month add	12		
	23 Days of the Moon's Age.		

Exam. 2. To find the Change in *October* 1754, and the Moon's Age the twenty-fourth Day.

Epact —	6	From	30
Number for *October* add	8	Subtract	14
	14		16 Day of Change in *Oct.* 1754
Given Day add —	24		
	38		
Deduct —	30		
	8 Days of the Moon's Age.		

To find the Time of the Moon's Southing, and of High Water at London *Bridge.*

Moon's Age multiplied by 4, divide
By 5 for Southing, add 3 for the Tide.

Exam. 1. The Moon's Age the twelfth of *July*, 1754, is 23 Days.

Multiply by 4

5) 92 ($18\frac{2}{5}$ Hours.
5
42
40
2

The Moon will be South eighteen Hours and twenty-four Minutes after Noon, that is, at twenty-four Minutes paſt Six, on the thirteenth of *July*, in the Morning; to which adding three Hours, it gives twenty-four Minutes paſt Nine in the Morning for the Time of High Water at *London* Bridge.

Exam. 2. The twenty-fourth of *October*, 1754, the Moon's Age is 8 Days.

Multiply by 4

5) 32 ($6\frac{2}{5}$ Hours.
30
2

Therefore the Moon ſouths twenty-four Minutes after Six at Night; to which three Hours being added, it gives twenty-four Minutes after Nine for the Tide.

To find the Day of Eaſter *for the New Style.*

It was ordered, by the *Nicene* Council, that *Eaſter Sunday* ſhould be kept on the firſt *Sunday* after the firſt full Moon which happened upon or after the twenty-firſt Day of *March*, the Day on which they thought the Vernal Æquinox happened.

Tho' this was a Miſtake, for the Vernal Æquinox that Year fell on the twentieth of *March.* But yet, the full Moon that fell on or next after the twenty-firſt of *March* they called the *Paſchal* full Moon. And, by the late Introduction of the *Gregorian* or New Style, the Æquinox will now always happen on the twentieth or twenty-firſt of *March.* And the Feaſt of *Eaſter* is now to be kept on the next *Sunday* after the Paſchal full Moon, or the full Moon that happens after the twenty-firſt of *March*; but, if the full Moon happens on a *Sunday*, *Eaſter-Day* is to be the next *Sunday* after.

All the other moveable Feaſts depending on *Eaſter*, I ſhall endeavour to ſhew the true Method of finding it. This will be beſt done, for any future Time, by the Table compoſed by the Right Honourable GEORGE Earl of *Macclesfield*; which ſhews, by the *Golden Number*, the Days of the Paſchal full Moons; by which, and the *Dominical Letter*, the Day on which *Eaſter* will fall may be found. His Table is calculated from the Time the Calendar was firſt corrected in the Year 1583; but, as *Eaſter* has been always kept in *Great Britain* according to the Old Style 'till the Year 1753, I ſhall begin the Table from thence.

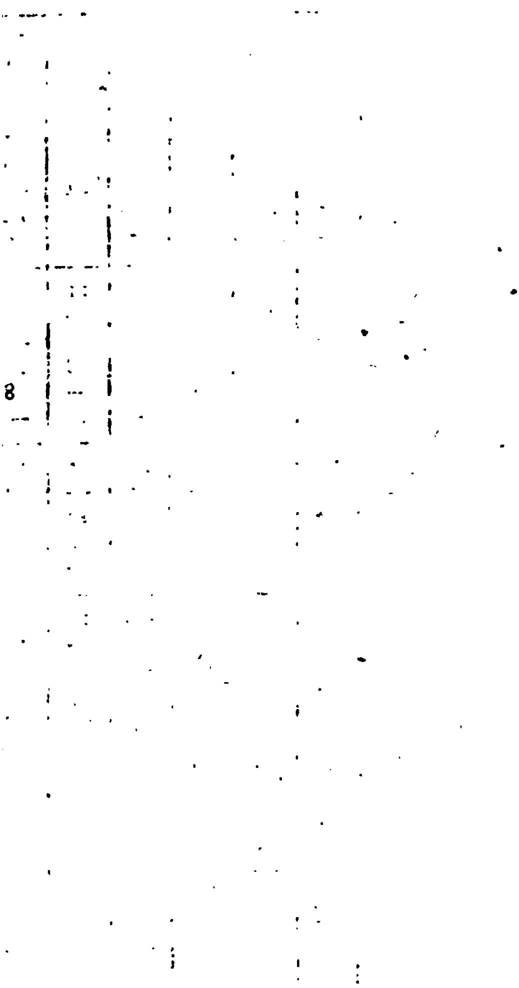
The Uſe of the Table.

Firſt find the Golden Number as before taught, which ſeek in the Column of Golden Numbers under the Time in which the given Year is included; right againſt the Golden Number of the Year, in the laſt Column but one, you have the Day of the Month on which the Paſchal full Moon happens, which is the Limit of *Eaſter*; from thence run your Eye down among the Dominical Letters 'till you come to the Letter for the given Year, and againſt it you have the Day of the Month on which *Eaſter* falls that Year.

Exam. 1. To know when *Eaſter* falls in the Year 1754: The Golden Number for the Year we have already found to be 7, and the Dominical Letter F; therefore ſeek in the firſt Column (the given Year being included between the Years 1753 and 1899) for the Golden Number; then caſt your Eye along to the laſt Column but one, under the Title Paſchal full ●, and you will find the ſeventh of *April* to be the Day of the full Moon, or *Eaſter* Limit; againſt which, in the laſt Column, ſtands F, which ſhews it to be *Sunday*, therefore the next *Sunday* following is *Eaſter Sunday*, which, by going down the Column of Letters to the next F, you will find to be the fourteenth of *April.*

yfive.

Dom. Letter.	Days of Month.		Paschal full ● 4100 to 4199	800 to 899
C	21	March.	4	—
D	22		—	12
E	23		12	1
F	24		1	—
G	25		—	9
A	26		9	—
B	27		—	17
C	28		17	6
D	29		6	—
E	30		—	14
F	31		14	3
G	1	April.	3	—
A	2		—	11
B	3		11	—
C	4		—	19
D	5		19	8
E	6		8	—
F	7		—	16
G	8		16	5
A	9		5	—
B	10		—	13
C	11		13	2
D	12		2	—
E	13		—	10
F	14		10	—
G	15		—	18
A	16		18	7
B	17		7	15
C	18		15	4
D	19			
E	20			
F	21			
G	22			
A	23			
B	24			
C	25			



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Exam. 2. To know when *Eaſter* will fall in the Year 2765: Look in the ſeventh Column of Golden Numbers under 2600 to 2899, in which the given Year in included, for 11, the Golden Number for the Year, and right againſt it, in the laſt Column but one, you have the twenty-eighth of *March* for the Limit, or Full-Moon Day; then, F being the Dominical Letter for the Year, go down the Letters to F, which ſtands againſt the thirty-firſt of *March*, for *Eaſter-Day*.

From this Table may be made a particular Table, for any Century included in either of the Columns. Thus, the firſt Column including the Remainder of this Century and the next, the following Table, made from thence, will ſhew the Time of *Eaſter* to the End of the Year 1899.

A Table for finding Eaſter *to the End of the nineteenth Century.*

Gold. Num.	A	B	C	D	E	F	G
1	Apr. 16	Apr. 17	Apr. 18	Apr. 19	Apr. 20	Apr. 14	Apr. 15
2	Apr. 9	Apr. 3	Apr. 4	Apr. 5	Apr. 6	Apr. 7	Apr. 8
3	Mar. 26	Mar. 27	Mar. 28	Mar. 29	Mar. 23	Mar. 24	Mar. 25
4	Apr. 16	Apr. 17	Apr. 11	Apr. 12	Apr. 13	Apr. 14	Apr. 15
5	Apr. 2	Apr. 3	Apr. 4	Apr. 5	Apr. 6	Mar. 31	Apr. 1
6	Apr. 23	Apr. 24	Apr. 25	Apr. 19	Apr. 20	Apr. 21	Apr. 22
7	Apr. 9	Apr. 10	Apr. 11	Apr. 12	Apr. 13	Apr. 14	Apr. 8
8	Apr. 2	Apr. 3	Mar. 28	Mar. 29	Mar. 30	Mar. 31	Apr. 1
9	Apr. 16	Apr. 17	Apr. 18	Apr. 19	Apr. 20	Apr. 21	Apr. 22
10	Apr. 9	Apr. 10	Apr. 11	Apr. 5	Apr. 6	Apr. 7	Apr. 8
11	Mar. 26	Mar. 27	Mar. 28	Mar. 29	Mar. 30	Mar. 31	Mar. 25
12	Apr. 16	Apr. 17	Apr. 18	Apr. 19	Apr. 13	Apr. 14	Apr. 15
13	Apr. 2	Apr. 3	Apr. 4	Apr. 5	Apr. 6	Apr. 7	Apr. 8
14	Mar. 26	Mar. 27	Mar. 28	Mar. 22	Mar. 23	Mar. 24	Mar. 25
15	Apr. 16	Apr. 10	Apr. 11	Apr. 12	Apr. 13	Apr. 14	Apr. 15
16	Apr. 2	Apr. 3	Apr. 4	Apr. 5	Mar. 30	Mar. 31	Apr. 1
17	Apr. 23	Apr. 24	Apr. 18	Apr. 19	Apr. 20	Apr. 21	Apr. 22
18	Apr. 9	Apr. 10	Apr. 11	Apr. 12	Apr. 13	Apr. 7	Apr. 8
19	Apr. 2	Mar. 27	Mar. 28	Mar. 29	Mar. 30	Mar. 31	Apr. 1

To find *Eaſter* by this Table, find the Golden Number in the firſt Column, and the Dominical Letter at the Top of the Table; then guide your Eye from the Golden Number, in a ſtraight Line, to the Column where you found the Dominical Letter.

Exam. 1. To find *Eaſter* for the Year 1754: Look for 7, the Golden Number, in the firſt Column, and for F, at the Head of the Table, in the ſeventh Column; then, right againſt 7 in the firſt Column, you will find the fourteenth of *April* in the ſeventh Column, under F, which is *Eaſter-Day.*

Exam. 2. To find *Eaſter* for the Year 1766: The Golden Number for the Year is 19, and the Dominical Letter E; in the Column under E, right againſt 19, you will find the thirtieth of *March*, which is *Eaſter-Sunday.*

As to the *Roman Indiction*, in the Diſtich for finding the Golden Number and Cycle of the Sun, it is of no Uſe but in the Court of *Rome*. But *Joſephus Scaliger*, by multiplying the Golden Number, Cycle of the Sun, and Indiction, one into another, made a Period of 7980 Years, which he called the *Julian* Period. Thus,

Cycle of the Sun ——	28
Multiplied by the Gold. Numb.	19
	252
	28
	532
Multiplied by the Indiction	15
	2660
	532
	7980

This Period he makes to begin ſeven hundred and ſixty-four Years before the Creation, according to the *Jewiſh* Chronology.

AN APPENDIX, CONTAINING

Cycles for the Motions of the Primary Planets, *and Mr.* Whiſton's *Cycle for calculating* Solar Eclipſes *without* Parallaxes.

NEAR thirty Years ago Mr. *Whiſton* publiſhed a Cycle for calculating Eclipſes. This firſt induced me to think the Motions of the Planets might be computed by a Cycle, ſince their apparent Motions, as beheld from the Earth, depend only on their own Motions in their Orbs and the Motion of the Earth, whilſt Eclipſes depend on divers variable, unequal Motions. On Examination, I found that the Motions of each of the primary Planets do return, after a Series of Years proper to each, the ſame in both theſe Reſpects; and that their Motions will be nearly the ſame in a future, as in a paſt Number of Years, thro' ſuch a reſpective Cycle.

The Planets moving in elliptic Orbs, in one of whoſe *Foci* the Sun is placed, their Motions are ſlower when at a greater Diſtance from the Sun, and ſwifter when nearer. This Variation is the ſame in every Revolution of the Planet, and is readily adjuſted by Tables of the Æquation of their Anomaly: Hence their heliocentric Motions, that is, their Places in the Zodiac as beheld from the Sun, are the ſame in entire Revolutions. But, the Earth being a Planet that moves round the Sun in its own Orb, the other Planets Motions, as beheld from the Earth, vary according to the different Situation of the Earth with regard to the particular Planet, as well as from the Planet's Situation in its Orb: Hence, for the geocentric Motion of any Planet, a Period is required, in which the Planet has the ſame Situation, with regard to the Earth, when he returns to the ſame Part of his Orb.

Since the Earth's Motion in her Orb, in any Number of Years, is nearly entire Revolutions, it requires only a Series of Years for each Planet, in which the Planet performs entire Revolutions; becauſe all the Variations in the Motion of ſuch Planet, whether ariſing from its own Motion or the Motion of the Earth, will, after ſuch a Series, return again in the ſame Order. Hence the Places of the Planets may be computed from old *Ephemerides*, or *Almanacks*, without Calculation.

Saturn, the moſt remote Planet of the Solar Syſtem, performs his Revolution in twenty-nine Years, one hundred and ſixty-ſeven Days, four Hours, and thirty-ſix Minutes. In fifty-nine Years he goes two Revolutions and 1° . 1′ . 8″, and the Earth in the ſame Time falls ſhort of entire Revolutions but 1° . 4′ . 57″; whence the apparent Motion of *Saturn*, after fifty-nine Years, will be nearly the ſame in all Reſpects thro' ſuch another Period. The mean Anomaly of the Earth and *Saturn*, according to Mr. *Street*'s *Aſtronomia Carolina*, is as follows:

		Anomaly of the Earth.	Anomaly of *Saturn*.
40	Years	$11^s . 29^\circ . 46' . 2''$	$4^s . 8^\circ . 50' . 36''$
19	is	11 . 29 . 9 . 1	7 . 22 . 10 . 32
		11 . 29 . 55 . 3	Exceſs 1 . 1 . 8
Diff. from 12 Signs		1 . 4 . 57	

Theſe Differences make very little Variation in *Saturn*'s Motion, as will appear by comparing two Years at fifty-nine Years Diſtance. Of this I ſhall give two Examples: The firſt, of the Year 1694 compared with the preſent Year 1753: The other, the Year 1675 compared with 1734. But, as in each of theſe Examples there are fifteen Leap-Years intervening, one Day muſt be abated in the fifty-nine Years. Then, with Regard to the firſt Example, the Style having been altered, the firſt Day of each Month in 1694 anſwers to the twelfth in 1753; only, as one Day is abated on Account of the fifteen Leap-Years, the firſt of the Months will anſwer to the eleventh. I ſhall only ſet down the firſt of each Month in the Examples of the ſuperior Planets, as their Motions are but ſlow.

Gadbury's Ephemerides, 1694.		*White's Ephemeris,* 1753.	
	♄ in ♐		♄ in ♐
Jan. 1	27° 36′	Jan. 11	29° 49′
Feb. 1	0 ♑ 40	Feb. 11	3 ♑ 1
Mar. 1	2 34	Mar. 11	5 2
Apr. 1	3 ℞ 22	Apr. 11	5 ℞ 57
May 1	2 41	May 11	5 2
June 1	0 48	June 11	3 35
July 1	28 ♐ 38	July 11	1 2
Aug. 1	27 4	Aug. 11	29 ♐ 46
Sep. 1	26 D 51	Sep. 11	29 D 44
Oct. 1	28 6	Oct. 11	0 ♑ 31
Nov. 1	0 ♑ 40	Nov. 11	2 59
Dec. 1	3 55	Dec. 11	6 10
21	6 16	31	8 31

Gadbury's Ephemerides, 1675.		*Parker's Ephemeris,* 1734.	
	♄ in ♈		♄ in ♈
Jan. 2	22° 1[illegible]	Jan. 1	23° 36′
Feb. 2	24 8	Feb. 1	25 24
Mar. 2	26 59	Mar. 1	28 9
Apr. 2	0 ♉ 43	Apr. 1	1 ♉ 54
May 2	4 32	May 1	5 44
June 2	8 8	June 1	9 22
July 2	10 40	July 1	12 6
Aug. 2	12 ℞ 17	Aug. 1	13 ℞ 39
Sep. 2	12 6	Sep. 1	13 33
Oct. 2	10 28	Oct. 1	11 59
Nov. 2	8 0	Nov. 1	9 32
Dec. 2	6 4	Dec. 1	7 32
31	5 D 29	30	6 D 52

By theſe Examples it appears, that the Differences are from one to about two Degrees and an Half more in the ſucceeding Period than in the preceding; alſo the Retrogradation happens a Day later in the ſucceeding than in the preceeding Period. Theſe Differences being known, the Computation may be made to a ſufficient Degree of Exactneſs.

Jupiter's periodic Time is eleven Years, three hundred and fifteen Days, twelve Hours, and twenty Minutes. In eighty-three Years he falls ſhort 1°. 2′. 49″ of four Revolutions; in which Time the Earth falls ſhort 1°. 13′. 50″ of entire Revolutions. Therefore, if we add one Day, and make the Period eighty-three Years and one Day, the Differences will be very inconſiderable, as appears by the following Calculation:

Anomaly of the Earth.		Anomaly of *Jupiter.*	
80 } Yrs.	11^{s}. 29°. 32′. 4″	80 } Yrs.	8^{s}. 27°. 58′. 0″
3 } is	11 . 29 . 14 . 36	3 } is	3 . 0 . 59 . 11
1 Day is	59 . 8	1 Day is	4 . 59
	11 . 29 . 55 . 48		11 . 29 . 2 . 10
Diff. from 12 Signs	4 . 12	Diff. from 12 Signs	57 . 50

The Differences hence ariſing are little more than Half a Degree, as appears by the following Examples. Where obſerve, that, in the firſt Example, twenty-one Leap-Years have intervened; in the other, the Year 1664 is Leap-Year.

Wing's *Ephemerides*, 1670.			*White*'s *Ephemeris*, 1753.		
	♃ in ♋			♃ in ♋	
Jan. 1	8 ℞	52	Jan. 12	9 ℞	14
Feb. 1	5	50	Feb. 12	6	13
Mar. 1	5 D	30	Mar. 12	5 D	56
Apr. 1	7	59	Apr. 12	8	26
May 1	12	29	May 12	12	59
June 1	18	31	June 12	19	2
July 1	25	1	July 12	25	33
Aug. 1	1 ♌	52	Aug. 12	2 ♌	25
Sep. 1	8	17	Sep. 12	8	48
Oct. 1	13	24	Oct. 12	13	53
Nov. 1	16	47	Nov. 12	17	15
Dec. 1	17 ℞	27	Nec: 12	17 ℞	52
26	10	20	31	16	46

Wing's *Ephemerides*, 1664.			*Weaver*'s *Ephemeris*, 1747.		
	♃ in ♑			♃ in ♑	
Jan. 1	1°	46	Jan. 2	2°	22
Feb. 1	8	20	Feb. 2	8	42
Mar. 1	13	20	Mar. 3	13	48
Apr. 1	16	45	Apr. 3	17	2
May 1	17 ℞	29	May 3	17 ℞	41
June 1	15	23	June 3	15	25
July 1	11	43	July 3	11	44
Aug. 1	8	31	Aug. 3	8	38
Sep. 1	7 D	50	Sep. 3	8 D	5
Oct. 1	9	59	Oct. 3	10	25
Nov. 1	14	36	Nov. 3	15	10
Dec. 1	20	39	Dec. 3	21	16
29	27	5	31	27	42

Mars performs his Revolution in one Year, three hundred and twenty-one Days, and twenty-three Hours. In ſeventy-nine Years he goes but 29′. 48″ more than entire Revolutions; and the Earth in the ſame Time falls ſhort 1°. 11′. 56″ of entire Revolutions.

Anomaly of the Earth.		Anomaly of *Mars*.	
60 Yrs. is	11ˢ. 29°. 39′. 3″	60 Yrs. is	10ˢ. 24°. 13′. 12″
19 Yrs. is	11 . 29 . 9 . 1	19 Yrs. is	1 . 6 . 16 . 36
	11 . 28 . 48 . 4	Exceſs	29 . 48
Diff. from 12 Signs	1 . 11 . 56		

The Differences hence ariſing will appear by the following Examples:

Wing's

Wing's Ephemerides, 1671.	♂ in ♓	*White's Ephemeris,* 1750.	♂ in ♓
Jan. 2	28° 7′	Jan. 1	29° 15′
Feb. 2	20 ♈ 5	Feb. 1	21 ♈ 9
Mar. 2	9 ♉ 33	Mar. 1	10 ♉ 34
Apr. 2	0 ♊ 37	Apr. 1	1 ♊ 37
May 2	20 34	May 1	21 50
June 2	10 ♋ 44	June 1	11 ♋ 41
July 2	0 ♌ 0	July 1	0 ♌ 55
Aug. 2	19 45	Aug. 1	20 43
Sep. 2	9 ♍ 30	Sep. 1	10 ♍ 2
Oct. 2	28 41	Oct. 1	29 3
Nov. 2	18 ♎ 41	Nov. 1	19 ♎ 38
Dec. 2	7 ♏ 34	Dec. 1	9 ♏ 3
31	27 15	30	28 53

Wing's Ephemerides, 1665.	♂ in ♒	*Weaver's Ephemeris,* 1744.	♂ in ♒
Jan. 1	5° 41′	Jan. 1	6° 34′
Feb. 1	0 ♓ 13	Feb. 1	1 ♓ 5
Mar. 2	22 59	Mar. 1	23 50
Apr. 2	16 ♈ 52	Apr. 1	17 ♈ 44
May 2	9 ♉ 14	May 1	10 ♉ 3
June 2	1 ♊ 32	June 1	2 ♊ 26
July 2	22 16	July 1	23 14
Aug. 2	12 51	Aug. 1	13 ♋ 51
Sep. 2	2 ♌ 33	Sep. 1	3 ♌ 33
Oct. 2	20 38	Oct. 1	21 42
Nov. 2	7 ♍ 57	Nov. 1	9 ♍ 7
Dec. 2	22 8	Dec. 1	24 1
31	3 17	30	5 ♎ 8

In the firſt Example, twenty Leap-Years having intervened, the ſecond of each Month in 1671 anſwers to the firſt in 1750. In the other, there had but nineteen Leap-Years intervened, therefore the firſt of *January* and *February* in 1665 anſwers to the firſt of *January* and *February* in 1744; but, that being Leap-Year, the twenty-ninth of *February* anſwers to the firſt of *March*; therefore the ſecond of *March*, and all the ſucceeding Months in 1665, agree with the firſt of each correſponding Month the Remainder of the Year.

Venus revolves round the Sun in two hundred and twenty-four Days and ſeventeen Hours. In eight Years ſhe goes 1°, 15′. 18″ more than entire Revolutions; in which Time the Earth falls ſhort of entire Revolutions 2′. 48″.

Anomaly of the Earth.		Anomaly of *Venus*.	
8 Years is	11ˢ. 29°. 57′. 12″	8 Years Exceſs is	1°. 25′. 18″.
Diff. from 12 Signs is	2 . 48		

In this Period there can be neither more or leſs than two Leap-Years. The Variations in the Motion of *Venus*, from one Period to another, will be ſeen by the following Examples of three Months Motion in each Year.

The Motion of Venus *for three Months in the Year* 1693 *compared with three Months of the Year* 1701; *both from* Gadbury's Ephemerides.

	Jan. 1693. ♀ in ♐	Jan. 1701. ♀ in ♐	Feb. 1693. ♀ in ♑	Feb. 1701. ♀ in ♑	March 1693. ♀ in ♒	March 1701. ♀ in ♒
1	12° 7′	12° 39′	19° 42′	20° 18′	23° 34′	24° 35′
2	13 20	13 52	20 55	21 31	25 20	25 48
3	14 33	15 4	22 9	22 44	26 7	27 2
4	15 45	16 16	23 22	23 58	27 54	28 15
5	16 57	17 29	24 35	25 11	28 40	29 29
6	18 9	18 41	25 49	26 24	0♓27	0♓42
7	19 21	19 54	27 2	27 37	1 17	1 56
8	20 33	21 6	28 16	28 51	2 31	3 9
9	21 45	22 19	29 29	0♒ 4	3 45	4 23
10	22 58	23 31	0♒42	1 18	4 58	5 36
11	24 11	24 44	1 55	2 31	6 12	6 50
12	25 23	25 56	3 8	3 46	7 25	8 4
13	26 36	27 10	4 21	4 58	8 38	9 13
14	27 48	28 22	5 35	6 11	9 51	10 31
15	29 1	29 35	6 48	7 25	11 5	11 45
16	0♑14	0♑48	8 1	8 39	12 19	12 58
17	1 27	2 1	9 15	9 52	13 32	14 12
18	2 40	3 14	10 28	11 5	14 46	15 25
19	3 53	4 27	11 42	12 19	15 59	16 29
20	5 6	5 40	12 55	13 33	17 13	17 32
21	6 18	6 53	14 9	14 46	18 27	19 6
22	7 31	8 6	15 23	16 0	19 40	20 19
23	8 44	9 19	16 36	17 13	20 53	21 32
24	9 57	10 32	17 50	18 27	22 7	22 46
25	11 10	11 45	19 3	19 40	23 20	23 59
26	12 23	12 59	20 17	20 54	24 34	25 13
27	13 36	14 12	21 30	22 7	25 47	26 27
28	14 49	15 25	22 43	23 21	27 2	27 41
29	16 2	16 38			28 19	28 54
30	17 15	17 51			29 28	0♈ 8
31	18 29	19 5			0♈42	1 20

The Motion of Venus, *according to* Weaver's Ephemeris 1744, *from the twenty-firſt of* December *to the End, and the Year* 1745, *from the firſt of* January *to the twenty-ninth of* March *incluſive, compared with the firſt three Months of this preſent Year* 1753, *according to* White's Ephemeris.

1744. December.	♀ in ♒		1753. January.	♀ in ♒	
21	14°	13	1	14°	51
22	15	26	2	16	5
23	16	40	3	17	18
24	17	54	4	18	32
25	19	7	5	19	45
26	20	21	6	20	59
27	21	35	7	22	12
28	22	48	8	23	25
29	24	2	9	24	39
30	25	11	10	25	52
31	26	29	11	27	5
January.					
1	27	42	12	28	18
2	28	55	13	29	31
3	0♓	8	14	0♓	44
4	1	21	15	1	57
5	2	34	16	3	10
6	3	47	17	4	23
7	5	0	18	5	36
8	6	12	19	6	49
9	7	25	20	8	1
10	8	38	21	9	14
11	9	51	22	10	26
12	11	4	23	11	38
13	12	17	24	12	51
14	13	29	25	14	3
15	14	41	26	15	15
16	15	54	27	16	27
17	17	6	28	17	39
18	18	18	29	18	50
19	19	30	30	20	2
20	20	42	31	21	14

1745. January.	♀ in ♓		1753. February	♀ in ♓	
21	21°	53′	1	22°	26′
22	23	5	2	23	37
23	24	17	3	24	48
28	25	23	4	25	59
25	26	40	5	27	11
26	27	52	6	28	22
27	29	4	7	29	33
28	0♈	15	8	0♈	44
29	1	26	9	1	55
20	2	36	10	3	5
31	3	46	11	4	16
February.					
1	4	57	12	5	26
2	6	7	13	6	36
3	7	17	14	7	46
4	8	28	15	8	55
5	9	38	16	10	5
6	10	48	17	11	15
3	11	57	18	12	25
8	13	7	19	13	34
9	14	16	20	14	44
10	15	26	21	15	53
11	16	35	22	17	2
12	17	44	23	18	11
13	18	53	24	19	19
14	20	1	25	20	28
15	21	10	26	21	36
16	22	18	27	22	44
17	23	26	28	23	53
			March.		
18	24	34	1	25	0
19	25	42	2	26	8
20	26	49	3	27	15

1745. February.	♀ in ♈		1753. March.	♀ in ♈	
21	27°	56	4	28°	22
22	29	3	5	29	29
23	0♉	10	6	0♉	35
24	1	17	7	1	42
25	2	24	8	2	48
26	3	31	9	3	54
27	4	38	10	4	59
28	5	44	11	6	4
March.					
1	6	50	12	7	10
2	7	55	13	8	15
3	9	0	14	9	20
4	10	5	15	10	24
5	11	10	17	11	28
6	12	14	17	12	31
7	13	18	18	13	34
8	14	22	19	14	37
9	15	25	20	15	39
10	16	28	21	16	41
11	17	31	22	17	44
12	18	33	23	18	46
13	19	35	24	19	48
14	20	37	25	20	49
15	21	38	26	21	50
16	22	39	27	22	50
17	23	40	28	23	50
18	24	41	29	24	49
19	25	41	30	25	48
20	26	40	31	26	46

Mercury's periodic Time is eighty-ſeven Days and twenty-three Hours. In ſeventy-nine Years his mean Anomaly exceeds entire Revolutions 12′.21″ only; and the Earth falls ſhort 1°.11′.56″, as we have ſeen in *Mars*, whoſe Cycle is the ſame.

Anomaly of *Mercury.*	
60 Years	1ˢ.13°.30′.0″
19 Years	10.16.42.21
Excefs	12.21

Mercury's *Motion, according to* Gadbury's Ephemerides, *from the twenty-ſecond of* December 1673 *to the twenty-firſt of* March 1674. *(there having twenty Leap-Years intervened) compared with the firſt three Months of the preſent Year* 1753, *according to* White's Ephemeris.

1673. December.	☿ in ♑	1753. January.	☿ in ♑	1674. January.	☿ in ♑	1753. February.	☿ in ♑	1674. February.	☿ in ♓	1753. March.	☿ in ♓
22	28° 15′	1	27° 20′	22	17° 40′	1	17° 21′	22	0° 48′	4	0° 29′
23	28 ℞ 1	2	27 ℞ 9	23	18 38	2	18 20	23	2 7	5	2 12
24	27 40	3	26 47	24	19 39	3	19 22	24	3 48	6	3 56
25	27 12	4	26 13	25	20 41	4	20 26	25	5 31	7	5 41
26	26 32	5	25 27	26	21 48	5	21 32	26	7 16	8	7 27
27	25 40	6	24 31	27	22 55	6	22 40	27	9 4	9	9 24
28	24 39	7	23 26	28	24 4	7	23 50	28	10 50	10	11 2
29	23 30	8	22 15	29	25 16	8	25 2	March.			
30	22 14	9	20 59	30	26 29	9	26 15	1	12 39	11	12 51
31	20 55	10	19 39	31	27 43	10	27 30	2	14 29	12	14 42
January.				February.				3	16 20	13	16 34
1	19 38	11	18 20	1	28 59	11	28 46	4	18 12	14	18 27
2	18 16	12	17 3	2	0 ♒ 15	12	0 ♒ 5	5	20 7	15	20 22
3	17 0	13	15 52	3	1 35	13	1 25	6	22 0	16	22 17
4	15 50	14	14 47	4	2 55	14	2 47	7	23 56	17	24 13
5	14 48	15	13 50	5	4 17	15	4 12	8	25 53	18	26 10
6	13 54	16	13 1	6	5 40	16	5 37	9	27 51	19	28 9
7	13 10	17	12 22	7	7 5	17	7 3	10	29 50	20	0 ♈ 9
8	12 34	18	11 51	8	8 30	18	8 30	11	1 ♈ 50	21	2 10
9	12 9	19	11 30	9	9 57	19	9 57	12	3 51	22	4 10
10	11 52	20	11 17	10	11 25	20	11 25	13	5 52	23	6 12
11	11 46	21	11 13	11	12 53	21	12 54	14	7 55	24	8 15
12	11 D 45	22	11 D 18	12	14 23	22	14 25	15	9 57	25	10 18
13	11 54	23	11 29	13	15 54	23	15 57	16	12 0	26	12 21
14	12 22	24	11 48	14	17 26	24	17 30	17	14 3	27	14 23
15	12 44	25	12 13	15	19 0	25	19 4	18	16 6	28	16 25
16	13 12	26	12 43	16	20 34	26	20 38	19	18 9	29	18 26
17	13 46	27	13 19	17	22 9	27	22 14	20	20 10	30	20 27
18	14 24	28	13 59	18	24 28	28	23 51	21	22 9	31	22 26
19	15 7	29	14 44			March.					
20	15 55	30	15 33	19	26 27	1	25 29				
21	16 46	31	16 25	20	28 4	2	27 8				
				21	29 43	3	28 48				

Venus's *Motion for the firſt three Months of the Year* 1668, *according to* Wing's Ephemerides, *compared with the firſt three Months of the Year* 1747, *according to* Weaver's Ephemeris. *But, there having been twenty Leap-Days between* January 1668 *and* January 1747, *the ſecond of* January *in the former correſponds with the firſt in the latter, which continues to the twenty-ninth of* February 1668, *which anſwers to the twenty-eighth in* 1747.

1668. January.	♀ in ♑		1747. January.	♀ in ♑	
2	2°	23′	1	1°	51′
3	2	26	2	1	57
4	2	37	3	2	10
5	2	55	4	2	37
6	3	20	5	2	59
7	3	51	6	3	33
8	4	28	7	4	10
9	5	10	8	4	54
10	5	56	9	5	41
11	6	45	16	6	32
12	7	38	11	7	28
13	8	34	12	8	26
14	9	34	13	9	28
18	10	37	14	10	31
16	11	42	15	11	36
17	12	49	16	12	45
18	13	58	17	13	55
19	15	9	18	15	8
20	16	22	19	16	22
21	17	37	20	17	38
22	18	54	21	18	55
33	20	11	22	20	14
24	21	30	23	21	34
25	22	51	24	22	54
26	24	13	25	24	16
27	25	36	26	25	40
28	27	0	27	27	5
29	28	25	28	28	30
30	29	51	29	29	56
31	1 ♒	19	30	1 ♒	24

1668. February.	♀ in ♒		1747. January.	♀ in ♒	
1	2°	47′	31	2°	53′
			February.		
2	4	16	1	4	22
3	5	47	2	5	53
4	7	18	3	7	25
5	8	50	4	8	57
6	10	23	5	10	30
7	11	57	6	12	4
8	13	32	7	13	39
9	15	8	8	15	16
10	16	44	9	16	52
11	18	21	10	18	30
12	19	59	11	20	10
13	21	37	12	21	50
14	23	17	13	23	32
15	24	59	14	25	15
16	26	43	15	26	58
17	28	27	16	28	41
18	0 ♓	12	17	0 ♓	25
19	1	58	18	2	18
20	3	45	19	4	4
21	5	33	20	5	50
22	7	21	21	7	30
23	9	11	22	9	28
24	11	3	23	11	20
25	12	56	24	13	13
26	14	50	25	15	8
27	16	45	26	17	2
28	18	41	27	18	56
29	20	38	28	20	49

1668. March.	♀ in ♓		1747. March. ♀ in ♓	
1	22°	37′	22°	53′
2	24	36	24	51
3	26	36	26	50
4	28	36	28	50
5	0 ♈	37	0 ♈	50
6	2	38	2	51
7	4	39	4	51
8	6	40	6	50
9	8	41	8	50
10	10	40	10	50
11	12	38	12	49
12	14	34	14	45
13	16	28	16	39
14	18	20	18	30
15	20	10	20	18
16	21	57	22	4
17	23	40	23	46
18	25	19	25	24
19	26	54	26	58
20	28	24	28	27
21	29	50	29	50
22	1 ♉	11	1 ♉	7
23	2	26	2	20
24	3	35	3	29
25	4	38	4	32
26	5	36	5	25
27	6	28	6	14
28	7	13	6	57
29	7	52	7	34
30	8	24	8	4
31	8	50	8	27

The Differences appear, by theſe Examples, to be but ſmall, and may be allowed for, according to what each Example ſhews to be requiſite. And, as to the Place of the Sun, that is well known to be the ſame, taking a Cycle of four Years, on Account of the Leap-Years. So that the Sun's Place for four ſucceſſive Years will ſerve for a Century, adding one Minute and thirty-ſix Seconds for every Cycle of four Years. And when, at the End of a Century, the Leap-Year is omitted, the Cycle muſt be made four Years and one Day.

Of Mr. Whiſton's *Cycle for* Eclipſes.

The moſt uſeful Cycle for Eclipſes is that publiſhed by *Pliny*, of two hundred and twenty-three Lunations, which are made in eighteen Years, eleven Days, ſeven Hours, forty-three Minutes, and fifteen Seconds. By Reaſon of a Miſtake of a Figure in *Pliny*'s Book, it being two hundred and twenty-two inſtead of two hundred and twenty-three, this Cycle had been taken no Notice of. Moſt of his Readers finding it, by Trial, to be a Miſtake, enquired no farther. But the great Dr. *Halley*, preſuming on the Sagacity of *Pliny*, made farther Enquiry, and found out the Miſtake. Perceiving the Uſefulneſs of this Cycle, he looked on it as a valuable Diſcovery, and, I ſuppoſe, intended to publiſh it to the World. In the mean Time, Mr. *Whiſton* learning the Doctor had made a Diſcovery of a Cycle for Eclipſes, and that he called it the *Chaldean Saros*, he recollected that *Pliny* had made Uſe of that Word, and thence began to ſuſpect the Doctor had his Cycle from thence. On ſearching *Pliny*, and finding the Number wrong, he ſoon found out the Miſtake, as Dr. *Halley* had done before, and immediately publiſhed it. In this Cycle, the mean Motion of the Moon from the Sun is entire Revolutions, and the other mean Motions are nearly ſo: Hence the Eclipſes of the Sun and Moon, in this Period, return with nearly the ſame Circumſtances, except what ariſes from the Exceſs above equal Days. I ſhall ſet down the mean Motions from Mr. *Whiſton*.

	Mean Motion of the Sun or Earth.	Mean Motion of the Moon in the Ecliptic.
18 Years	$11^{s} . 29^{\circ} . 38' . 32''$	$7^{s} . 11^{\circ} . 37' . 22''$
11 Days	10 . 50 . 32	4 . 24 . 56 . 25
7 Hours	17 . 15	3 . 50 . 35
43 Minutes	1 . 46	23 . 36
15 Seconds	1	8
Sum	10 . 48 . 6	Sum 10 . 48 : 6

Mean

	Mean Motion of the Apogee.	Mean Motion of the Node backwards.
18 Years	12°. 23′. 53″	11^{s}. 18°. 7′. 39″
11 Days	1. 13. 32	34. 57
7 Hours	1. 57	56
43¼ Minutes	12	6
Sum	13. 39. 34	Sum 11. 18. 43. 38

From the mean Motion of the Apogee	13°. 39′. 34″
Subtract that of the Moon in the Ecliptic	10. 48. 6
Remains	2. 51. 28

To the mean Motion of the Moon	10°. 48′. 6″
Add that of the Node —	11^{s}. 18. 43. 38
Sum	11. 29. 31. 44
Difference from 12 Signs	28. 16

Thus the Difference of the mean Motion of the Apogee from that of the Moon is but 2°. 51′. 28″, which is about $\frac{1}{63}$ of the whole Difference of 180°: That of the Node is but 28′. 44″, or $\frac{1}{383}$ Part of the whole Difference: And the mean Motion of the Sun differs little more than $\frac{1}{18}$ of the entire Difference. Which Quantities, being but ſmall, cannot occaſion any great Variations in the Time or Circumſtances of the Change or Full of the Moon, or even in Eclipſes, which will therefore return nearly the ſame in different Periods.

The Limits for Eclipſes are, for Lunar Eclipſes 11°. 40′, or 700′, on each Side of the Node; and for Solar Eclipſes 16°. 40′, or 1000′, on each Side. If we divide 700′ by 28′. 16″, we ſhall have 25 for the Number of Periods, after a central Eclipſe of the Moon in either of the Nodes, before the Moon gets clear of the Earth's Shadow entirely; that is, 450 Years, for 18, being multipled by 25, produces 450; and it is double that Time, or 900 Years, from the Moon's firſt getting within the ecliptic Limits on one Side of the Node, before ſhe gets clear on the other. And if 1000′, the Limits of Solar Eclipſes, be divided by 28′. 16″, we ſhall have 35 for the Number of Cycles after a central Eclipſe of the Sun in one of the Nodes, before the Earth gets clear of the Shadow of the Moon; that is, 630 Years, the Product of 18 multiplied into 35; and double that Time, or 1260 Years, from the Earth's firſt coming within the ecliptic Limits on one Side, 'till it gets entirely clear of the other. During which long Periods there will be Eclipſes ſomewhere.

Hence the Alteration in each Period, with Regard to Lunar Eclipſes, will be $\frac{1}{25}$ Part, or nearly $\frac{1}{2}$ a Digit; and in Solar Eclipſes $\frac{1}{35}$, or ſomewhat more than $\frac{1}{3}$ of a Digit: Tho' Mr. *Whiſton*, by Miſtake, makes the Alteration about a Digit each Cycle.

With Regard to the Alteration in Time, by which any Eclipſe may fall before or after the Concluſion of the Period, this depends on the Situation of the Earth and Moon in their Orbits. For, if the Earth be nearer its Aphelion than the Moon is to the Apogee, the Earth moving comparatively ſlower, and the Moon comparatively ſwifter, the Moon will arrive at the Conjunction or Oppoſition ſooner than the Concluſion of the Period. If the Earth be farther from the Aphelion than the Moon is from the Apogee, the contrary will happen, and the Eclipſe will fall ſomewhat after the Expiration of the Period.

The Quantity of Time to be allowed in theſe Caſes muſt be determined by the Anomaly of the Earth and Moon. We have ſeen that the Earth goes 10°. 48′. 6″ of Anomaly, and the Moon 2°. 51′. 28″ more than entire Revolutions. Therefore the Difference will be the Time the Moon takes to go ſuch Diſtance as is the Equation for 10°. 48′. 6″ of the Earth's Anomaly, in ſuch Part of the Orb in which the Earth then is, with Regard to the Earth's Motion; and ſuch Time as the Moon takes in going the Equation of 2°. 51′. 28″, in the Part of the Orb the Moon is then in, for the Moon's Motion.

This may be determined by the two following Tables, where the Time to be added to or ſubtracted from the Period of eighteen Years, eleven Days, ſeven Hours, forty-three Minutes, and fifteen Seconds, is calculated, with Regard to the Sun, for the particular Month in which any Eclipſe happens; (but then it muſt be obſerved, that this Table is made according to the New Style, and therefore the Time to be added, or ſubtracted, muſt be ſought by the Month and Day of the New Style on which any Eclipſe happened;) and, for the Moon, according to her mean Anomaly at the Time of the Eclipſe.

A Table of the Difference with Regard to the Sun.

Months.	DAYS. 1	6	11	16	21	26
January add	44′ 54″	44′ 11″	43′ 5″	41′ 46″	40′ 4″	38′ 5″
February add	36 15	33 16	30 20	27 14	24 3	20 34
March add	17 43	13 17	9 28	5 40	1 44	0 4
April ſubtract	5 5	9 37	13 15	16 48	20 10	23 27
May ſubtract	25 53	29 16	31 53	34 17	36 25	38 15
June ſubtract	39 30	41 5	42 5	42 45	43 24	44 25
July ſubtract	43 1	42 25	41 52	40 23	38 57	37 14
Aug. ſubtract	35 38	32 58	30 26	28 3	24 44	21 37
Sept. ſubtract	18 58	14 48	11 11	7 32	3 42	0 3
October add	3 13	7 50	11 40	15 23	19 1	23 45
Novemb. add	25 18	29 47	32 1	34 41	37 8	39 16
December add	40 45	42 35	43 46	44 35	45 1	45 5

A Table of the Difference according to the Anomal of the Moon.

Dg	Sign 0 add. Sign 6 ſub.	Sign 1 add. Sign 7 ſub.	Sign 2 add. Sign 8 ſub.	Sign 3 ſub. Sign 9 add.	Sign 4 ſub. Sign 10 add.	Sign 5 ſub. Sign 11 add
0	29′ 33″	25′ 16″	14′ 10″	0′ 45″	15′ 29″	26′ 1″
5	29 23	23 49	11 51	3 19	17 37	27 10
10	29 0	22 12	9 27	5 58	19 37	28 5
15	28 25	20 24	6 57	8 26	21 29	28 46
20	27 33	18 28	4 24	10 51	23 9	29 18
25	26 27	16 24	1 0	13 11	24 42	29 32

The Uſe of theſe Tables will be beſt explained by Examples; for which Purpoſe we may take the remarkable Solar Eclipſe which happened at the Siege of *Barcelona* in the Year 1706, on the firſt of *May*, at nine Minutes and forty-five Seconds paſt Nine in the Morning; that is, according to Aſtronomical Computation, *May* $0^{d}.21^{h}.45'$; to which in eighteen Years, that is, from *May* 1706 to *May* 1724, five Leap-Days intervening, there muſt, in this Caſe, be only added eighteen Years, ten Days, ſeven Hours, forty-three Minutes, and fifteen Seconds.

To	1706^{y}	*May*	$0^{d}.21^{h}.9'.45''$
Add	18		10 . 7 . 43 . 15
	1724	*May*	11 . 4 . 53 . 0

Hence the next correſponding Eclipſe would happen at fifty-three Minutes paſt Four in the Afternoon, on the eleventh of *May* 1724. Then finding, by the Table, the former Eclipſe happened on the firſt of *May* Old Style, that is, the twentieth of *April* New Style, there muſt be $19'.40''$ ſubtracted on Account of the Sun's Motion; and the Anomaly of the Moon for the ſame Time I find, by the Aſtronomical Tables, to be $8^{s}.3^{\circ}$, for which, by the Lunar Table, $12'.47''$ muſt be ſubtracted from the Time before found.

To	$19'.40''$	From *May*	$11^{d}.4^{h}.53'.0''$
Add	12 . 47	Subtract	32 . 27
Sum	32 . 27		11 . 4 . 20 . 33

Therefore the true Time for this Eclipſe was twenty Minutes and thirty-three Seconds after Four in the Afternoon. But then it is to be obſerved with Regard to Solar Eclipſes, that, when they fall in the Afternoon, they come ſomewhat later, and in the Forenoon ſomewhat ſooner, than the true Conjunction. So that, on this Account, the Eclipſe happened about one Hour and three Quarters later at *London* than the Expiration of the Time, on Account that the Eclipſe in 1706 was in the Forenoon, and that in 1724 in the Afternoon: So that the true Time of the Middle of the Eclipſe at *London* was on the eleventh of *May*, at Six in the Afternoon.

To 1724^{y} *May* 11^{d}. 4^{h}. 20′. 33″
Add 18 11 . 7 . 43 . 15

1742 22 . 12 . 3 . 48

Which brings the next correſponding Eclipſe to the twenty-ſecond of *May* 1742, at three Minutes and forty-eight Seconds paſt Midnight. From which ſubtract the following Time, found by the Tables:

For the Sun's Motion *May* 11, or *April* 30, ſubtract 25′. 30″
For the Anomaly of the Moon, 8^{s}. 6°, ſubtract 11 . 22

Sum to ſubtract 26 . 42

From *May* 22^{d}. 12^{h}. 3′. 48″
Subtract 26 . 42

1742 *May* 22 . 11 . 37 . 6 for the true Time
To which add 18 10 . 7 . 43 . 15 (of the Middle
(of the Eclipſe.

Gives 1760 *June* 1 . 19 . 20 . 21
Add for the New Style 11

June 12 . 19 . 20 . 21

From which ſubtract as follows:

For the Sun's Motion *May* 22, or 11, 31′ . 53″
For the Anomaly of the Moon, 8^{s}. 14°, 7 . 27

Sum to ſubtract 39 . 20

From *June* 12^{d}. 19^{h}. 20′. 21″
Subtract 39 . 20

12 . 18 . 41 . 1

That is, the thirteenth of *June*, at forty-one Minutes paſt Six in the Morning.

For another Example of a Solar Eclipſe, let it be the very remarkable one that happened the twenty-ſecond of *April* 1715, at twenty-five Minutes and forty-three Seconds paſt Nine in the Morning.

That is	—	1715^{y}	*April*	21^{d}.	21^{h}.	$25'$.	$43''$
To which add,		18		10 .	7 .	43 .	15
Gives	—	1733	*May*	2 .	5 .	8 .	58

That is, the ſecond of *May* 1733, about Five in the Afternoon. But here the Time of the Day being altered from the Forenoon to the Afternoon, it will be near two Hours later on that Account. The Alteration with Regard to the Motion of the Sun and Moon will be as follows:

For the Sun, *April* 22, or 11 N. S. ſubtract	$13'$. $15''$
Anomaly of the Moon, 6^{s} . 20°, ſubtract	27 . 33
	40 . 48

Therefore the true Time is about Six in the Afternoon; for

From the Time before found —	5^{h}. $8'$. $58''$
Subtract —— ——	40 . 48
Remains ——	4 . 28 . 10
To which add, for the Alteration from Forenoon to Afternoon,	1 . 45 . 0
	6 . 13 . 10

To which Time,		1733^{y}	*May*	2^{d}.	4^{h}.	$28'$.	$10''$
Add	——	18		11 .	7 .	43 .	15
Gives	—	1751	*May*	13 .	12 .	11 .	25

From which ſubtract as follows:

For the Sun's Motion, *May* 2, or *April* 21 New Style, ſubtract —— ——	$20'$. $10''$
For the Anomaly of the Moon, 6^{s} . 23°, ſubtract	27 . 7
Sum to ſubtract	47 . 17

From	*May*	$13^d . 12^h . 11' . 25''$
Subtract		47 . 17
		.13 . 11 . 24 . 8

Which happening at Midnight, was of Courſe inviſible to us. The next will happen as follows:

To	1751^y	*May*	$13^d . 11^h . 24' . 8''$
Add	18		10 . 7 . 43 . 15
	1769		23 . 19 . 7 . 23

From which ſubtract as follows:

For the Sun's Motion, *May* 13, or 2 N. S. ſubtract	$26' . 33''$
For the Moon's Anomaly, $6^s . 23^\circ$, ſubtract —	26 . 44
	53 . 17

From	*May*	$23^d . 19^h . 7' . 23''$
Subtract		53 . 17
		23 . 18 . 14 . 6

That is, the twenty-fourth of *May* 1769, at fourteen Minutes and ſix Seconds paſt Six in the Morning, Old Style; but, by Reaſon of the Alteration of the Style, this Eclipſe will fall on the fourth of *June*, at the ſame Time of the Day.

* The next Example ſhall be of a total Eclipſe of the Moon, which happened on the twenty-third of *February* 1700, at twenty-four Minutes and forty-ſix Seconds paſt Seven in the Morning: To which adding eighteen Years, ten or eleven Days, (according as there have happened either five or four Leap-Days) ſeven Hours, forty-three Minutes, and fifteen Seconds, three times, we ſhall have the three ſucceſſive correſpondent Lunar Eclipſes as follows:

To ———	1700^y	*Feb.*	$22^d . 19^h . 24' . 46''$
Add ———	18		10 . 7 . 43 . 15
Gives for the ſecond	1718	*March*	5 . 3 . 8 . 1
To which add —	18		10 . 7 . 43 . 15
Gives for the third	1736	*March*	15 . 10 . 51 . 16
To which add —	18		11 . 7 . 43 . 15
Gives for the fourth	1754	*March*	26 . 18 . 34 . 31

That is, the ſeventh of *April*, New Style, at thirty-four Minutes and thirty-one Seconds after Six in the Morning. To each of which adding or ſubtracting the Time in the foregoing Tables, we ſhall have the true Time of each Eclipſe: Thus, the firſt Eclipſe happening on the twenty-third of *February*, Old Style, and the Anomaly of the Moon being then $7^s . 8^\circ$, one being add, the other ſubtract; their Difference being added, as that is the greater Part, gives the true Time:

For the Sun's Motion, *Feb.* 23, or 12 N. S. add	29′. 43″
For the Anomaly of the Moon, $7^s . 8^\circ$, ſubtract	22 . 51
Difference to add	6 . 52

What is to be added or ſubtracted for each of the other two is as follows:

For the Sun's Motion, *March* 5, or *February* 22 New Style, add	23′. 21″
For the Anomaly of the Moon, $7^s . 11^\circ$, ſubtract	21 . 50
Difference to add	1 . 31

For the Anomaly of the Moon, $7^s . 14^\circ$, ſubtract	21′. 46″
For the Sun's Motion, *March* 15, or 4 N. S. add	14 . 39
Difference to ſubtract	7 . 7

To the mean Time of the 2d,	1718 *March*	$5^d . 3^h . 8' . 1''$
Add		6 . 52
Gives the true Time,	1718 *March*	5 . 3 . 14 . 53

To mean Time of the 3d,	1736 *March*	$15^d . 10^h . 51' . 16''$
Add 6′. 52″ and 1′. 31″, *i. e.*		8 . 23
Gives the true Time,	1736 *March*	15 . 10 . 59 . 39

To mean Time of the 4th,	1754 *April* N.S.	$6^d . 18^h . 34' . 31''$
Add the Diff. of 8′. 23″ and 7′. 7″, *i. e.*		1 . 16
Gives the true Time,	1754 *April* N.S.	6 . 18 . 35 . 46

The principal Alteration in this Period is with Regard to the Time of the Day, on Account of the ſeven Hours, forty-three Minutes, and fifteen Seconds; which not only makes a great Difference with Regard to the Places where thoſe Eclipſes are viſible, but, in Solar Eclipſes, makes a more conſiderable Alteration with Regard to the Time and Quantity; tho' Lunar Eclipſes are abſolutely the ſame in all Places where they are viſible. To remove theſe Inconveniences, Mr. *Whiſton* propoſed the putting three of theſe Cycles together, which he calls the *grand Cycle* for Eclipſes, in which the odd Hours and Minutes amount nearly to a whole Day: By which Means the Eclipſes will be brought to nearly the ſame Time of the Day, in the ſame Places, with only fifty Minutes Anticipation. This Cycle conſiſts of fifty-four Years and thirty-two or thirty-three Days, (that is, thirty-two Days when fourteen Leap-Days have intervened, and thirty-three Days when there has been but thirteen Leap-Days) abating fifty Minutes. This Cycle brings the Eclipſes to almoſt the ſame Time of the Day, and with nearly the ſame Circumſtances, and is therefore preferable to the ſingle Cycle, eſpecially for Solar Eclipſes: This has been already ſhewn in the former Examples. Thus the Solar Eclipſe on the firſt of *May* 1706, at Nine in the Morning, by the Addition of fifty-four Years and thirty-two Days, wanting fifty Minutes, (as there are fourteen Leap-Days) brings us to the ſecond of *June*, Old Style; that is, the thirteenth of *June* 1760, at about Seven in the Morning. Thus alſo the great Eclipſe of the Year 1715 will happen in the Year 1769, within about two Hours of the ſame Time of the Day. And the laſt Example of the Lunar Eclipſe will be but little more than Half an Hour ſooner than it happened in the Year 1700.

Eclipſes of the Sun that are central, are either total, when the whole Body of the Sun is hid by the Moon; or annular, when the Moon's apparent Diameter, being leſs than the apparent Diameter of the Sun, cannot hide the whole Body of the Sun, but a Ring of Light is ſeen round the Moon's Body. Alſo the Duration of total Darkneſs, in total Eclipſes of the Sun, is longer or ſhorter, in Proportion as the apparent Diameter of the Moon exceeds that of the Sun. Theſe Differences ariſe from the different real Diſtances of the Earth, Moon, and Sun: For, if, at the Time of a central Solar Eclipſe, the Earth be in the Aphelion, at the fartheſt Diſtance from the Sun, the apparent Diameter of the Sun being then leaſt, and the Moon in the Perigee, or neareſt Diſtance, and of Courſe its apparent Diameter greateſt, the total Darkneſs will be of longer Duration: On the contrary, the Earth in its Perihelion, the apparent Diameter of the Sun being then greateſt, and the Moon in her Apogee, in a central Solar Eclipſe, there can be no total Darkneſs; but the Eclipſe will be annular, that is, a Ring of the Solar Light will be ſeen round the Moon's Body. Therefore, if, at any central Eclipſe, the Earth is going from the Aphelion to the Perihelion, the Sun's apparent Diameter will gradually increaſe every Cycle; and, from the Perihelion, decreaſe 'till arrived at the Aphelion. The ſame is the Caſe with Regard to the apparent Diameter of the Moon, which increaſes from the Apogee to the Perigee, and decreaſes from the Perigee to the Apogee. Now, if the Moon's apparent Diameter be increaſing, and the Sun's decreaſing, the Eclipſes will every Cycle increaſe; but, if the Moon's apparent Diameter decreaſes, and the Sun's increaſes, there will be every Cycle leſs total Obſcurity, 'till the Eclipſe becomes annular: Laſtly, if the Diameters of the Sun and Moon are both increaſing, or both decreaſing, the Eclipſes will be nearly the ſame thro' a long Succeſſion of Periods.

The Situation of the Earth in its Orbit may be known by the Time of the Year; for the Earth is in its Aphelion about the thirtieth of *June*, and in the Perihelion about the thirtieth of *December*; and increaſes one Sign of Anomaly in about thirty Days, or nearly a Degree each Day. And, as to the Anomaly of the Moon, the Days ſhe is in her Apogee and Perigee are noted in all Ephemerides; and her Motion of Anomaly being about thirteen Degrees a Day, that is, a Sign in ſomewhat leſs than two Days and an Half, it will not be difficult to determine the Anomaly of the Moon when any Eclipſe happened by an old Ephemeris, without Aſtronomical Tables, near enough for the Purpoſe of determining the Variations of any future Eclipſe, either as to their true Time of happening, by the Table above given, or the Increaſe or Decreaſe of the Obſcurity of central Eclipſes. Only obſerve, that the Anomaly of the Moon is reckoned from the Apogee, and of the Earth from the Aphelion.

Thus may the Full, Change, and Quarters of the Moon, and Eclipſes both of the Sun and Moon, be found by the ſingle or grand Cycle, with little Trouble, to a good Degree of Exactneſs. And by the ſame may alſo be found the Moon's Place in the Zodiac; for, the mean Motion of the Moon from the Sun exactly correſponding with theſe Periods, the Diſtance of the Moon from the Sun will vary very little thro' any ſucceeding Period from what it was in a former.

To find the Moon's Place in the Zodiac by the grand Cycle of fifty-four Years and thirty-two or thirty-three Days, abating fifty Minutes.

Rule. From the given Time ſubtract fifty-four Years and thirty-two or thirty-three Days, according as fourteen or thirteen Leap-Days have intervened; then, by an Ephemeris for that Year, take out the Moon's Place in the Zodiac for the given Day, and alſo the Sun's Place; from the Place of the Moon ſubtract the Place of the Sun, adding twelve Signs if neceſſary, and the Remainder is the Moon's Diſtance from the Sun; this being added to the Sun's Place for the given Day, gives the Moon's Place for the ſame Day very nearly, making Allowance for the fifty Minutes.

Exam. 1. The preſent Year 1753, *January* 1.

This being ſince the Alteration of the Style, eleven Days muſt firſt be ſubtracted.

To *January*	1		
Add ——	31	Days for *December.*	
	32		
Subtract —	11		
December	21.	Old Style, 1752.	
To which add	30	Days for *November.*	
	51		
Subtract —	32	Days	54 Years.
	19	*November*	1698.

Moon's Place *Nov.* 19, 1698, is ♏ 3°.19′, *i. e.*	7^{s}.	3°.	19′
To which add —— ——	12		
Sum ——	19	3	19
From which ſubtract the Sun's Place the ſame Day, ♐ 7°.50′, *i. e.* ——	8	7	50
Remainder is the Moon's Diſtance from the Sun	10	25	29
To which add the Sun's Place *Jan.* 1, 1753,	9	11	21
Gives the Moon's Place the ſame Day ——	8	6	50

That is, ♐ 6°.50′, the Moon's Place at ten Minutes after Eleven o'Clock; to which adding 27′ for the Moon's Motion in fifty Minutes of Time, gives ♐ 7°.17′ for the Moon's Place at Noon, on the firſt of *January* 1753: And by *White*'s Ephemeris it is ♐ 7°.22′.

Exam. 2. To find the Moon's Place for *January* 5, 1753, which correſponds with *November* 23, 1698.

Moon's Place *November* 23, 1698, is	8ˢ. 21°. 8′
Sun's Place on the ſame Day —	8 . 11 . 54
Diſtance of the Moon from the Sun	0 . 9 . 14
To which add the Sun's Place for *January* 5, 1753, ♑ 15°.26′, *i. e.*	9 . 15 . 26
	9 . 24 . 40
To this Sum add for the fifty Minutes	27
	9 . 25 . 7

That is ♑ 25°.7′: And by *White*'s Ephemeris it is ♑ 25°.5′: In the ſame Manner may the Moon's Place for any aſſigned Time be computed from an Ephemeris fifty-four Years back. But, for the farther Satiſfaction of the Reader, I will here give the Moon's Motion in the Zodiac, computed, as in theſe Examples, from *Gadbury*'s Ephemerides, from the nineteenth of *November* 1698 to the twentieth of *December* incluſive, which anſwers to the Month of *January* 1753; and, by ſetting down the Moon's Motion in the ſame Month of *January* from *White*'s Ephemeris, it will appear how near the Computation correſponds with the Aſtronomical Calculation of that Author.

Jan.	☾'s Place computed.	☾'s Place by *White*.	Jan.	☾'s Place computed.	☾'s Place by *White*.
1	7° ♐ 17′	7° ♐ 22′	16	17° 13′	14° 59′
2	19 18	19 24	17	2 ♋ 20	29 50
3	1 ♑ 16	1 ♑ 19	18	17 36	15 ♋ 0
4	13 12	13 12	19	2 ♌ 52	0 ♌ 22
5	25 7	25 5	20	17 58	15 42
6	7 ♒ 3	6 ♒ 57	21	2 ♍ 44	0 ♍ 49
7	19 1	18 50	22	17 3	15 33
8	1 ♓ 3	0 ♓ 48	23	6 ♎ 54	29 49
9	13 11	12 51	24	14 17	13 ♎ 32
10	25 29	25 5	25	27 4	26 48
11	8 ♈ 8	7 ♈ 29	26	9 ♏ 40	9 ♏ 39
12	21 5	20 11	27	22 8	22 8
13	4 ♉ 26	3 ♉ 13	28	4 ♐ 15	4 ♐ 21
14	18 15	16 39	29	16 12	16 24
15	2 ♊ 31	0 ♊ 35	30	28 6	28 20
			31	9 ♑ 59	10 ♑ 12

The Latitude of the Moon.

1698. Nov.	☾ Lat. North.	1753. Jan.	☾ Lat. North.	1698. Dec.	☾ Lat. South	1753. Jan.	☾ Lat. South.
19	2° 14′	1	2° 4′	6	4° 49′	18	4° 28′
20	3 11	2	2 59	7	5 13	19	4 54
21	4 0	3	3 46	8	5 15	20	4 59
22	4 36	4	4 23	9	4 56	21	4 43
23	5 3	5	4 48	10	4 16	22	4 8
24	5 16	6	5 1	11	3 28	23	3 19
25	5 15	7	5 1	12	2 24	24	2 19
26	5 0	8	4 48	13	1 17	25	1 14
27	4 32	9	4 21	14	0 6	26	0 7
28	3 52	10	3 43	15	1 N. 0	27	0 N. 59
29	2 57	11	2 54	16	2 4	28	2 0
30	1 53	12	1 55	17	3 6	29	2 55
Dec. 1	0 39	13	0 48	18	3 52	30	3 42
2	0 S. 39	14	0 S. 23	19	4 23	31	4 19
3	1 53	15	1 35				
4	3 6	16	2 44				
5	4 6	17	3 43				

The Differences that appear in this Example might be accounted for by the Anomaly of the Earth and Moon, ſo as to bring it within a few Minutes in the Longitude: And by that the Latitude might alſo be brought to the greateſt Exactneſs. By theſe Cycles, the Motions of the Sun, Moon, and Planets, may be computed for a whole Year, with leſs Trouble than is required for one ſingle Day by the Aſtronomical Tables.

FINIS.

E R R A T A.

Page.

6, Line 2, *read* $\frac{5}{7}$; Line 24, *read* *Exam.* 1. The two Numerators 1 and 3.

43, In the Quotient for Pence of the ſecond Example read 6, and the Remainder 6; and the Anſwer 3 *l.* 15 *s.* 6 *d.* $\frac{1}{2}$.

116, l. 3, *read* dividing the 8 Feet by.

155, l. 1, *for* A C *read* B D, and *for* B D *read* A C.

157, in the ſecond Example, read the Work thus,

```
6)144(24
  12
  ——
   24
   24
   ——
```

159, l. 6, put the Dot after the 1, thus, 1.625; and on the other Side, *for* 12.9900, *read* 13.0000.

163, l. 5, *read* Remainder.

174, laſt Line but one, *read* Art. 64.

181, dele the Dot in the firſt Operation.

188, in Maſons' Work, l. 3, *read* Length, Breadth, and Thickneſs.

Page.

195, l. 15, *read* B A C.

204, Exam. 2, l. 3, *read* 3.532499.

205, l. 5, dele .6.

206, l. 14, *read* .454235.

211, l. 27, *read* Prob. 9.

214, l. 13, *read* .552133.

215, l. 18 and 20, *read* .552133; l. 22, *read* 1.773586.

222, l. 9, *read* 11.676694.

254, l. 29, *read* the Circle H Z R N.

In the References in Spherical Trigonometry, and Aſtronomy, read one leſs than is printed, as for Prob. 41 read Prob. 40, &c.

275, l. 3, *read* ſecond Rule.

282, l. 9, *read* 9.705469.

285, l. 34, *read* 9.529140.

286, l. 12, *read* Prob. 87.

306, l. ult. *for* Perpendicular *read* Side.

321, l. 32, *dele* at right Angles.

350, l. 20, *read* M *u s*.

353, l. 42, *read* one Foot in *c*.

371, l. 3, *read* is included.

381, l. 1, *for* Venus's *read* Mercury's.

In the Deſcription of the Tables of the Logarithms of Numbers, Page 200, is mentioned a Repetition of the Numbers in the firſt Column of the right-hand Page, but they are printed without ſuch Repetition.

A
TABLE
OF
Logarithm Numbers,

From One to Ten Thouſand:

WHEREBY

The Logarithm of any Number
under 400000 may be readily
diſcovered.

A
TABLE
OF
LOGARITHMS.

N	Log.	N	Log.	N	Log.
1	000000	34	531479	67	826075
2	301030	35	544068	68	832509
3	477121	36	556303	69	838849
4	602060	37	568202	70	845098
5	698970	38	579783	71	851258
6	778151	39	591064	72	857332
7	845098	40	602060	73	863323
8	903090	41	612784	74	869232
9	954242	42	623249	75	875061
10	000000	43	633468	76	880813
11	041393	44	643452	77	886491
12	079181	45	653212	78	892094
13	113943	46	662758	79	897627
14	146128	47	672098	80	903090
15	176091	48	681241	81	908485
16	204120	49	690196	82	913814
17	230449	50	698970	83	919078
18	255272	51	707570	84	924279
19	278753	52	716003	85	929419
20	301030	53	724276	86	934498
21	322219	54	732394	87	939519
22	342422	55	740362	88	944482
23	361728	56	748188	89	949390
24	380211	57	755875	90	954242
25	397940	58	763428	91	959041
26	414973	59	770852	92	963788
27	431364	60	778151	93	968483
28	447158	61	785330	94	973128
29	462398	62	792391	95	977723
30	477121	63	799340	96	982271
31	491361	64	806180	97	986772
32	505150	65	812913	98	991226
33	518514	66	819544	99	995635

The Table of Logarithms.

N	0	1	2	3	4
100	000000	000434	000868	001301	001734
101	004321	004751	005181	005609	006038
102	008600	009026	009451	009876	010299
103	012837	013259	013679	014100	014521
104	017033	017451	017868	018284	018700
105	021189	021603	022016	022428	022841
106	025306	025715	026125	026533	026942
107	029384	029789	030195	030599	031004
108	033424	033826	034227	034628	035029
109	037426	037825	038223	038620	039017
110	041393	041787	042182	042576	042969
111	045323	045714	046105	046495	046885
112	049218	049606	049993	050379	050766
113	053078	053463	053846	054229	054613
114	056905	057286	057666	058046	058426
115	060698	061075	061452	061829	062206
116	064458	064832	065206	065579	065953
117	068186	068557	068928	069298	069668
118	071882	072249	072617	072985	073352
119	075547	075912	076276	076640	077004
120	079181	079543	079904	080266	080626
121	082785	083144	083503	083861	084219
122	086359	086716	087071	087426	087781
123	089905	090258	090611	090963	091315
124	093422	093772	094122	094471	094820
125	096910	097257	097604	097951	098298
126	100371	100715	101059	101403	101747
127	103804	104146	104487	104828	105169
128	107209	107549	107888	108227	108565
129	110589	110926	111263	111599	111934
130	113943	114277	114611	114944	115278
131	117271	117603	117934	118265	118595
132	120574	120903	121231	121559	121888
133	123852	124178	124504	124830	125156
134	127105	127429	127753	128076	128399
135	130334	130655	130977	131298	131619
136	133539	133858	134177	134496	134814
137	136721	137037	137354	137671	137987
138	139879	140194	140508	140822	141136
139	143015	143327	143639	143951	144263
140	146128	146438	146748	147058	147367
141	149219	149527	149835	150142	150449
142	152288	152594	152899	153205	153509
143	155336	155639	155943	156246	156549
144	158362	158664	158965	159266	159567

The Table of Logarithms.

5	6	7	8	9	D
002166	002598	003029	003461	003891	432
006466	006894	007321	007748	008174	428
010724	011147	011570	011993	012415	424
014940	015359	015779	016197	016616	419
019116	019532	019947	020361	020775	416
023252	023664	024075	024486	024896	412
027349	027757	028164	028571	028978	408
031408	031812	032216	032619	033021	404
035429	035829	036229	036629	037028	400
039414	039811	040207	040602	040998	396
043362	043755	044148	044539	044932	393
047275	047664	048053	048442	048830	389
051153	051538	051924	052309	052694	386
054996	055378	055760	056142	056524	382
058805	059185	059563	059942	060320	379
062582	062958	063333	063709	064083	376
066326	066699	067071	067443	067815	372
070038	070407	070776	071145	071514	369
073718	074085	074451	074816	075182	366
077368	077731	078094	078457	078819	363
080987	081347	081707	082067	082426	360
084576	084934	085291	085647	086004	357
088136	088490	088845	089198	089552	354
091667	092018	092369	092721	093071	351
095169	095518	095866	096215	096562	349
098644	098989	099335	099681	100026	346
102091	102434	102777	103119	103462	343
105510	105851	106191	106531	106871	340
108903	109241	109579	109916	110253	338
112269	112605	112939	113275	113609	335
115611	115943	116276	116608	116939	333
118926	119256	119586	119915	120245	330
122216	122544	122871	123198	123525	328
125481	125806	126131	126456	126781	325
128722	129045	129368	129689	130012	323
131939	132259	132579	132899	133219	321
135133	135451	135769	136086	136403	318
138303	138618	138934	139249	139564	315
141449	141763	142076	142389	142702	314
144574	144885	145196	145507	145818	311
147676	147985	148294	148603	148911	309
150756	151063	151369	151676	151982	307
153815	154119	154423	154728	155032	305
156852	157154	157457	157759	158061	303
159868	160168	160469	160769	161068	301

The Table of Logarithms.

N	0	1	2	3	4
145	161368	161667	161967	162266	162564
146	164353	164650	164947	165244	165541
147	167317	167613	167908	168203	168497
148	170262	170555	170848	171144	171434
149	173186	173478	173769	174059	174351
150	176091	176381	176669	176959	177248
151	178977	179264	179552	179839	180126
152	181844	182129	182415	182699	182985
153	184691	184975	185259	185542	185825
154	187521	187803	188084	188316	188647
155	190332	190612	190892	191171	191451
156	193125	193681	193681	193959	194237
157	195899	196176	196453	196729	197005
158	198657	199206	199206	199481	199755
159	201398	201943	201943	202216	202488
160	204119	204391	204663	204934	205204
161	206826	207096	207365	207634	207904
162	209515	209783	210051	210319	210586
163	212187	212454	212720	212986	213252
164	214844	215109	215373	215638	215902
165	217484	217747	218010	218273	218536
166	220108	220369	220631	220892	221153
167	222716	222976	223236	223496	223755
168	225309	225568	225826	226084	226342
169	227887	228142	228400	228657	228913
170	230449	230704	230959	231215	231469
171	232996	233250	233504	233757	234011
172	235528	235781	236033	236285	236537
173	238046	238297	238548	238799	239049
174	240549	240799	241048	241297	241546
175	243038	243286	243534	243534	244029
176	245513	245759	246006	246252	246499
177	247973	248219	248464	248709	248954
178	250420	250664	250908	251151	251395
179	252853	253096	253334	253580	253822
180	255273	255514	255755	255996	256237
181	257679	257918	258158	258398	258637
182	260071	260309	260548	260787	261025
183	262451	262688	262925	263162	263399
184	264818	265054	265289	265525	265761
185	267172	267406	267641	267875	268109
186	269513	269746	269979	270213	270446
187	271842	272074	272306	272538	272769
188	274158	274389	274619	274850	275081
189	276462	276692	276921	277151	277379

5	6	7	8	9	D
162863	163161	163459	163758	164055	299
165838	166134	166430	166726	167022	297
168792	169086	169380	169674	169968	295
171726	172019	172311	172603	172895	293
174641	174932	175222	175512	175802	291
177536	177825	178113	178401	178689	289
180413	180699	180986	181272	181558	287
183269	183555	183839	184123	184407	285
186108	186391	186674	186956	187239	283
188928	189209	189490	189771	190051	281
191730	192009	192289	192567	192846	279
194514	194792	195069	195346	195623	278
197281	197556	197832	198107	198382	276
200029	200303	200577	200850	201124	274
202761	203033	203303	203577	203848	272
205475	205746	206016	206286	206556	271
208173	208441	208710	208978	209247	269
210853	211121	211388	211654	211921	267
213518	213783	214049	214314	214579	266
216166	216429	216694	216957	217221	264
218798	219060	219323	219585	229846	262
221414	221675	221936	222196	222456	261
224015	224274	224533	224791	225051	259
226599	226858	227115	227372	227629	258
229169	229426	229682	229938	230192	256
231724	231979	232234	232488	232742	254
234264	234517	234770	235023	235276	253
236789	237041	237292	237544	237795	252
239299	239549	239799	240049	240299	250
241795	242044	242293	242541	242789	249
244277	244525	244772	245019	245266	248
246745	246991	247237	247482	247728	246
249198	249443	249687	249932	250176	245
251638	251881	252125	252368	252610	243
254064	254306	254548	254789	255031	242
256477	256718	256958	257198	257438	241
258877	259116	259355	259594	259833	239
261263	261501	261739	261976	262214	238
263636	263873	264109	264346	264582	237
265996	266232	266467	266702	266937	235
268344	268578	268812	269046	269279	234
270679	270912	271114	271377	271609	233
273001	273233	273464	273696	273927	232
275311	275542	275772	276002	276232	230
277609	277838	278067	278296	278525	229

The Table of Logarithms.

N	0	1	2	3	4
190	278754	278982	279211	279439	279667
191	281033	281261	281488	281714	281942
192	283301	283527	283753	283979	284205
193	285557	285782	286007	286232	286456
194	287802	288026	288249	288473	288696
195	290035	290257	290479	290702	290925
196	292256	292478	292699	292920	293141
197	294466	294687	294907	295127	295347
198	296665	296884	297104	297323	297542
199	298853	299071	299289	299507	299725
200	301029	301247	301464	301681	301898
201	303196	303412	303628	303844	304059
202	305351	305566	305781	305996	306211
203	307496	307709	307924	308137	308351
204	309630	309843	310056	310268	310481
205	311754	311966	312177	312389	312600
206	313867	314078	314289	314499	314709
207	315970	316180	316389	316599	316809
208	318063	318272	318481	318689	318898
209	320146	320354	320562	320769	320977
210	322219	322426	322633	322839	323046
211	324282	324488	324694	324899	325105
212	326336	326541	326745	326949	327155
213	328379	328583	328787	328991	329194
214	330414	330617	330819	331022	331225
215	332438	332640	332842	333044	333246
216	334454	334655	334856	335057	335257
217	336459	336659	336859	337059	337259
218	338456	338656	338856	339054	339253
219	340444	340642	340841	341039	341237
220	342422	342620	342817	343014	343212
221	344392	344589	344785	344981	345178
222	346353	346549	346744	346939	347135
223	348305	348499	348694	348889	349083
224	350248	350442	350636	350829	351023
225	352183	352375	352568	352761	352954
226	354108	354301	354493	354685	354876
227	356026	356217	356408	356599	356790
228	357935	358125	358316	358506	358696
229	359835	360025	360215	360404	360593
230	361728	361917	362105	362294	362482
231	363612	363799	363988	364176	364363
232	365488	365675	365862	366049	366236
233	367356	367542	367729	367915	368101
234	369216	369401	369587	369772	369958

5	6	7	8	9	D
279895	280123	280351	280578	280806	228
282169	282396	282622	282849	283075	227
284431	284656	284882	285107	285332	226
286681	286905	287129	287354	287578	225
288919	289143	289366	289589	289812	223
291147	291369	291591	291813	292034	222
293362	293584	293804	294025	294246	221
295567	295787	296007	296226	296446	220
297761	297979	298198	298416	298634	219
299943	300161	300378	300595	300813	218
302114	302331	302547	302764	302979	217
304275	304491	304706	304921	305136	216
306425	306639	306854	307068	307282	215
308564	308778	308991	309204	309417	213
310693	310906	311118	311329	311542	212
312812	313023	313234	313445	313656	211
314920	315130	315340	315551	315760	210
317018	317227	317436	317645	317854	209
319106	319314	319522	319730	319938	208
321184	321391	321598	321805	322012	207
323252	323458	323665	323871	324077	206
325310	325516	325721	325926	326131	205
327359	327563	327767	327972	328176	204
329398	329601	329805	330008	330211	203
331427	331629	331832	332034	332236	202
333447	333649	333850	334051	334253	202
335458	335658	335859	336059	336259	201
337459	337659	337859	338058	338257	200
339451	339650	339849	340047	340246	199
341435	341632	341830	342028	342225	198
343409	343606	343802	343999	344196	197
345373	345569	345766	345962	346157	196
347330	347525	347720	347915	348110	195
349278	349472	349666	349860	350054	194
351216	351409	351603	351796	351989	193
353147	353339	353532	353724	353916	193
355068	355260	355452	355643	355834	192
356981	357172	357363	357554	357744	191
358886	359076	359266	359456	359646	190
360783	360972	361161	361350	361539	189
362671	362859	363048	363236	363424	188
364551	364739	364926	365113	365301	188
366423	366609	366796	366983	367169	187
368287	368473	368659	368845	369030	186
370143	370328	370513	370698	370882	185

The Table of Logarithms.

N	0	1	2	3	4
235	371068	371253	371437	371622	371806
236	372912	373096	373279	373464	373647
237	374748	374932	375115	375298	375481
238	376577	376759	376942	377124	377306
239	378398	378579	378761	378943	379124
240	380211	380392	380573	380754	380934
241	382017	382197	382377	382557	382737
242	383815	383995	384174	384353	384533
243	385606	385785	385964	386142	386321
244	387389	387568	387746	387923	388101
245	389166	389343	389520	389698	389875
246	390935	391112	391288	391464	391641
247	392697	392873	393048	393224	393399
248	394452	394627	394802	394977	395152
249	396199	396374	396548	396722	396896
250	397940	398114	398287	398461	398634
251	399674	399847	400019	400192	400365
252	401401	401573	401745	401917	402089
253	403121	403292	403464	403635	403807
254	404834	405005	405176	405346	405517
255	406540	406710	406881	407051	407221
256	408239	408409	408579	408749	408918
257	409933	410102	410271	410439	410609
258	411619	411788	411956	412124	412293
259	413299	413467	413635	413803	413969
260	414973	415140	415307	415474	415641
261	416641	416807	416973	417139	417306
262	418301	418467	418633	418798	418964
263	419956	420121	420286	420451	420616
264	421604	421768	421933	422097	422261
265	423246	423410	423574	423737	423901
266	424882	425045	425208	425371	425534
267	426511	426674	426836	426999	427161
268	428135	428297	408459	428621	428783
269	429752	429914	430075	430236	430398
270	431364	431525	431685	431846	432007
271	432969	433129	433289	433449	433609
272	434569	434729	434888	435048	435207
273	436163	436322	436481	436639	436799
274	437751	437909	438067	438226	438384
275	439333	439491	439648	439806	439964
276	440909	441066	441224	441381	441538
277	442479	442637	442793	442949	443106
278	444045	444201	444357	444513	444669
279	445604	445759	445915	446071	446226

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5	6	7	8	9	D
371991	372175	372359	372544	372728	184
373831	374015	374198	374382	374565	184
375664	375846	376029	376212	376394	183
377488	377670	377852	378034	378216	182
379306	379487	379668	379849	380030	181
381115	381296	381476	381656	381837	181
382917	383097	383277	383456	383636	180
384712	384891	385069	385249	385428	179
386499	386677	386856	387034	387212	178
388279	388456	388634	388811	388989	178
390051	390228	390405	390582	390759	177
391817	391993	392169	392345	392521	176
393575	393751	393926	394101	394277	176
395326	395501	395676	395850	396025	175
397071	397245	397419	397592	397766	174
398808	398981	399154	399328	399501	173
400538	400711	400883	401056	401228	173
402261	402433	402605	402777	402949	172
403978	404149	404320	404492	404663	171
405688	405858	406029	406199	406369	171
407391	407561	407731	407901	408070	170
409087	409257	409426	409595	409764	169
410777	410946	411114	411283	411451	169
412461	412629	412796	412964	413132	168
414137	414305	414472	414639	414806	167
415808	415974	416141	416308	416474	167
417472	417638	417804	417969	418135	166
419129	419295	419460	419625	419791	165
420781	420945	421110	421275	421439	165
422426	422589	422754	422918	423082	164
424065	424228	424392	424555	424718	164
425697	425860	426023	426186	426349	163
427324	427486	427648	427811	427973	162
428944	429106	429268	429429	429591	162
430559	430719	430881	431042	431203	161
432167	432328	432488	432649	432809	161
433769	433929	434089	434249	434409	160
435366	435526	435685	435844	436004	159
436957	437116	437275	437433	437592	159
438542	438701	438859	439017	439175	158
440122	440279	440437	440594	440752	158
441695	441852	442009	442166	442323	157
443263	443419	443576	443732	443889	157
444825	444981	445137	445293	445449	156
446382	446537	446692	446848	447003	155

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N	0	1	2	3	4
280	447158	447313	447468	447623	447778
281	448706	448861	449015	449169	449324
282	450249	450403	450557	450711	450865
283	451786	451940	452093	452246	452399
284	453318	453471	453624	453777	453929
285	454845	454997	455149	455302	455454
286	456366	456518	456669	456821	456973
287	457882	458033	458184	458336	458487
288	459392	459543	459694	459845	459995
289	460898	461048	461198	461348	461499
290	462398	462548	462697	462847	462997
291	463893	464042	464191	464340	464489
292	465383	465532	465680	465829	465977
293	466868	467016	467164	467312	467460
294	468347	468495	468643	468790	468938
295	469822	469969	470116	470263	470410
296	471292	471438	471585	471732	471878
297	472756	472903	473049	473195	473341
298	474216	474362	474508	474653	474799
299	475671	475816	475962	476107	476252
300	477121	477266	477411	477555	477699
301	478566	478711	478855	478999	479143
302	480007	480151	480294	480438	480582
303	481443	481586	481729	481872	482016
304	482874	483016	483159	483302	483445
305	484299	484442	484585	484727	484869
306	485721	485863	486005	486147	486289
307	487138	487279	487421	487563	487704
308	488551	488692	488833	488974	489114
309	489958	490099	490239	490379	490520
310	491362	491502	491642	491782	491922
311	492760	492900	493039	493179	493319
312	494155	494294	494433	494572	494711
313	495544	495683	495822	495960	496099
314	496929	497068	497206	497344	497483
315	498311	498448	498586	498724	498862
316	499687	499824	499962	500099	500236
317	501059	501196	501333	501470	501607
318	502427	502564	502700	502837	502973
319	503791	503927	504063	504199	504335
320	505149	505286	505421	505557	505693
321	506505	506640	506776	506911	507046
322	507856	507991	508126	508260	508395
323	509203	509337	509471	509606	509740
324	510545	510679	510813	510947	511081

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5	6	7	8	9	D
447932	448088	448242	448397	448552	155
449478	449633	449787	449941	450095	154
451018	451172	451326	451479	451633	154
452553	452706	452859	453012	453165	153
454082	454235	454387	454539	454692	153
455606	455758	455910	456062	456214	152
457125	457276	457428	457579	457731	152
458638	458789	458939	459091	459242	151
460146	460296	460447	460597	460748	151
461649	461799	461948	462098	462248	150
463146	463296	463445	463594	463744	150
464639	464788	464936	465085	465234	149
466126	466274	466423	466571	466719	149
467608	467756	467904	468052	468199	148
469085	469233	469380	469527	469675	147
470557	470704	470851	470998	471145	147
472025	472171	472318	472464	472610	146
473487	473633	473779	473925	474071	146
474944	475089	475235	475381	475526	146
476397	476542	476687	476832	476976	145
477844	477989	478133	478278	478422	145
479287	479431	479575	479719	479863	144
480725	480869	481012	481156	481299	144
482159	482302	482445	482588	482731	143
483587	483729	483872	484015	484157	143
485011	485153	485295	485437	485579	142
486430	486572	486714	486855	486997	142
487845	487986	488127	488269	488409	141
489255	489396	489537	489677	489818	141
490661	490801	490941	491081	491222	140
492062	492201	492341	492481	492621	140
493458	493597	493737	493876	494015	139
494850	494989	495128	495267	495406	139
496238	496376	496515	496653	496791	139
497621	497759	497897	498035	498173	138
498999	499137	499275	499412	499549	138
500374	500511	500648	500785	500922	137
501744	501880	502017	502154	502291	137
503109	503246	503382	503518	503655	136
504471	504607	504743	504878	505014	136
505828	505964	506099	506234	506369	136
507181	507316	507451	507586	507721	135
508529	508664	508799	508934	509068	135
509874	510009	510143	510277	510411	134
511215	511349	511482	511616	511749	134

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N	0	1	2	3	4
325	511883	512017	512151	512284	512418
326	513218	513351	513484	513617	513750
327	514548	514681	514813	514946	515079
328	515874	516006	516139	516271	516403
329	517196	517328	517459	517592	517724
330	518514	518646	518777	518909	519040
331	519828	519959	520090	520221	520353
332	521138	521269	521399	521530	521661
333	522444	522575	522705	522835	522966
334	523746	523876	524006	524136	524266
335	525045	525174	535304	525434	525563
336	526339	526469	526598	526727	526856
337	527629	527759	527888	528016	528145
338	528916	529045	529174	529302	529430
339	530199	530328	530456	530584	530711
340	531479	531607	531734	531862	531989
341	532754	532882	533009	533136	533264
342	534026	534153	534280	534407	534534
343	535294	535421	535547	535674	535800
344	536558	536685	536811	536937	537063
345	537819	537945	538071	538197	538322
346	539076	539202	539327	539452	539578
347	540329	540454	540579	540704	540829
348	541579	541704	541829	541953	542078
349	542825	542949	543074	543199	543323
350	544068	544192	544316	544440	544564
351	545307	545431	545555	545678	545802
352	546543	546666	546789	546913	547036
353	547775	547898	548021	548144	548267
354	549003	549126	549249	549371	549494
355	550228	550351	550473	550595	550717
356	551449	551572	551694	551816	551938
357	552668	552789	552911	553033	553155
358	553883	554004	554126	554247	554368
359	555094	555215	555336	555457	555578
360	556303	556423	556544	556664	556785
361	557507	557627	557748	557868	557988
362	558709	558729	558948	559068	559188
363	559907	560026	560146	560265	560385
364	561101	561221	561340	561459	561578
365	562293	562412	562531	562650	562769
366	563481	563599	563718	563837	563955
367	564666	564784	564903	565021	565139
368	565848	565966	566084	566202	566319
369	567026	567144	567262	567379	567497

5	6	7	8	9	D
512551	512684	512818	512951	513084	133
513883	514016	514149	514282	514415	133
515211	515344	515476	515609	515741	133
516535	516667	516799	516932	517064	132
517855	517987	518119	518251	518382	132
519171	519303	519434	519566	519697	131
520484	520615	520745	520876	521007	131
521792	521922	522053	522183	522314	131
523096	523226	523356	523486	523616	130
524396	524526	524656	524785	524915	130
525693	525822	525951	526081	526210	129
526985	527114	527243	527372	527501	129
528274	528402	528531	528659	528788	129
529559	529687	529815	529943	530072	128
530839	530968	531096	531223	531351	128
532117	532245	532372	532499	532627	128
533391	533518	533645	533772	533899	127
534661	534787	534914	535041	535167	127
535927	536053	536179	536305	536432	126
537189	537315	537441	537567	537693	126
538448	538574	538699	538825	538951	126
539703	539829	539954	540079	540204	125
540955	541079	541205	541329	541454	125
542203	542327	542452	542576	542701	125
543447	543571	543696	543819	543944	124
544688	544812	544935	545059	545183	124
545925	546049	546172	546296	546419	124
547159	547282	547405	547529	547652	123
548389	548512	548635	548758	548881	123
549616	549739	549861	549984	550106	123
550839	550962	551084	551206	551328	122
552059	552181	552303	552425	552547	122
553276	553398	553519	553640	553762	121
554489	554610	554731	554852	554973	121
555699	555819	555940	556061	556182	121
556905	557026	557146	557267	557387	120
558108	558228	558349	558469	558589	120
559308	559428	559548	559667	559787	120
560504	560624	560743	560863	560982	119
561698	561817	561936	562055	562174	119
562887	563006	563125	563244	563362	119
564074	564192	564311	564429	564548	119
565257	565376	565494	565612	565729	118
566437	566555	566673	566791	566909	118
567614	567732	567849	567967	568084	118

N	0	1	2	3	4
370	568202	568319	568436	568554	568671
371	569374	569491	569608	569725	569842
372	570543	570659	570776	570893	571009
373	571709	571825	571942	572058	572174
374	572872	572988	573104	573219	573336
375	574031	574147	574263	574379	574494
376	575188	575303	575419	575534	575649
377	576341	576457	576572	576687	576802
378	577492	577607	577722	577836	577951
379	578639	578754	578868	578983	579097
380	579784	579898	580012	580126	580241
381	580925	581039	581153	581267	581381
382	582063	582177	582291	582404	582518
383	583199	583312	583426	583539	583652
384	584331	584444	584557	584670	584782
385	585461	585574	585686	585799	585912
386	586587	586699	586812	586925	587037
387	587711	587823	587935	588047	588159
388	588832	588944	589056	589167	589279
389	589949	590061	590173	590284	590396
390	591065	591176	591287	591399	591509
391	592177	592288	592399	592509	592621
392	593286	593397	593508	593618	593729
393	594393	594503	594614	594724	594834
394	595496	595606	595717	595827	595937
395	596597	596707	596817	596927	597037
396	597695	597805	597914	598024	598134
397	598790	598899	599009	599119	599228
398	599883	599992	600101	600210	600319
399	600973	601082	601191	601299	601408
400	602059	602169	602277	602386	602494
401	603144	603253	603361	603469	603577
402	604226	604334	604442	604550	604658
403	605305	605413	605521	605628	605736
404	606381	606489	606596	606704	606811
405	607455	607562	607669	607777	607884
406	608526	608633	608739	608847	608954
407	609594	609701	609808	609914	610021
408	610660	610767	610873	610979	611086
409	611723	611829	611936	612042	612148
410	612784	612889	612996	613102	613207
411	613842	613947	614053	614159	614264
412	614897	615003	615108	615213	615319
413	615950	616055	616160	616265	616370
414	617000	617105	617210	617315	617419

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5	6	7	8	9	D
568788	568905	569023	569139	569257	117
569959	570076	570193	570309	570426	117
571126	571243	571359	571476	571592	117
572291	572407	572523	572639	572755	116
573452	573568	573684	573799	573915	116
574609	574726	574841	574957	575072	116
575765	575880	575996	576111	576226	115
576917	577032	577147	577262	577377	115
578066	578181	578295	578409	578525	115
579212	579326	579441	579555	579669	114
580355	580469	580583	580697	580811	114
581495	581608	581722	581836	581949	114
582631	582745	582858	582972	583085	114
583765	583879	583992	584105	584218	113
584896	585009	585122	585235	585348	113
586024	586137	586249	586362	586475	113
587149	587262	587374	587486	587599	112
588272	588384	588496	588608	588719	112
589391	589503	589615	589726	589838	112
590597	590619	590730	590842	590953	112
591621	591732	591843	591955	592066	111
592732	592843	592954	593064	593175	111
593839	593950	594061	594171	594282	111
594945	595055	595165	595276	595386	110
596047	596157	596267	596377	596487	110
597146	597256	597366	597476	597586	110
598243	598353	598462	598572	598681	110
599337	599446	599556	599665	599774	109
600428	600537	600646	600755	600864	109
601517	601625	601734	601843	601951	109
602603	602711	602819	602928	603036	108
603686	603794	603902	604009	604118	108
604766	604874	604982	605089	605197	108
605844	605951	606059	606166	606274	108
606919	607026	607133	607241	607348	107
607991	608098	608205	608312	608419	107
609061	609167	609274	609381	609488	107
610128	610234	610341	610447	610554	107
611192	611298	611405	611511	611617	106
612254	612359	612466	612572	612678	106
613313	613419	613525	613030	613736	106
614369	614475	614581	614686	614792	106
615424	615529	615634	615739	615845	105
616476	616581	616686	616790	616895	105
617525	617629	617734	617839	617943	105

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N	0	1	2	3	4
415	618048	618153	618257	618362	618466
416	619093	619198	619302	619406	619511
417	620136	620240	620344	620448	620552
418	621176	621280	621384	621488	621592
419	622214	622318	622421	622525	622628
420	623249	623353	623456	623559	623663
421	624282	624385	624488	624591	624695
422	625312	625415	625518	625621	625724
423	626340	626443	626546	626648	626751
424	627366	627468	627571	627673	627775
425	628389	628491	628593	628695	628797
426	629409	629512	629613	629715	629817
427	630428	630529	630631	630733	630835
428	631444	631545	631647	631748	631849
429	632457	632559	632659	632761	632862
430	633468	633569	633670	633771	633872
431	634477	634578	634679	634779	634880
432	635484	635584	635685	635785	635886
433	636488	636588	636688	636789	636889
434	637489	637589	637689	637789	637889
435	638489	638589	638689	638789	638888
436	639486	639586	639686	639785	639885
437	640481	640581	640680	640779	640879
438	641475	641573	641672	641771	641871
439	642465	642563	642662	642761	642860
440	643453	643551	643650	643749	643847
441	644429	644537	644636	644734	644832
442	645422	645521	645619	645717	645815
443	646404	646502	646599	646698	646796
444	647383	647481	647579	647676	647774
445	648360	648458	648555	648653	648750
446	649335	649432	649529	649627	649724
447	650308	650405	650502	650599	650696
448	651278	651375	651473	651569	651666
449	652246	652343	652439	652536	652633
450	653213	653309	653405	653502	653598
451	654177	654273	654369	654465	654562
452	655138	655235	655331	655427	655523
453	656098	656194	656289	656386	656482
454	657056	657152	657247	657343	657438
455	658011	658107	658202	658298	658393
456	658965	659060	659155	659250	659346
457	659916	660011	660106	660201	660296
458	660865	660960	661055	661149	661245
459	661813	661907	662002	662096	662191

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618571	618676	618780	618884	618989	105
619615	619719	619824	619928	620032	104
620656	620760	620864	620968	621072	104
621695	621799	621902	622007	622110	104
622732	622835	622939	623042	623146	104
623766	623869	623973	624076	624179	103
624798	624901	625004	625107	625209	103
625827	625929	626032	626135	626237	103
626853	626956	627058	627161	627263	103
627878	627979	628082	628185	628287	102
628899	629002	629104	629206	629308	102
629919	630021	630123	630224	630326	102
630936	631038	631139	631241	631342	102
631951	632052	632153	632235	632356	101
632963	633064	633165	633266	633367	101
633973	634075	634975	634276	634376	100
634981	635081	635182	635283	635383	100
635986	636087	636187	636288	636388	100
636989	637089	637189	637289	637389	100
637989	638089	638189	638289	638389	99
638988	639088	639188	639287	639387	99
639984	640084	640183	640283	640382	99
640978	641077	641177	641276	641375	99
641969	642069	642168	642267	642366	99
642959	643058	643156	643255	643354	99
643946	644044	644143	644242	944340	98
644931	645029	645127	645226	645324	98
645913	646011	646109	646208	646306	98
646894	646992	647089	647187	647285	98
647872	647969	648067	648165	648262	98
648848	648945	649043	649140	649237	97
649821	649919	650016	650113	650210	97
650793	650890	650987	651084	651181	97
651762	651859	651956	652053	652149	97
652729	652826	652923	653019	653116	97
653695	653791	653888	653984	654086	96
654658	654754	654850	654946	655042	96
655619	655715	655810	655906	656002	96
656577	656673	656769	656864	656960	96
657534	657629	657725	657820	657916	96
658488	658584	658679	658774	658869	95
659441	659536	659630	659726	659821	95
660391	660486	660581	660676	660771	95
661339	661434	661529	661623	661718	95
662286	662380	662475	662569	662663	95

The Table of Logarithms.

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461	663701	663795	663889	663983	664078
462	664642	664736	664829	664924	665018
463	665581	665675	665769	665862	665956
464	666518	666611	666705	666799	666892
465	667453	667546	667639	667733	657826
466	668386	668479	668572	668665	668759
467	669317	669410	669503	669596	669689
468	670246	670339	670431	670524	670617
469	671173	671265	671358	671451	671543
470	672098	672190	672283	672375	672467
471	673021	673113	673205	673297	673389
472	673942	674034	674126	674218	674309
473	674861	674953	675045	675137	675228
474	675778	675869	675962	676053	676145
475	676694	676785	676876	676968	677059
476	677607	677698	677789	677881	677972
477	678518	678609	678700	678791	678882
478	679428	679519	679609	679700	679791
479	680336	680426	680517	680607	680698
480	681241	681332	681422	681513	681603
481	682145	682235	682326	682416	682506
482	683047	683137	683227	683317	683407
483	683947	684037	684127	684217	684307
484	684845	684935	685025	685114	685204
485	685742	685831	685921	686010	686099
485	686636	686726	686815	686904	686994
487	687529	687618	687707	687796	687885
488	688419	688509	688598	688687	688776
489	689309	689398	689486	689575	689664
490	690196	690285	690373	690462	690550
491	691081	691169	691258	691347	691435
492	691965	692053	692142	692229	692318
493	692847	692935	693023	693111	693199
494	693727	693815	693903	693991	694078
495	694605	694693	694781	694868	694956
496	695482	695569	695657	695744	695832
497	696356	696444	696531	696618	696706
498	697229	697317	697404	697491	697578
499	698101	698188	698275	698362	698449
500	698970	699057	699144	699231	699317
501	699838	699924	700011	700098	700184
502	700704	700790	700877	700963	701049
503	701568	701654	701741	701827	701913
504	702430	702517	702603	702689	702775

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663229	663324	663418	663512	663607	94
664172	664266	664359	664454	664548	94
665112	665206	665299	665393	665487	94
666049	666143	666237	666331	666424	94
666986	667079	667173	667266	667359	94
667919	668013	668106	668199	668293	93
668852	668945	669038	669131	669224	93
669782	669875	669967	670060	670153	93
670709	670802	670895	670988	671080	93
671636	671728	671821	671913	672005	93
672559	672652	672744	672836	672929	92
673482	673574	673666	673758	673849	92
674402	674494	674586	674677	674769	92
675319	675412	675503	675595	675687	92
676236	676328	676419	676511	676602	92
677151	677242	677333	677424	677516	91
678063	678154	678245	678335	678427	91
678973	679064	679155	679246	679337	91
679882	679972	680063	680154	680245	91
680789	680879	680969	681060	681151	91
681693	681784	681874	681964	682055	90
682596	682686	682777	682867	682957	90
683497	683587	683677	683767	683857	90
684396	684486	684576	684666	684756	90
685294	685383	685473	685563	685652	90
686189	686279	686368	686458	686547	89
687083	687172	687261	687350	687439	89
687975	688064	688153	688242	688331	89
688865	688953	689042	689131	689220	89
689753	689841	689930	690019	690107	89
690639	690728	690816	690905	690993	89
691524	691612	691700	691789	691877	88
692406	692494	692583	692671	692759	88
693287	693375	693463	693551	693639	88
694166	694254	694342	694429	694517	88
695044	695131	695219	695307	695394	88
695919	696007	696094	696182	696269	87
696793	696880	696968	697055	697142	87
697665	697752	697839	697926	698014	87
698535	698622	698709	698796	698883	87
699404	699491	699578	699664	699751	87
700271	700358	700444	700531	700617	87
701136	701222	701309	701395	701482	86
701999	702086	702172	702258	702344	86
702861	702947	703033	703119	703205	86

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506	704151	704236	704322	704408	704494
507	705008	705094	705179	705265	705350
508	705863	705949	706035	706120	706206
509	706718	706803	706888	706974	707059
510	707570	707655	707740	707826	707911
511	708421	708506	708591	708676	708761
512	709269	709355	709439	709524	709609
513	710117	710202	710287	710371	710456
514	710963	711048	711132	711217	711301
515	711807	711892	711976	712060	712144
516	712649	712734	712818	712902	712986
517	713491	713575	713659	713742	713826
518	714329	714413	714497	714581	714665
519	715167	715251	715335	715418	715501
520	716003	716087	716170	716254	716337
521	716838	716921	717004	717088	717171
522	717671	717754	717837	717920	718003
523	718502	718585	718668	718751	718834
524	719331	719414	719497	719579	719663
525	720159	720242	720325	720407	720490
526	720986	721068	721151	721233	721316
527	721811	721893	721975	722058	722140
528	722634	722716	722798	722881	722963
529	723456	723538	723619	723702	723784
530	724276	724358	724439	724522	724604
531	725095	725176	725258	725339	725422
532	725912	725993	726075	726156	726238
533	726727	726809	726890	726972	727053
534	727541	727623	727704	727785	727866
535	728354	728435	728516	728597	728678
536	729165	729246	729327	729408	729489
537	729974	730055	730136	730217	730298
538	730782	730863	730944	731024	731105
539	731589	731669	731749	731830	731911
540	732394	732474	732555	732635	732715
541	733197	733278	733358	733438	733518
542	733999	734079	734159	734239	734319
543	734799	734879	734959	735039	735119
544	735599	735679	735759	735838	735918
545	736397	736476	736556	736635	736715
546	737192	737272	737352	737431	737511
547	737987	738067	738146	738225	738305
548	738781	738859	738939	739018	739097
549	739572	739651	739731	739809	739889

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703721	703807	703893	703979	704065	86
704579	704665	704751	704837	704922	86
705436	705522	705607	705693	705778	86
706291	706376	706462	706547	706632	85
707144	707229	707315	707399	707485	85
707996	708081	708166	708251	708336	85
708846	708931	709015	709100	709185	85
709694	709779	709863	709948	710033	85
710540	710625	710709	710794	710879	85
711385	711469	711554	711639	711723	84
712229	712313	712397	712481	712566	84
713070	713154	713238	713323	713407	84
713910	713994	714078	714162	714246	84
714749	714833	714916	714999	715084	84
715586	715669	715753	715836	715919	84
716421	716504	716588	716671	716754	83
717254	717338	717421	717504	717587	83
718086	718169	718253	718336	718419	83
718917	718999	719083	719165	719248	83
719745	719828	719911	719994	720077	83
720573	720655	720738	720821	720903	83
721398	721481	721563	721646	721728	82
722222	722305	722387	722469	722552	82
723045	723127	723209	723291	723374	82
723866	723948	724029	724112	724194	82
724685	724767	724849	724931	725013	82
725503	725585	725667	725748	725829	82
726319	726401	726483	726564	726646	82
727134	727216	727297	727379	727459	81
727948	728029	728110	728191	728273	81
728759	728841	728922	729003	729084	81
729569	729651	729732	729813	729893	81
730378	730459	730540	730621	730701	81
731186	731266	731347	731428	731508	81
731991	732072	732152	732233	732313	81
732796	732876	732956	733037	733117	80
733598	733679	733759	733839	733919	80
734399	734479	734559	734639	734719	80
735199	735279	735359	735439	735519	80
735998	736078	736157	736237	736317	80
736795	736874	736954	737034	737113	80
737590	737669	737749	737829	737908	79
738384	738463	738543	738622	738701	79
739177	739256	739335	739414	739493	79
739968	740047	740126	740205	740284	79

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551	741152	741230	741309	741388	741467
552	741939	742018	742096	742175	742254
553	742725	742803	742882	742961	743039
554	743509	743588	743667	743745	743823
555	744293	744371	744449	744528	744606
556	745075	745153	745231	745309	745387
557	745855	745933	746011	746089	746167
558	746634	746712	746789	746868	746945
559	747412	747489	747567	747645	747722
560	748188	748266	748343	748421	748498
561	748963	749040	749118	749195	749272
562	749736	749814	749891	749968	750045
563	750508	750586	750663	750739	750817
564	751279	751356	751433	751510	751587
565	752048	752125	752202	752279	752356
566	752816	752893	752969	753047	753123
567	753583	753659	753736	753813	753889
568	754348	754425	754501	754578	754654
569	755112	755189	755265	755341	755417
570	755875	755951	756027	756103	756179
571	756636	756712	756788	756864	756940
572	757396	757472	757548	757624	757699
573	758155	758230	758306	758382	758458
574	758912	758988	759063	759139	759214
575	759668	759743	759819	759894	759969
576	760422	760498	760573	760649	760724
577	761176	761251	761326	761402	761477
578	761928	762003	762078	762153	762228
579	762679	762754	762829	762904	762978
580	763428	763503	763578	763653	763727
581	764176	764251	764326	764400	764475
582	764923	764998	765072	765147	765221
583	765669	765743	765818	765892	765966
584	766413	766487	766562	766636	766710
585	767156	767230	767304	767379	767453
586	767898	767972	768046	768119	768194
587	768638	768712	768786	768860	768934
588	769377	769451	769525	769599	769673
589	770115	770189	770263	770336	770410
590	770852	770926	770999	771073	771146
591	771587	771661	771734	771808	771881
592	772322	772395	772468	772542	772615
593	773055	773128	773201	773274	773348
594	773786	773859	773933	774006	774079

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740757	740836	740915	740994	741073	79
741546	741624	741703	741782	741860	79
742332	742411	742489	742568	742647	79
743118	743196	743275	743353	743431	78
743902	743979	744058	744136	744215	78
744684	744762	744840	744919	744997	78
745465	745543	745621	745699	745777	78
746245	746323	746401	746479	746556	78
747023	747101	747179	747256	747334	78
747800	747878	747955	748033	748110	78
748576	748653	748731	748808	748885	77
749349	749427	749504	749582	749659	77
750123	750199	750277	750354	750431	77
750894	750971	751048	751125	751202	77
751664	751741	751818	751895	751972	77
752433	752509	752586	752663	752739	77
753199	753277	753353	753429	753506	77
753966	754042	754119	754195	754272	77
754730	754807	754883	754959	755036	76
755494	755569	755646	755722	755799	76
756256	756332	756408	756484	756560	76
757016	757092	757168	757244	757320	76
757775	757851	757927	758003	758079	76
758533	758609	758685	758761	758836	76
759290	759366	759441	759517	759592	76
760045	760121	760196	760272	760347	75
760799	760875	760949	761025	761101	75
761552	761627	761702	761778	761853	75
762303	762378	762453	762529	762604	75
763053	763128	763203	763279	763353	75
763802	763877	763952	764027	764101	75
764549	764624	764699	764774	764848	75
765296	765370	765445	765519	765594	75
766041	766115	766189	766264	766338	74
766785	766859	766933	767007	767082	74
767527	767601	767675	767749	767823	74
768268	768342	768416	768490	768564	74
769008	769082	769156	769229	769303	74
769746	769820	769894	769968	770042	74
770484	770557	770631	770705	770778	74
771219	771293	771367	771440	771514	74
771955	772028	772102	772175	772248	73
772688	772762	772835	772908	772981	73
773421	773494	773567	773640	773713	73
774152	774225	774298	774371	774444	73

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597	775974	776047	776119	776193	776265
598	776701	776774	776846	776919	776992
599	777427	777499	777572	777644	777717
600	778151	778224	778296	778368	778441
601	778874	778947	779019	779091	779163
602	779596	779669	779741	779813	779885
603	780317	780389	780461	780533	780605
604	781037	781109	781181	781253	781324
605	781755	781827	781899	781971	782042
606	782473	782544	782616	782688	782759
607	783189	783260	783332	783403	783475
608	783904	783975	784046	784118	784189
609	784617	784689	784759	784831	784902
610	785329	785401	785472	785543	785615
611	786041	786112	786183	786254	786325
612	786751	786822	786893	786964	787035
613	787460	787531	787602	787673	787744
614	788164	788239	788309	788381	788451
615	788875	788946	789016	789087	789157
616	789581	789651	789722	789792	789863
617	790285	790356	790426	790496	790567
618	790988	791059	791129	791199	791269
619	791691	791761	791831	791901	791971
620	792392	792462	792532	792602	792672
621	793092	793162	793231	793301	793371
622	793791	793860	793930	793999	794069
623	794488	794558	794627	794697	794767
624	795185	795254	795324	795393	795463
625	795880	795949	796019	796088	796158
626	796574	796644	796713	796782	796852
627	797268	797337	797406	797475	797545
628	797959	798029	798098	798167	798236
629	798651	798719	798789	798858	798927
630	799341	799409	799478	799547	799616
631	800029	800098	800167	800236	800305
632	800717	800786	800854	800923	800992
633	801404	801472	801541	801609	801678
634	802089	802158	802226	802295	802363
635	802774	802842	802910	802979	803047
636	803457	803525	803594	803662	803730
637	804139	804208	804276	804344	804412
638	804821	804889	804957	805025	805093
639	805501	805569	805637	805705	805773

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5	6	7	8	9	D
774882	774955	775028	775100	775173	73
775610	775683	775756	775829	775902	73
776338	776411	776483	776556	776629	73
777064	777137	777209	777282	777354	73
777789	777862	777934	778006	778079	72
778513	778585	778658	778729	778802	72
779236	779308	779380	779452	779524	72
779957	780029	780101	780173	780245	72
780677	780749	780821	780893	780965	72
781396	781468	781539	781612	781684	72
782114	782186	782258	782329	782401	72
782831	782902	782974	783046	783117	72
783546	783618	783689	783761	783832	71
784261	784332	784403	784475	784546	71
784974	785045	785116	785187	785259	71
785686	785757	785828	785899	785970	71
786396	786467	786538	786609	786680	71
787106	787177	787248	787319	787389	71
787815	787885	787956	788027	788098	71
788522	788593	788663	788734	788804	71
789228	789299	789369	789439	789510	71
789933	790004	790074	790144	790215	70
790637	790707	790778	790848	790918	70
791339	791409	791480	791550	791620	70
792041	792111	792181	792252	792322	70
792742	792812	792882	792952	793022	70
793441	793511	793581	793651	793721	70
794139	794209	794279	794349	794418	70
794836	794906	794976	795045	795115	70
795532	795602	795672	795741	795810	70
796227	796297	796366	796436	796505	69
796921	796990	797059	797129	797198	69
797614	797683	797752	797821	797890	69
798305	798374	798443	798513	798582	69
798996	799065	799134	799203	799272	69
799685	799754	799823	799892	799961	69
800373	800442	800511	800579	800648	69
801061	801129	801198	801266	801335	69
801747	801815	801884	801952	802021	69
802432	802500	802568	802637	802705	68
803116	803184	803252	803321	803389	68
803798	803867	803935	804003	804071	68
804480	804548	804616	804685	804753	68
805161	805229	805297	805365	805433	68
805841	805908	805976	806044	806112	68

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N	0	1	2	3	4
640	806179	806248	806316	806384	806451
641	806858	806926	806994	807061	807129
642	807535	807603	807670	807738	807806
643	808211	808279	808346	808414	808481
644	808886	808953	809021	809088	809156
645	809559	809627	809694	809762	809829
646	810233	810299	810367	810434	810501
647	810904	810971	811039	811106	811173
648	811575	811642	811709	811776	811843
649	812245	812312	812379	812445	812512
650	812913	812980	813047	813114	813181
651	813581	813648	813714	813781	813848
652	814248	814314	814381	814447	814514
653	814913	814979	815046	815113	815179
654	815578	815644	815711	815777	815843
655	816241	816308	816374	816440	816506
656	816904	816970	817036	817102	817169
657	817565	817631	817698	817764	817829
658	818226	818292	818358	818424	818489
659	818885	818951	819017	819083	819149
660	819543	819609	819676	819741	819807
961	820201	820267	820333	820399	820464
662	820858	820924	820989	821055	821120
663	821514	821579	821645	821709	821775
664	822168	822233	822299	822364	822429
665	822822	822887	822952	823018	823083
666	823474	823539	823605	823669	823735
667	824126	824191	824256	824321	824386
668	824776	824841	824906	824971	825036
669	825426	825491	825556	825621	825686
670	826075	826139	826204	826209	826334
671	826723	826787	826852	826917	826981
672	827369	827434	827499	827563	827628
673	828015	828079	828144	828209	828273
674	828659	828724	828789	828853	828918
675	829304	829368	829432	829497	829561
676	829947	830011	830075	830139	830204
677	830589	830653	830717	830781	830845
678	831229	831294	831358	831422	831486
679	831869	831934	831998	832062	832126
680	832509	832573	832637	832700	832764
681	833147	833211	833275	833338	833402
682	833784	833848	833912	833975	834039
683	834421	834484	834548	834611	834675
684	835056	835119	835183	835247	835310

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5	6	7	8	9	D
806519	806587	806655	806723	806790	68
807197	807264	807332	807399	807467	68
807873	807941	808008	808076	808143	68
808549	808616	808684	808751	808818	67
809223	809290	809358	809425	809492	67
809896	809964	810031	810098	810165	67
810569	810636	810703	810770	810837	67
811239	811307	811374	811441	811508	67
811909	811977	812044	812111	812178	67
812579	812646	812713	812779	812847	67
813247	813314	813381	813448	813514	67
813914	813981	814048	814114	814181	67
814581	814647	814714	814780	814847	67
815246	815312	815378	815445	815511	66
815909	815976	816042	816109	816175	66
816573	816639	816705	816771	816838	66
817235	817301	817367	817433	817499	66
817896	817962	818028	818094	818159	66
818556	818622	818688	818754	818819	66
819215	819281	819346	819412	819478	66
819873	819939	820004	820070	820136	66
820529	820595	820661	820727	820792	66
821186	821251	821317	821382	821448	66
821841	821906	821972	822037	822103	65
822495	822560	822626	822691	822756	65
823148	823213	823279	823344	823409	65
823800	823865	823930	823996	824061	65
824451	824516	824581	824646	824711	65
825101	825166	825231	825296	825361	65
825751	825815	825880	825945	826009	65
826399	826464	826528	826593	826658	65
827046	827111	827175	827239	827305	65
827692	827757	827822	827886	827951	65
828338	828402	828467	828531	828595	64
828982	829046	829111	829175	829239	64
829625	829689	829754	829818	829882	64
830268	830332	830396	830460	830525	64
830909	830973	831037	831102	831166	64
831549	831614	831678	831742	831806	64
832189	832253	832317	832381	832445	64
832828	832892	832956	833019	833083	64
833466	833529	833593	833657	833721	64
834103	834166	834229	834294	834357	64
834739	834802	834866	834929	834993	64
835373	835437	835500	835564	835627	63

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N	0	1	2	3	4
685	835691	835754	835817	835881	835944
686	836324	836387	836451	836514	836577
687	836957	837019	837083	837146	837209
888	837588	837652	837715	837777	837841
689	838219	838282	838345	838408	838471
690	838849	838912	838975	839038	839101
691	839478	839541	839604	839667	839729
692	840106	840169	840232	840294	840357
693	840733	840796	840859	840921	840984
694	841359	841423	841485	841547	841609
695	841985	842047	842109	842172	842235
696	842609	842672	842734	842796	842859
697	843233	843295	843357	843419	843482
698	843855	843918	843979	844042	844104
699	844477	844539	844601	844664	844726
700	845098	845160	845222	845284	845346
701	845718	845779	845842	845904	845966
702	846337	846399	846461	846523	846585
703	846955	847017	847079	847141	847202
704	847573	847634	847696	847758	847819
705	848189	848251	848312	848374	848435
706	848805	848866	848928	848989	849051
707	849419	849481	849542	849604	849665
708	850033	850095	850156	850217	850279
709	850646	850707	850769	850829	850891
710	851258	851319	851381	851442	851503
711	851869	851931	851992	852053	852114
712	852479	852541	852602	852663	852724
713	853089	853150	853211	853272	853333
714	853698	853759	853819	853881	853941
715	854306	854367	854428	854488	854549
716	854913	854974	855034	855095	855156
717	855519	855579	855640	855701	855761
718	856124	856185	856245	856306	856366
719	856729	856789	856849	856910	856970
720	857332	857393	857453	857513	857574
721	857935	857995	858056	858116	858176
722	858537	858597	858657	858718	858778
723	859138	859198	859258	859318	859379
724	859739	859799	859859	859918	859978
725	860338	860398	860458	860518	860578
726	860937	860996	861056	861116	861176
727	861534	861594	861654	861714	861773
728	862131	862191	862251	862310	862369
729	862728	862787	862847	862906	862966

5	6	7	8	9	D
836007	836071	836134	836197	836261	63
836641	836704	836767	836830	836894	63
837273	837336	837399	837462	837525	63
837904	837967	838030	838093	838156	63
838534	838597	838660	838723	838786	63
839164	839227	839289	839352	839415	63
839792	839855	839918	839981	840043	63
840419	840482	840545	840608	840671	63
841049	841109	841172	841234	841297	63
841672	841735	841797	841859	841922	63
842297	842359	842422	842484	842547	62
842921	842983	843046	843108	843170	62
843544	843606	843669	843731	843793	62
844166	844229	844291	844353	844415	62
844788	844849	844912	844974	845036	62
845408	845470	845532	845594	845656	62
846028	846089	846151	846213	846275	62
846646	846708	846769	846832	846894	62
847264	847326	847388	847449	847511	62
847881	847943	848004	848067	848128	62
848497	848559	848620	848682	848743	62
849112	849174	849235	849297	849358	61
849726	849788	849849	849911	849972	61
850339	850401	850462	850524	850585	61
850952	851014	851075	351136	851197	61
851564	851625	851686	851747	851809	61
852175	852236	852297	852358	852419	61
852785	852846	852907	852968	853029	61
853394	853455	853516	853577	853637	61
854002	854063	854124	854185	854245	61
854609	854670	854731	854792	854852	61
855216	855277	855337	855398	855459	61
855822	855882	855943	856003	856064	61
856427	856487	856548	856608	856668	60
857031	857091	857152	857212	857272	60
857634	857694	857755	857815	857875	60
858236	858297	858357	858417	858477	60
858838	858898	858958	859018	859078	60
859439	859499	859559	859619	859679	60
860038	860098	860158	860218	860278	60
860637	860697	860757	860817	860877	60
861236	861295	861355	861415	861475	60
861833	861893	861952	862012	862072	60
862429	862489	862549	862608	862668	60
863025	863085	863144	863204	863263	60

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N	0	1	2	3	4
730	863323	863382	863442	863501	863561
731	863917	863977	864036	864096	864155
732	864511	864570	864629	864689	864748
733	865104	865163	865222	865282	865341
734	865696	865755	865814	865874	865933
735	866287	866346	866405	866465	866524
736	866878	866937	866996	867055	867114
737	867467	867526	867585	867644	867703
738	868056	868115	868174	868233	868292
739	868643	868703	868762	868821	868879
740	869232	869290	869349	869408	869466
741	869818	869877	869935	869994	870053
742	870404	870462	870521	870579	870638
743	870989	871047	871106	871164	871223
744	871573	871631	871689	871748	871806
745	872156	872215	872273	872331	872389
746	872749	872797	872855	872913	872972
747	873321	873379	873437	873495	873553
748	873902	873959	874018	874076	874134
749	874482	874539	874598	874656	874714
750	875061	875119	875177	875235	875293
751	875639	875698	875756	875813	875871
752	876218	876276	876333	876391	876449
753	876795	876853	876910	876968	877026
754	877371	877429	877487	877544	877602
755	877947	878004	878062	878119	878177
756	878522	878579	878637	878694	878752
757	879096	879153	879211	879268	879325
758	879669	879726	879784	879841	879898
759	880242	880299	880356	880413	880471
760	880814	880871	880928	880985	881042
761	881385	881442	881499	881556	881613
762	881955	882012	882069	882126	882183
763	882525	882581	882638	882695	882752
764	883093	883150	883207	883264	883321
765	883661	883718	883775	883832	883888
766	884229	884285	884342	884399	884455
767	884795	884852	884909	884965	885022
768	885361	885418	885474	885531	885587
769	885926	885983	886039	886096	886152
770	886491	886547	886604	886659	886716
771	887054	887111	887167	887223	887279
772	887617	887674	887720	887786	887842
773	888179	888236	888292	888348	888404
774	888741	888797	888853	888909	888965

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5	6	7	8	9	D
863620	863679	863739	863799	863858	59
864214	864274	864333	864392	864452	59
864808	864867	864926	864985	865045	59
865400	865459	865519	865578	865637	59
865992	866051	866110	866169	866228	59
866583	866642	866701	866759	866819	59
867173	867232	867291	867349	867409	59
867762	867821	867879	867939	867998	59
868350	868409	868468	868527	868586	59
868938	868997	869056	869114	869173	59
869525	869584	869642	869701	869759	59
870111	870169	870228	870287	870345	59
870696	870755	870813	870872	870930	58
871281	871339	871398	871456	871515	58
871865	871923	871981	872039	872098	58
872448	872506	872564	872622	872681	58
873029	873088	873146	873204	873262	58
873611	873669	873727	873785	873844	58
874192	874249	874308	874366	874424	58
874772	874829	874888	874945	875003	58
875351	875409	875466	875524	875582	58
875929	875987	876045	876102	876160	58
876507	876564	876622	876679	876737	58
877083	877141	877199	877256	877314	58
877659	877717	877774	877832	877889	58
878234	878292	878349	878407	878464	57
878808	878866	878924	878981	879039	57
879382	879439	879497	879555	879612	57
879955	880013	880070	880127	880185	57
880527	880585	880642	880699	880756	57
881099	881156	881213	881271	881328	57
881669	881727	881784	881841	881898	57
882239	882297	882354	882411	882468	57
882809	882866	882923	882979	883037	57
883377	883434	883491	883548	883605	57
883945	884002	884059	884115	884172	57
884512	884569	884625	884682	884739	57
885078	885135	885192	885248	885305	57
885644	885700	885757	885813	885869	57
886209	886265	886321	886378	886434	56
886773	886829	886885	886941	886998	56
887336	887392	887449	887505	887561	56
887898	887955	888011	888067	888123	56
888460	888516	888573	888629	888685	56
889021	889077	889134	889189	889246	56

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N	0	1	2	3	4
775	889302	889358	889414	889469	889526
776	889862	889918	889974	890029	890086
777	890421	890477	890533	890589	890645
778	890979	891035	891091	891147	891203
779	891537	891593	891649	891705	891760
780	892095	892150	892206	892262	892317
781	892651	892707	892762	892818	892873
782	893207	893262	893318	893373	893429
783	893762	893817	893873	893928	893984
784	894316	894371	894427	894482	894538
785	894869	894925	894980	895036	895091
786	895423	895478	895533	895588	895644
787	895975	896029	896085	896140	896195
788	896526	896581	896636	896692	896747
789	897077	897132	897187	897242	897297
790	897627	897682	897737	897792	897847
791	898176	898231	898286	898341	898396
792	898725	898780	898835	898889	898944
793	899273	899328	899383	899437	899492
794	899821	899875	899929	899985	900039
795	900367	900422	900476	900531	900586
796	900913	900968	901022	901077	901131
797	901458	901513	901567	901622	901676
798	902003	902057	902112	902166	902221
799	902547	902601	902655	902709	902764
800	903089	903144	903199	903253	903307
801	903633	903687	903741	903795	903849
802	904174	904229	904282	904337	904391
803	904716	904769	904824	904878	904932
804	905256	905310	905364	905418	905472
805	905796	905849	905904	905958	906012
806	906335	906389	906443	906497	906551
807	906874	906927	906981	907035	907089
808	907411	907465	907519	907573	907626
809	907949	908002	908056	908109	908163
810	908485	908539	908592	908646	908699
811	909021	909074	909128	909181	909235
812	909556	909609	909663	909716	909769
813	910091	910144	910197	910251	910304
814	910624	910678	910731	910784	910838
815	911158	911211	911263	911317	911371
816	911690	911743	911797	911849	911903
817	912222	912275	912323	912381	912435
818	912753	912806	912859	912913	912966
819	913284	913337	913389	913443	913496

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5	6	7	8	9	D
889582	889638	889694	889749	889806	56
890141	890197	890253	890309	890365	56
890700	890756	890812	890868	890924	56
891259	891314	891370	891426	891482	56
891816	891872	891928	891983	892039	56
892373	892429	892484	892539	892595	56
892929	892985	893040	893096	893151	56
893484	893539	893595	893651	893706	56
894039	894094	894149	894205	894261	55
894593	894648	894704	894759	894814	55
895146	895201	895257	895312	895367	55
895699	895754	895809	895864	895919	55
896251	896306	896361	896416	896471	55
896802	896857	896912	896967	897022	55
897352	897407	897462	897517	897572	55
897902	897957	898012	898067	898122	55
898451	898506	898561	898615	898670	55
898999	899054	899109	899164	899218	55
899547	899602	899656	899711	899766	55
900094	900149	900203	900258	900312	55
900640	900695	900749	900804	900859	55
901186	901240	901295	901349	901404	55
901731	901785	901839	901894	901948	54
902275	902329	902384	902438	902492	54
902818	902873	902927	902981	903036	54
903361	903416	903469	903524	903578	54
903904	903958	904012	904066	904120	54
904445	904499	904553	904607	904661	54
904986	905039	905094	905148	905202	54
905526	905580	905634	905688	905742	54
906066	906119	906173	906227	906281	54
906604	906658	906712	906766	906819	54
907143	907196	907250	907304	907358	54
907680	907734	907787	907841	907895	54
908217	908270	908324	908378	908431	54
908753	908807	908860	908914	908967	54
909289	909342	909396	909449	909503	54
909823	909877	909930	909984	910037	53
910358	910411	910464	910518	910571	53
910891	910944	910998	911051	911104	53
911424	911477	911530	911584	911037	53
911956	912009	912063	912116	912169	53
912488	912541	912594	912647	912700	53
913019	913072	913125	913178	913231	53
913549	913602	913655	913708	913761	53

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820	913814	913867	913919	913973	914026
821	914343	914396	914449	914502	914555
822	914872	914925	914977	915030	915083
823	915399	915452	915505	915558	915611
824	915927	915979	916033	916085	916138
825	916454	916507	916559	916612	916664
826	916980	917033	917085	917138	917190
827	917506	917558	917611	917663	917716
828	918030	918083	918135	918188	918240
829	918555	918607	918659	918712	918764
830	919078	919130	919183	919235	919287
831	919601	919653	919706	919758	919810
832	920123	920176	920228	920279	920332
833	920645	920697	920749	920801	920853
834	921166	921218	921270	921322	921374
835	921686	921738	921790	921842	921894
836	922206	922258	922310	922362	922414
837	922725	922777	922829	922881	922933
838	923244	923296	923348	923399	923451
839	923762	923814	923865	923917	923969
840	924279	924331	924383	924434	924486
841	924796	924848	924899	924951	925003
842	925312	925364	925415	925467	925518
843	925828	925879	925931	925982	926034
844	926342	926394	926445	926497	926548
845	926857	926908	926959	927011	927062
846	927370	927422	927473	927524	927576
847	927883	927935	927986	928037	928088
848	928396	928447	928498	928549	928601
849	928908	928959	929009	929061	929112
850	929419	929470	929521	929572	929623
851	929929	929981	930032	930083	930134
852	930439	930491	930542	930592	930643
853	930949	930999	931051	931102	931153
854	931458	931509	931559	931610	931661
855	931966	932017	932068	932118	932169
856	932474	932524	932575	932626	932677
857	932981	933031	933082	933133	933183
858	933487	933538	933589	933639	933689
859	933993	934044	934094	934145	934195
860	934498	934549	934599	934649	934700
861	935003	935053	935104	935154	935205
862	935507	935558	935608	935658	935709
863	936011	936061	936111	936162	936212
864	936514	936564	936614	936665	936715

5	6	7	8	9	D
914079	914132	914184	914237	914290	53
914608	914660	914713	914766	914819	53
915136	915189	915241	915294	915347	53
915664	915716	915769	915822	915875	53
916191	916243	916296	916349	916401	53
916717	916769	916822	916875	916927	53
917243	917295	917348	917400	917453	53
917768	917820	917873	917925	917978	52
918293	918345	918397	918449	918502	52
918816	918869	918921	918973	919026	52
919339	919392	919444	919496	919549	52
919862	919914	919967	920019	920071	52
920384	920436	920489	920541	920593	52
920906	920958	921009	921062	921114	52
921426	921478	921530	921582	921634	52
921946	921998	922050	922102	922154	52
922466	922518	922569	922622	922674	52
922985	923037	923089	923140	923192	52
923503	923555	923607	923658	923710	52
924021	924072	924124	924176	924228	52
924538	924589	924641	924693	924744	52
925054	925106	925157	925209	925261	52
925569	925621	925673	925725	925776	52
926085	926137	926188	926239	926291	51
926599	926651	926702	926754	926805	51
927114	927165	927216	927268	927319	51
927627	927678	927729	927781	927832	51
928139	928191	928242	928293	928345	51
928652	928703	928754	928805	928857	51
929163	929215	929266	929317	929368	51
929674	929725	929776	929827	929879	51
930185	930236	930287	930338	930389	51
930694	930745	930796	930847	930898	51
931204	931254	931305	931356	931407	51
931712	931763	931814	931865	931915	51
932220	932271	932322	932372	932423	51
932727	932778	932829	932879	932930	51
933234	933285	933335	933386	933437	51
933740	933791	933841	933892	933943	51
934246	934296	934347	934397	934448	51
934751	934801	934852	934902	934953	50
935255	935306	935356	935406	935457	50
935759	935809	935859	935910	935960	50
936262	936313	936363	936413	936463	50
936765	936815	936865	936916	936966	50

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N	0	1	2	3	4
865	937016	937066	937117	937167	937217
866	937518	937568	937618	937668	937718
867	938019	938069	938119	938169	938219
868	938519	938569	938619	938669	938719
869	939019	939069	939119	939169	939219
870	939519	939569	939619	939669	939719
871	940018	940068	940118	940168	940218
872	940516	940566	940616	940666	940716
873	941014	941064	941114	941163	941213
874	941511	941561	941611	941660	941710
875	942008	942058	942107	942157	942207
876	942504	942554	942603	942653	942702
877	942999	943049	943099	943148	943198
878	943495	943544	943594	943643	943692
879	943989	944038	944088	944137	944186
880	944483	944532	944581	944631	944680
881	944976	945025	945074	945124	945173
882	945468	945518	945567	945616	945665
883	945961	946009	946059	946108	946157
884	946452	946501	946551	946599	946649
885	946943	946992	947041	947090	947139
886	947434	947483	947532	947581	947629
887	947924	947973	948022	948070	948119
888	948413	948462	948511	948559	948609
889	948902	948951	948999	949048	949097
890	949390	949439	949488	949536	949585
891	949878	949926	949975	950024	950073
892	950365	950414	950462	950511	950559
893	950851	950900	950949	950997	951046
894	951338	951386	951435	951483	951532
895	951823	951872	951920	951969	952017
896	952308	952356	952405	952453	952502
897	952792	952841	952889	952938	952986
898	953276	953325	953373	953421	953469
899	953759	953808	953856	953905	953953
900	954243	954291	954339	954387	954435
901	954725	954773	954821	954869	954918
902	955207	955255	955303	955351	955399
903	955688	955736	955784	955832	955880
904	956168	956216	956265	956313	956361
905	956649	956697	956745	956793	956840
906	957128	957176	957224	957272	957319
907	957607	957655	957703	957751	957799
908	958086	958134	958181	958229	958277
909	958564	958612	958659	958707	958755

5	6	7	8	9	D
937267	937317	937367	937418	937468	50
937769	937819	937869	937919	937969	50
938269	938319	938369	938419	938469	50
938769	938819	938869	938919	938969	50
939269	939319	939369	939419	939469	50
939769	939819	939869	939918	939968	50
940267	940317	940367	940417	940467	50
940765	940815	940865	940915	940964	50
941263	941313	941362	941412	941462	50
941759	941809	941859	941909	941958	50
942256	942306	942355	942405	942455	50
942752	942801	942851	942901	942950	50
943247	943297	943346	943396	943445	49
943742	943791	943841	943890	943939	49
944236	944285	944335	944384	944433	49
944729	944779	944828	944877	944927	49
945222	945272	945321	945370	945419	49
945715	945764	945813	945862	945912	49
946207	946256	946305	646354	946403	49
946698	946747	946796	946845	946894	49
947189	947238	947287	947336	947385	49
947679	947728	947777	947826	947875	49
948168	948217	948266	948315	948364	49
948657	948706	948755	948804	948853	49
949146	949195	949244	949292	949341	49
949633	949683	949731	949780	949829	49
950121	950170	950219	950267	950316	49
950608	950657	950706	950754	950803	49
951095	951143	951192	951240	951289	49
951580	951629	951677	951726	951775	49
952066	952114	952163	952211	952259	49
952550	952599	952647	952696	952744	48
953034	953083	953131	953179	953228	48
953518	953566	953615	953663	953711	48
954001	954049	954099	954146	954194	48
954484	954532	954580	954628	954677	48
954966	955014	955062	955110	955158	48
955447	955495	955543	955592	955639	48
955928	955976	956024	956072	956120	48
956409	956457	956505	956553	956601	48
956888	956936	956984	957032	957080	48
957368	957416	957464	957512	957559	48
957847	957894	957942	957990	958038	48
958325	958373	958421	958468	958516	48
958803	958850	958898	958946	958994	48

The Table of Logarithms.

N	0	1	2	3	4
910	959041	959089	959137	959185	959232
911	959518	959566	959614	959661	959709
912	959995	960042	960090	960138	960185
913	960471	960518	960566	960613	960661
914	960946	960994	961041	961089	961136
915	961421	961469	961516	961563	961611
916	961895	961943	961990	962038	962085
917	962369	962417	962464	962511	962559
918	962842	962889	962937	962985	963032
919	963315	963363	963410	963457	963504
920	963788	963835	963882	963929	963977
921	964259	964307	964354	964401	964448
922	964731	964778	964825	964872	964919
923	965202	965249	965296	965343	965389
924	965672	965719	965766	965813	965859
925	966142	966189	966239	966283	966329
926	966611	966658	966705	966752	966799
927	967079	967127	967173	967220	967267
928	967548	967595	967642	967688	967735
929	968016	968062	968109	968156	968202
930	968483	968529	968576	968623	968669
931	968949	968996	969043	969089	969136
932	969416	969463	969509	969556	969602
933	969882	969928	969975	970021	970068
934	970347	970393	970439	970486	970533
935	970812	970858	970904	970951	970997
936	971276	971322	971369	971415	971461
937	971739	971786	971832	971879	971925
938	972203	972249	972295	972342	972388
939	972666	972712	972758	972804	972851
940	973128	973174	973220	973266	973313
941	973589	973636	973682	973728	973774
942	974050	974097	974143	974189	974235
943	974512	974558	974604	974649	974695
944	974972	975018	975064	975109	975156
945	975432	975478	975524	975569	975616
946	975891	975937	975983	976029	976075
947	976349	976396	976442	976488	976533
948	676808	976854	976899	976946	976992
949	977266	977312	977358	977403	977449
950	977724	977769	977815	977861	977906
951	978181	978226	978272	978317	978363
952	978637	978683	978728	978774	978819
953	979093	979138	979184	979229	979275
954	979548	979594	979639	979685	979730

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5	6	7	8	9	D
959279	959328	959375	959423	959471	48
959757	959804	959852	959899	959947	48
960233	960280	960328	960376	960423	48
960709	960756	960804	960851	960899	48
961184	961231	961279	961326	961374	47
961658	961706	961753	961801	961848	47
962132	962179	962227	962275	962322	47
962606	962653	962701	962748	962795	47
963079	963126	963174	963221	963268	47
963552	963599	963646	963693	963741	47
964024	964071	964118	964165	964212	47
964495	964542	964589	964637	964684	47
964966	965013	965061	965108	965155	47
965437	965484	965531	965578	965624	47
965906	965954	966001	966048	966095	47
966376	966423	966470	966517	966564	47
966845	966892	966939	966986	967033	47
967314	967361	967408	967454	967501	47
967782	967829	967875	967922	967969	47
968249	968296	968343	968389	968436	47
968716	968763	968809	968856	968902	47
969183	969229	969276	969323	969369	47
969649	969695	969741	969789	969835	47
970114	970161	970207	970254	970300	47
970579	970626	970672	970719	970765	46
971044	971090	971137	971183	971229	46
971508	971554	971601	971647	971693	46
971971	972018	972064	972110	972157	46
972434	972481	972527	972573	972619	46
972897	972943	972989	973035	973082	46
973359	973405	973451	973497	973543	46
973820	973866	973913	973959	974005	46
974281	974327	974374	974419	974466	46
974742	974788	974834	974879	974926	46
975202	975248	975294	975339	975386	46
975662	975707	975753	975799	975845	46
976[illegible]21	976167	976212	976258	976304	46
976579	976625	976671	976717	976763	46
977037	977083	977129	977175	977220	46
977495	977541	977586	977632	977678	46
977952	977998	978042	978089	978135	46
978409	978454	978500	978546	978591	46
978865	978911	978956	979002	979047	46
979321	979366	979412	979457	979503	46
979776	979821	979867	979912	979958	46

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N	0	1	2	3	4
955	980003	980049	980094	980139	980185
956	980458	980503	980549	980594	980639
957	980912	980957	981003	981048	981093
958	981366	981411	981456	981501	981547
959	981819	981864	981909	981954	981999
960	982271	982316	982362	982407	982452
961	982723	982769	982814	982859	982904
962	983175	983220	983265	983310	983356
963	983626	983671	983716	983762	983807
964	984077	984122	984167	984212	984257
965	984527	984572	984617	984662	984707
966	984977	985022	985067	985112	985157
967	985426	985471	985516	985561	985606
968	985875	985920	985965	986009	986055
969	986324	986369	986414	986458	986504
970	986772	986817	986861	986906	986951
971	987219	987264	987309	987353	987398
972	987666	987711	987756	987800	987845
973	688113	988157	988202	988247	988291
974	988559	988604	988748	988693	988737
975	989005	989049	989094	989138	989183
976	989449	989494	989539	989584	989628
977	989895	989939	989983	990028	990072
978	990339	990383	990428	990472	990516
979	990783	990827	990871	990916	990960
980	991226	991270	991315	991359	991403
981	991669	991713	991758	991802	991846
982	992111	992156	992199	992244	992288
983	992554	992598	992642	992686	992730
984	992995	993039	993083	993127	993172
985	993436	993480	993524	993568	993613
986	993877	993921	993965	994009	994053
987	994317	994361	994405	994449	994493
988	994756	994801	994845	994889	994933
989	995196	995240	995284	995328	995372
990	995035	995679	995723	995764	995811
991	996074	996117	996161	996205	996249
992	996512	996555	996599	996643	996687
993	996949	996993	997037	997080	997124
994	997386	997430	997474	997517	997561
995	997823	997867	997910	997954	997998
996	998259	998303	998347	998390	998434
997	998695	998739	998783	998826	998869
998	999133	999176	999218	999261	999305
999	999565	999609	999652	999696	99973

5	6	7	8	9	D
980231	980276	980322	980367	980412	45
980685	980730	980776	980821	980867	45
981139	981184	981229	981275	981320	45
981592	981637	981683	981728	981773	45
982045	982090	982135	982181	982226	45
982497	982543	982588	982633	982678	45
982949	982994	983039	983085	983129	45
983401	983446	983490	983536	983581	45
983852	983897	983942	983987	984032	45
984302	984347	984392	984437	984482	45
984752	984797	984842	984887	984932	45
985202	985247	985292	985337	985382	45
985651	985696	985741	985786	985830	45
986099	986144	986189	986234	986279	45
986548	986593	986637	986682	986727	45
986996	987040	987085	987129	987175	45
987443	987488	987532	987577	987622	45
987889	987934	987979	988024	988068	45
988336	988381	988425	988469	988514	45
988782	988826	988871	988916	988960	45
989227	989272	989316	989361	989405	45
989672	989717	989761	989806	989850	44
990117	990161	990206	980250	990294	44
990561	990605	990649	990694	990738	44
991004	991049	991093	991137	991182	44
991448	991492	991530	991580	991625	44
991890	991935	991979	992023	992067	44
992333	992377	992421	992465	992509	44
992774	992819	992863	992907	992951	44
993216	993259	993304	993348	993392	44
993657	993701	993745	993789	993833	44
994097	994141	994185	994229	994273	44
994537	994581	994625	994669	994713	44
994977	995021	995065	995108	995152	44
995416	995459	995504	995547	995591	44
995854	995898	995942	995986	996029	44
996293	996337	996380	996424	996468	44
996731	996774	996818	996862	996906	44
997168	997212	997255	997299	997343	44
997605	997648	997692	997736	997779	44
998041	998085	998129	998172	998216	44
998477	998521	998564	998608	998652	44
998913	998956	998999	999043	999087	44
999348	999392	999435	999479	999522	44
999783	999826	999869	999913	999957	43

A

TABLE

OF

Artificial Sines and Tangents

To every

DEGREE and MINUTE

OF THE

QUADRANT.

The Common Radius being 10,000000.

The Table of Sines and Tangents.

Degree o.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	0.000000	10.000000	0.000000	*Infinita.*	60
1	6.463726	9.999999	6.463726	13.536274	59
2	6.764756	9.999999	6.764756	13.235244	58
3	6.940847	9.999999	6.940847	13.059153	57
4	7.065786	9.999999	7.065786	12.934214	56
5	7.162696	9.999999	7.162696	12.837304	55
6	7.241877	9.999999	7.241878	12.758122	54
7	7.308824	9.999999	7.308825	12.691175	53
8	7.366816	9.999999	7.366817	12.633183	52
9	7.417968	9.999999	7.417970	12.582030	51
10	7.463726	9.999998	7.463727	12.536273	50
11	7.505118	9.999998	7.505120	12.494880	49
12	7.542906	9.999997	7.542909	12.457091	48
13	7.577668	9.999997	7.577672	12.422328	47
14	7.609853	9.999996	7.609857	12.390143	46
15	7.639816	9.999996	7.639820	12.360180	45
16	7.667844	9.999995	7.667849	12.332151	44
17	7.694173	9.999995	7.694179	12.305821	43
18	7.718997	9.999994	7.719003	12.280997	42
19	7.742477	9.999993	7.742484	12.257516	41
20	7.764754	9.999993	7.764761	12.235239	40
21	7.785943	9.999992	7.785951	12.214049	39
22	7 806146	9.999991	7.806155	12 193845	38
23	7.825451	9.999990	7.825460	12.174540	37
24	7.843934	9.999989	7.843944	12.156056	36
25	7.861662	9.999989	7.861674	12.138326	35
26	7.878695	9.999988	7.878708	12.121292	34
27	7.895085	9.999987	7.895099	12.104901	33
28	7.910879	9.999986	7.910894	12.089106	32
29	7.926119	9.999985	7.926134	12.073866	31
30	7.940842	9.999983	7.940858	12.059142	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 89.

The Table of Sines and Tangents.

Degree 0.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	7.940842	9.999983	7.940858	12.059142	30
31	7.955082	9.999982	7.955100	12.044900	29
32	7.968870	9.999981	7.968889	12.031111	28
33	7.982233	9.999980	7.982253	12.017747	27
34	7.995198	9.999978	7.995219	12.004781	26
35	8.007787	9.999978	8.007810	11.992191	25
36	8.020021	9.999976	8.020044	11.979956	24
37	8.031919	9.999975	8.031945	11.968055	23
38	8.043501	9.999973	8.043527	11.956473	22
39	8.054781	9.999972	8.054809	11.945191	21
40	8.065776	9.999971	8.065806	11.934194	20
41	8.076500	9.999969	8.076531	11.923469	19
42	8.086965	9.999968	8.086997	11.913003	18
43	8.097183	9.999966	8.097217	11.902783	17
44	8.107167	9.999964	8.107203	11.892797	16
45	8.116926	9,999963	8.116963	11.883037	15
46	8.126471	9.999961	8.126510	11.873490	14
47	8.135810	9.999959	8.135851	11.864149	13
48	8.144953	9.999958	8.144996	11.855004	12
49	8.153907	9.999956	8.153952	11.846048	11
50	8.162681	9.999954	8.162727	11.837273	10
51	8.171280	9.999952	8.171328	11.828672	9
52	8.179713	9.999950	8.179763	11.820237	8
53	8.187985	9.999948	8.188036	11.811964	7
54	8.196102	9.999946	8.196156	11.803844	6
55	8.204070	9.999944	8.204126	11.795874	5
56	8.211895	9.999942	8.211953	11.788047	4
57	8.219581	9.999940	8.219641	11.780359	3
58	8.227134	9.999938	8.227195	11.772805	2
59	8.234557	9.999936	8.234621	11.765379	1
60	8.241855	9.999934	8.241921	11.758079	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 89.

The Table of Sines and Tangents.

Degree 1.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	8.241855	9.999934	8.241921	11.758079	60
1	8.249033	9.999932	8.249102	11.750898	59
2	8.256094	9.996929	8.256165	11.743835	58
3	8.263042	9.999927	8.263115	11.736885	57
4	8.269881	9.999925	8.269956	11.730044	56
5	8.276614	9.999922	8.276691	11.723309	55
6	8.283243	9.999920	8.283323	11.716677	54
7	8.289773	9.999918	8.289856	11.716144	53
8	8.296207	9.999915	8.296292	11.703708	52
9	8.302546	9.999913	8.302634	11.697366	51
10	8.308794	9.999910	8.308884	11.691116	50
11	8.314954	9.999907	8.315046	11.684954	49
12	8.321027	9.999905	8.321122	11 678878	48
13	8.327016	9.999902	8.327114	11.672886	47
14	8.332924	9.999899	8.333025	11.666975	46
15	8.338753	9.999897	8.338856	11.661144	45
16	8.344504	9.999894	8.344610	11.655390	44
17	8.350180	9.999891	8.350289	11.649711	43
18	8.355783	9.999888	8.355895	11.644105	42
19	8.361315	9.999885	8.361430	11.638570	41
20	8.366777	9.999882	8.366895	11.633105	40
21	8.372171	9.999879	8.372292	11.627708	39
22	8.377499	9.999876	8.377622	11.622378	38
23	8.382762	9.999873	8.382889	11,617111	37
24	8.387962	9.999870	8.388092	11.611908	36
25	8.393101	9.999867	8.393234	11.606766	35
26	8.398179	9.999864	8.398315	11.601685	34
27	8.403199	9.999861	8.403338	11.596662	33
28	8.408161	9.999858	8.408304	11.591696	32
29	8.413068	9.999854	8.413213	11.586787	31
30	8.417919	9.999851	8.418068	11.581932	30
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 88.

The Table of Sines and Tangents.

Degree 1.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
30	8.417919	9.999851		8.418068	11.581932	30
31	8.422717	9.999848		8.422869	11.577131	29
32	8.427462	9.999844		8.427618	11.572382	28
33	8.432156	9.999841		8.432315	11.567685	27
34	8.436800	9.999838		8.436962	11.563038	26
35	8.441394	9.999834		8.441560	11.558440	25
36	8.445941	9.999831		8.446110	11.553990	24
37	8.450440	9.999827		8.450613	11.549387	23
38	8.454893	9.999824		8 455070	11.544930	22
39	8.459301	9.999820		8.459481	11.540519	21
40	8.463665	9.999816		8.463849	11.536151	20
41	8.467985	9.999812		8.468172	11.531828	19
42	8.472263	9.999809		8.472454	11.527546	18
43	8.476498	9.999805		8.476693	11.523307	17
44	8.480693	9.999801		8.480892	11.519108	16
45	8.484848	9.999797		8.485050	11.514950	15
46	8.488963	9.999794		8.489170	11.510830	14
47	8.493040	9.999790		8.493250	11.506750	13
48	8.497078	9.999786		8.497293	11.502707	12
49	8.501080	9.999782		8.501298	11.498702	11
50	8.505045	9.999778		8.505267	11.494733	10
51	8.508974	9.999774		8.509200	11.490800	9
52	8.512867	9.999769		8.513098	11.486902	8
53	8.516726	9.999765		8.516961	11.483039	7
54	8.520551	9.999761		8.520790	11.479210	6
55	8.524343	9.999756		8.524586	11.475414	5
56	8.528102	9.999753		8.528349	11.471651	4
57	8.531828	9.999748		8.532080	11.467920	3
58	8.535523	9.999744		8.535779	11.464221	2
59	8.539186	9.999740		8.539447	11.460553	1
60	8.542819	9.999735		8.543084	11.456916	0
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 88.

The Table of Sines and Tangents.

Degree 2.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	8.542819	9.999735	8.543084	11.456916	60
1	8.546422	9.999731	8.546691	11.453309	59
2	8.549995	9.999726	8.550268	11.449732	58
3	8.553538	9.999722	8.553817	11.446183	57
4	8.557054	9.999717	8.557336	11.442664	56
5	8.560540	9.999713	8.560827	11.439172	55
6	8.563999	9.999708	8.564291	11.435709	54
7	8.567431	9.999703	8.567727	11.432272	53
8	8.570836	9.999699	8.571137	11.428863	52
9	8.574214	9.999694	8.574520	11.425480	51
10	8.577566	9.999689	8.577877	11.422123	50
11	8.580892	9.999685	8.581208	11.418792	49
12	8.584193	9.999680	8.584514	11.415486	48
13	8.587469	9.999675	8.587795	11.412205	47
14	8.590721	9.999670	8.591051	11.408949	46
15	8.593948	9.999665	8.594283	11.405717	45
16	8.597152	9.999660	8.597492	11.402508	44
17	8.600332	9.999655	8.600677	11.399323	43
18	8.603488	9.999650	8.603838	11.396161	42
19	8.606622	9.999645	8.606978	11.393022	41
20	8.609734	9.999640	8.610094	11.389906	40
21	8.612823	9.999635	8.613189	11.386811	39
22	8.615891	9.999629	8.616262	11.383738	38
23	8.618937	9.999624	8.619313	11.380687	37
24	8.621962	9.999619	8.622343	11.377657	36
25	8.624965	9.999614	8.625352	11.374648	35
26	8.627948	9.999608	8.628340	11.371660	34
27	8.630911	9.999603	8.631308	11.368692	33
28	8.633854	9.999597	8.634456	11.365744	32
29	8.636776	9.999592	8.637184	11.362816	31
30	8.639680	9.999586	8.640093	11.359907	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 87.

The Table of Sines and Tangents.

Degree 2.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
30	8.639679	9.999586		8.640093	11.359907	30
31	8.642563	9.999581		8.642982	11.357017	29
32	8.645428	9.999575		8.645853	11.354147	28
33	8.648274	9.999570		8.648704	11.351296	27
34	8.651102	9.999564		8.651538	11.348463	26
35	8.653911	9.999558		8.654352	11.345648	25
36	8.656702	9.999553		8.657149	11.342851	24
37	8.659475	9.999547		8.659928	11.340072	23
38	8.662230	9.999541		8.662689	11.337311	22
39	8.664968	9.999535		8.665433	11.334567	21
40	8.667689	9.999529		8.668160	11.331840	20
41	8.670393	9.999523		8.670869	11.329130	19
42	8.673080	9.999518		8.673563	11.326437	18
43	8.675751	9.999512		8.676239	11.323761	17
44	8.678405	9.999506		8.678899	11.921100	16
45	8.681043	9.999499		8.641544	11.318456	15
46	8.683665	9.999493		8.684172	11.315828	14
47	8.686272	9.999487		8.686784	11.313216	13
48	8.688862	9.999481		8.689381	11.310619	12
49	8.691438	9.999475		8.691963	11.308037	11
50	8.693998	9.999469		8.694529	11.305471	10
51	8.696543	9.999462		8.697081	11.302919	9
52	8.699073	9.999456		8.699617	11.300383	8
53	8.701589	9.999450		8.702139	11.297861	7
54	8.704090	9.999443		8.704646	11.295354	6
55	8.706576	9.999437		8.707139	11.292860	5
56	8.709049	9.999431		8.709618	11.290381	4
57	8.711507	9.999424		8.712083	11.287917	3
58	8.713952	9.999418		8.714534	11.285466	2
59	8.716383	9.999411		8.716972	11.283028	1
60	8.718800	9.999404		8.719396	11.280604	0
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 87.

The Table of Sines and Tangents.

Degree 3.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	8.718800	9.999404	8.719396	11.280604	60
1	8.721204	9.999398	8.721806	11.278194	59
2	8.723595	9.999391	8.724204	11.275796	58
3	8.725972	9.999384	8.726588	11.273412	57
4	8.728336	9.999378	8.728959	11.271041	56
5	8.730688	9.999371	8.731317	11.268683	55
6	8.733027	9.999364	8.733663	11.266337	54
7	8.735354	9.999357	8.735996	11.264004	53
8	8.737667	9.999350	8.738317	11.261683	52
9	8.739969	9.999343	8.740626	11.259374	51
10	8.742259	9.999336	8.742922	11.257078	50
11	8.744536	9.999329	8.745207	11.254793	49
12	8.746801	9.999322	8.747479	11.252521	48
13	8.749055	9.999315	8.749740	11.250260	47
14	8.751297	9.999308	8.751989	11.248011	46
15	8.753528	9.999301	8.754227	11.245773	45
16	8.755747	9.999294	8.756453	11.243547	44
17	8.757955	9.999286	8.758668	11.241332	43
18	8.760151	9.999279	8.760872	11.239128	42
19	8.762337	9.999272	8.763065	11.236935	41
20	8.764511	9.999265	8.765246	11.234754	40
21	8.766675	9.999257	8.767417	11.232583	39
22	8.768828	9.999250	8.769578	11.230422	38
23	8.770970	9.999242	8.771727	11.228273	37
24	8.773101	9.999235	8.773866	11.229134	36
25	8.775223	9.999227	8.775995	11.224005	35
26	8.777333	9.999220	8.778114	11.221886	34
27	8.779434	9.999212	8.783222	11.219778	33
28	8.781524	9.999204	8.782320	11.217680	32
29	8.783605	9.999197	8.784408	11.215592	31
30	8.785675	9.999189	8.786486	11.213514	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 86.

Degree 3.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	8.785675	9.999189	8.786486	11.213514	30
31	8.787736	9.999181	8.788554	11.211446	29
32	8.789787	9.999174	8.790613	11.209387	28
33	8.791828	9.999166	8.792662	11.207338	27
34	8.793859	9.999158	8.794701	11.205299	26
35	8.795881	9.999150	8.796731	11.203269	25
36	8.797894	9.999142	8.798752	11.201248	24
37	8.799897	9.999134	8.800763	11.199237	23
38	8.801891	9.999126	8.802765	11.197235	22
39	8.803876	9.999118	8.804758	11.195242	21
40	8.805852	9.999110	8.806742	11.193258	20
41	8.807819	9.999102	8.808717	11.191283	19
42	8.809777	9.999094	8.810683	11.189317	18
43	8.811726	9.999086	8.812641	11.187359	17
44	8.813667	9.999077	8.814589	11.185411	16
45	8.815598	9.999069	8.816529	11.183471	15
46	8.817522	9.999061	8.818461	11.181539	14
47	8.819436	9.999052	8.820384	11.179616	13
48	8.821342	9.999044	8.822298	11.177702	12
49	8.823240	9.999036	8.824205	11.175795	11
50	8.825130	9.999027	8.826103	11.173897	10
51	8.827011	9.999019	8.827992	11.172008	9
52	8.828884	9.999010	8.829874	11.170126	8
53	8.830749	9.999002	8.831748	11.168252	7
54	8.832606	9.998993	8.833613	11.166387	6
55	8.834456	9.998984	8.835471	11.164529	5
56	8.836297	9.998976	8.837321	11.162679	4
57	8.838130	9.998967	8.839163	11.160837	3
58	8.839956	9.998958	8.840998	11.159002	2
59	8.841774	9.998950	9.842825	11.157175	1
60	8.843585	9.998951	8.844644	11.155356	0
	Co-ſine	Sine.	*Co-tang.*	Tangent.	M

Degree 86.

The Table of Sines and Tangents.

Degree 4.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	8.843584	9.998941	8.844644	11.155356	60
1	8.845387	9.998931	8.846455	11.153545	59
2	8.847183	9.998923	8.848256	11.151740	58
3	8.848971	9.998914	8.850057	11.149943	57
4	8.850751	9.998905	8.851846	11.148154	56
5	8.852525	9.998896	8.853628	11.146372	55
6	8.854291	9.998887	8.855403	11.144597	54
7	8.856049	9.998878	8.857171	11.142829	53
8	8.857801	9.998869	8.858932	11.141068	52
9	8.859546	9.998860	8.860686	11.139314	51
10	8.861283	9.998851	8.862433	11.137567	50
11	8.863014	9.998841	8.864173	11.135827	49
12	8.864738	9.998832	8.865906	11.134094	48
13	8.866454	9.998823	8.867632	11.132368	47
14	8.868165	9.998813	8.869351	11.130649	46
15	8.869868	9.998804	8.871064	11.128936	45
16	8.871565	9.998795	8.872770	11.127230	44
17	8.873255	9.998785	8.874469	11.125531	43
18	8.874938	9.998776	8.876162	11.123838	42
19	8.876615	9.998766	8.877849	11.122151	41
20	8.878285	9.998757	8.879529	11.120471	40
21	8.879949	9.998747	8.881202	11.118798	39
22	8.881607	9.998738	8.882869	11.117131	38
23	8.883258	9.998728	8.884530	11.115470	37
24	8.884903	9.998718	8.886185	11.113815	36
25	8.886542	9.998708	8.887833	11.112167	35
26	8.888174	9.998699	8.889476	11.110524	34
27	8.889801	9.998689	8.891112	11.108888	33
28	8.891421	9.998679	8.892742	11.107258	32
29	8.893035	9.998669	8.894366	11.105634	31
30	8.894643	9.998659	8.895984	11.104016	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 85.

The Table of Sines and Tangents.

Degree 4.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	8.894643	9.998659	8.895984	11.104016	30
31	8.896246	9.998649	8.897596	11.102404	29
32	8.897842	9.998639	8.899203	11.100797	28
33	8.899432	9.998629	8.900803	11.099197	27
34	8.901017	9.998619	8.902398	11.097602	26
35	8.902596	9.998609	8.903987	11.096013	25
36	8.904169	9.998599	8.905570	11.094430	24
37	8.905736	9.998589	8.907147	11.092853	23
38	8.907297	9.998577	8.908719	11.091281	22
39	8.908853	9.998568	8.910285	11.089715	21
40	8.910404	9.998558	8.911846	11.088154	20
41	8.911949	9.998548	8.913401	11.086599	19
42	8.913488	9.998537	8.914951	11.085049	18
43	8.915022	9.998527	8.916495	11.083505	17
44	8.916550	9.998516	8.918034	11.081966	16
45	8.918073	9.998506	8.919568	11.080432	15
46	8.919591	9.998495	8.921096	11.078904	14
47	8.921103	9.998485	8.922619	11.077381	13
48	8.922610	9.998474	8.924136	11.075864	12
49	8.924112	9.998464	8.925649	11.074351	11
50	8.925609	9.998453	8.927156	11.072844	10
51	8.927100	9.998442	8.928658	11.071342	9
52	8.928587	9.998431	8.930155	11.069845	8
53	8.930068	9.998421	8.931647	11.068353	7
54	8.931544	9.998410	8.933134	11.066866	6
55	8.933015	9.998399	8.934616	11.065384	5
56	8.934481	9.998388	8.936093	11.063907	4
57	8.935942	9.998377	8.937565	11.062435	3
58	8.937398	9.998366	8.939032	11.060968	2
59	8.938850	9.998355	8.940494	11.059506	1
60	8.940296	9.998344	8.941952	11.058048	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 85.

The Table of Sines and Tangents.

Degree 5.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	8.940296	9.998344	8.941952	11.058048	60
1	8.941738	9.998333	8.943404	11.056596	59
2	8.943174	9.998322	8.944852	11.055148	58
3	8.944606	9.998311	8.946295	11.053705	57
4	8.946034	9.998300	8.947734	11.052266	56
5	8.947456	9.998289	8.949168	11.050832	55
6	8.948874	9.998277	8.950597	11.049403	54
7	8.950287	9.998266	8.952021	11.047979	53
8	8.951696	9.998255	8.953441	11.046559	52
9	8.953099	9.998243	8.954856	11.045144	51
10	8.954499	9.998232	8.956267	11.043733	50
11	8.955894	9.998220	8.957674	11.042326	49
12	8.957284	9.998209	8.959075	11.040925	48
13	8.958670	9.998197	8.960473	11.039527	47
14	8.960052	9.998186	8.961866	11.038134	46
15	8.961429	9.998174	8.963254	11.036746	45
16	8.962801	9.998163	8.964639	11.035361	44
17	8.964170	9.998151	8.966019	11.033981	43
18	8.965534	9.998139	8.967394	11.032606	42
19	8.966893	9.998128	8.968766	11.031234	41
20	8.968249	9.998116	8.970133	11.029867	40
21	8.969600	9.998104	8.971495	11.028505	39
22	8.970947	9.998092	8.972855	11.027145	38
23	8.972289	9.998080	8.974209	11.025791	37
24	8.973626	9.998068	8.975560	11.024440	36
25	8.974962	9.998056	8.976906	11.023094	35
26	8.976293	9.998044	8.978248	11.021752	34
27	8.977619	9.998032	8.979586	11.020414	33
28	8.978941	9.998020	8.980921	11.019079	32
29	8.980259	9.998008	8.982251	11.017749	31
30	8.981573	9.997996	8.983577	11.016423	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 84.

The Table of Sines and Tangents.

Degree 5.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
30	8.981573	9.997996		8.983577	11.016423	30
31	8.982883	9.997984		8.984899	11.015101	29
32	8.984189	9.997971		8.986217	11.013783	28
33	8.985491	9.997959		8.987532	11.012468	27
34	8.986789	9.997947		8.988842	11.011158	26
35	8.988083	9.997935		8.990149	11.009851	25
36	8.989374	9.997922		8.991451	11.008549	24
37	8.990660	9.997910		8.992750	11.007250	23
38	8.991943	9.997897		8.994045	11.005955	22
39	8.993222	9.997885		8.995337	11.004663	21
40	8 994497	9.997873		8.996624	11.003376	20
41	8.995768	9.997860		8.997908	11.002092	19
42	8.997036	9.997847		8.999188	11.000812	18
43	8.998299	9.997835		9.000465	10.999535	17
44	8.999560	9.997822		9.001738	10.998262	16
45	9.000816	9.997809		9.003007	10.996993	15
46	9.002069	9.997797		9.004272	10.995728	14
47	9.003318	9.997784		9.005534	10.994466	13
48	9.004563	9.997771		9.006792	10.993208	12
49	9.005805	9.997758		9.008047	10.991953	11
50	9.007044	9.997745		9.009298	10.990702	10
51	9.008278	9.997732		9.010546	10.989454	9
52	9.009510	9.997719		9.011790	10.988210	8
53	9.010737	9.997706		9.013031	10.986969	7
54	9.011962	9.997693		9.014268	10.985732	6
55	9.013182	9.997680		9.015502	10.984498	5
56	9.014399	9.997667		9.016732	10.983268	4
57	9.015613	9.997654		9.017959	10.982041	3
58	9.016824	9.997641		9.019183	10.980817	2
59	9.018031	9.997628		9.020403	10.979597	1
60	9.019235	9.997614		9.021620	10.978380	0
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 84.

The Table of Sines and Tangent.

Degree 6.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
0	9.019235	9.997614		9.021620	10.978380	60
1	9.020435	9.997601		9.022834	10.977166	59
2	9.021632	9.997588		9.024044	10.975956	58
3	9.022825	9.997574		9.025251	10.974749	57
4	9.024016	9.997561		9.026455	10.973545	56
5	9.025203	9.997548		9.027655	10.972345	55
6	9.026386	9.997534		9.028852	10.971148	54
7	9.027567	9.997520		9.030046	10.969954	53
8	9.028744	9.997507		9.031237	10.968763	52
9	9.029918	9.997493		9.032425	10.967575	51
10	9.031089	9.997480		9.033609	10.966391	50
11	9.032257	9.997466		9.034791	10.965209	49
12	9.033421	9.997452		9.035969	10.964031	48
13	9.034582	9.997439		9.037144	10.962856	47
14	9.035741	9.997425		9.038316	10.961684	46
15	9.036896	9.997411		9.039485	10.960515	45
16	9.038048	9.997397		9.040651	10.959349	44
17	9.039197	9.997383		9.041813	10.958187	43
18	9.040342	9.997369		9.042973	10.957027	42
19	9.041485	9.997355		9.044130	10.955870	41
20	9.042625	9.997341		9.045284	10.954716	40
21	9.043762	9.997327		9.046434	10.953566	39
22	9.044895	9.997313		9.047582	10.952418	38
23	9.046026	9.997299		9.048727	10.951273	37
24	9.047154	9.997285		9.049869	10.950131	36
25	9.048279	9.997271		9.051008	10.948992	35
26	9.049400	9.997256		9.052144	10.947856	34
27	9.050519	9.997242		9.053277	10.946723	33
28	9.051635	9.997228		9.054408	10.945593	32
29	9.052749	9.997214		9.055535	10.947465	31
30	9.053859	9.997199		9.056660	10.943340	30
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 83.

The Table of Sines and Tangents.

Degree 6.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
30	9.053859	9.997199	9.056660	10.943340	30
31	9.054966	9.997185	9.057781	10.942219	29
32	9.056071	9.997170	9.058900	10.941100	28
33	9.057172	9.997156	9.060016	10.939984	27
34	9.058271	9.997141	9.061130	10.938870	26
35	9.059367	9.997127	9.062240	10.937760	25
36	9.060460	9.997112	9.063348	10.936652	24
37	9.061551	9.997098	9.064453	10.935547	23
38	9.062638	9.997083	9.065556	10.934444	22
39	9.063723	9.997068	9.066655	10.933345	21
40	9.064806	9.997053	9.067752	10.932248	20
41	9.065885	9.997039	9.068847	10.931153	19
42	9.066962	9.997024	9.069938	10.930062	18
43	9.068036	9.997009	9.071027	10.928973	17
44	9.069107	9.996994	9.072113	10.927887	16
45	9.070176	9.996979	9.073197	10.926803	15
46	9.071242	9.996964	9.074278	10.925722	14
47	9.072306	9.996949	9.075356	10.924644	13
48	9.073366	9.996934	9.076432	10.923568	12
49	9.074424	9.996919	9.077505	10.922495	11
50	9.075480	9.996904	9.078576	10.921424	10
51	9.076533	9.996889	9.079644	10.920356	9
52	9.077583	9.996874	9.080710	10.919290	8
53	9.078631	9.996858	9.081773	10.918227	7
54	9.079676	9.996843	9.082833	10.917167	6
55	9.080719	9.996828	9.083891	10.916109	5
56	9.081759	9.996812	9.084947	10.915053	4
57	9.082797	9.996797	9.085999	10.914000	3
58	9.083832	9.996782	9.087050	10.912950	2
59	9.084864	9.996766	9.088098	10.911902	1
60	9.085894	9.996751	9.089144	10.910856	0
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 83.

Degree 7.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.085894	9.996751	9.089144	10.910856	60
1	9.086922	9.996735	9.090187	10.909813	59
2	9.087947	9.996720	9.091228	10.908772	58
3	9.088970	9.996704	9.092266	10.907734	57
4	9.089990	9.996688	9.093302	10.906698	56
5	9.091008	9.996673	9.094336	10.905664	55
6	9.092024	9.996657	9.095367	10.904633	54
7	9.093037	9.996641	9.096395	10.903604	53
8	9.094047	9.996625	9.097422	10.902578	52
9	9.095056	9.996610	9.098446	10.901554	51
10	9.096062	9.996594	9.099468	10.900532	50
11	9.097065	9.999578	9.100487	10.899513	49
12	9.098066	9.996562	9.101504	10.898496	48
13	9.099065	9.996546	9.102519	10.897481	47
14	9.100062	9.996530	9.103532	10.896468	46
15	9.101056	9.996514	9.104542	10.895458	45
16	9.102048	9.996498	9.105550	10.894450	44
17	9.103037	9.996482	9.106556	10.893444	43
18	9.104025	9.996465	9.107559	10.892441	42
19	9.105010	9.996449	9.108560	10.891440	41
20	9.105992	9.996433	9.109559	10.890441	40
21	9.106973	9.996417	9.110556	10.889444	39
22	9.107951	9.996400	9.111551	10.888449	38
23	9.108927	9.996384	9.112543	10.887457	37
24	9.109901	9.996368	9.113533	10.886467	36
25	9.110873	9.996351	9.114521	10.885478	35
26	9.111842	9.996335	9.115507	10.884493	34
27	9.112809	9.996318	9.116491	10.883509	33
28	9.113774	9.996302	9.117472	10.882528	32
29	9.114737	9.996285	9.118452	10.881548	31
30	9.115698	9.996269	9.119429	10.880571	30
	Co-ſine.	Sine.	*Co-tang:*	Tangent.	M

Degree 82.

The Table of Sines and Tangents.

Degree 7.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.115698	9.996269	9.119429	10.880571	30
31	9.116656	9.996252	9.120404	10.879596	29
32	9.117612	9.996235	9.121377	10.878623	28
33	9.118567	9.996218	9.122348	10.877652	27
34	9.119519	9.996202	9.123317	10.876683	26
35	9.120469	9.996185	9.124284	10.875716	25
36	9.121417	9.996168	9.125248	10.874751	24
37	9.122362	9.996151	9.126211	10.873789	23
38	9.123306	9.996134	9.127172	10 872828	22
39	9.124248	9.996117	9.128130	10.871870	21
40	9.125187	9.996100	9.129087	10.870913	20
41	9.126125	9.996083	9.130041	10.869959	19
42	9.127060	9.996066	9.130994	10.869006	18
43	9.127993	9.996049	9.131944	10.868056	17
44	9.128925	9.996032	9.132893	10.867107	16
45	9.129854	9.996015	9.133839	10.866161	15
46	9.130781	9.995998	9.134784	10.865216	14
47	9.131706	9.995980	9.135726	10.864274	13
48	9.132630	9.995963	9.136666	10.863334	12
49	9.133551	9.995946	9.137605	10.862395	11
50	9.134470	9.995928	9.138542	10.861458	10
51	9.135387	9.995911	9.139476	10.860524	9
52	9.136303	9.995894	9.140409	10.859591	8
53	9.137216	9.995876	9.141340	10.858660	7
54	9.138127	9.995859	9.142269	10.857731	6
55	9.139037	9.995841	9.143196	10.856804	5
56	9.139944	9.995824	9.144121	10.855879	4
57	9.140850	9.995806	9.145044	10.854956	3
58	9.141754	9.995788	9.145965	10.854035	2
59	9.142655	9.995770	9.146885	10.853115	1
60	9.142555	9.995753	9.147803	10.852197	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 82.

The Table of Sines and Tangents.

Degree 8.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.143555	9.995753	9.147803	10.852197	60
1	9.144453	9.995735	9.148718	10.851282	59
2	9.145349	9.995717	9.149632	10.850368	58
3	9.146243	9.995699	9.159544	10.849456	57
4	9.147136	9.995681	9.151454	10.848546	56
5	9.148026	9.995664	9.152363	10.847637	55
6	9.148915	9.995646	9.153269	10.846731	54
7	9.149801	9.995628	9.154174	10.845826	53
8	9.150686	9.995610	9.155077	10.844923	52
9	9.151569	9.995591	9.155978	10.844022	51
10	9.152451	9.995573	9.156877	10.843123	50
11	9.153330	9.995555	9.157775	10.842225	49
12	9.154208	9.995537	9.158671	10.841329	48
13	9.155083	9.995519	9.159565	10.840435	47
14	9.155957	9.995501	9.160457	10.839543	46
15	9.156830	9.995482	9.161347	10.838653	45
16	9.157700	9.995464	9.162236	10.837764	44
17	9.158569	9.995446	9.163123	10.836877	43
18	9.159436	9.995427	9.164008	10.835992	42
19	9.160301	9.995409	9.164892	10.835108	41
20	9.161164	9.995390	9.165773	10.834226	40
21	9.162025	9.995372	9.166654	10.833346	39
22	9.162885	9.995353	9.167532	10.832468	38
23	9.163743	9.995334	9.168409	10.831591	37
24	9.164600	9.995316	9.169284	10.830716	36
25	9.165454	9.995297	9.170157	10.829843	35
26	9.166307	9.995278	9.171029	10.828971	34
27	9.167158	9.995260	9.171899	10.828101	33
28	9.168008	9.995241	9.172767	10.827233	32
29	9.168856	9.995222	9.173634	10.826366	31
30	9.169702	9.995203	9.174499	10.825501	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 81.

Degree 8.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.169702	9.995203	9.174499	10.825501	30
31	9.170546	9.995184	9.175362	10.824638	29
32	9.171389	9.995165	9.176224	10.823776	28
33	9.172230	9.995146	9.176084	10.822916	27
34	9.173070	9.995127	9.177942	10.822057	26
35	9.173908	9.995108	9.178799	10.821201	25
36	9.174744	9.995089	9.179655	10.820345	24
37	9.175578	9.995070	9.180508	10.819492	23
38	9.176411	9.995051	9.181360	10.818640	22
39	9.177242	9.995032	9.182211	10.817789	21
40	9.178072	9.995012	9.183060	10.816940	20
41	9.178900	9.994993	9.183907	10.816093	19
42	9.179726	9.994974	9.184752	10.815248	18
43	9.180551	9.994955	9.185597	10.814403	17
44	9.181374	9.994935	9.188439	10.813561	16
45	9.182196	9.999916	9.187280	10.812720	15
46	9.183016	9.994896	9.188120	10.811880	14
47	9.183834	9.994876	9.188957	10.811042	13
48	9.184651	9.994857	9.189794	10.810206	12
49	9.185466	9.994838	9.190629	10.809371	11
50	9.186280	9.994818	9.191462	10.808538	10
51	9.187092	9.994798	9.192294	10.807706	9
52	9.187903	9.994779	9.193124	10.806876	8
53	9.188712	9.994759	9.193953	10.806047	7
54	9.189519	9.994739	9.194780	10·805220	6
55	9.190325	9.994719	9 195606	10.804394	5
56	9.191130	9.994699	9.196430	10.803569	4
57	9.191933	9.994680	9.197253	10.802747	3
58	9.192734	9.994660	9.198674	10.801926	2
59	9.193534	9.994640	9.198894	10.801106	1
60	9.194332	9.994620	9.199712	10.800287	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 81.

The Table of Sines and Tangents.

Degree 9.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.194332	9.994620	9.199712	10.800287	60
1	9.195129	9.994600	9.200529	10.799470	59
2	9.195925	9.994580	9.201345	10.798655	58
3	9.196718	9.994560	9.202159	10.797841	57
4	9.197511	9.994540	9.202971	10.797029	56
5	9.198302	9.994519	9.203782	10.796218	55
6	9.199091	9.994499	9.204592	10.795408	54
7	9.199879	9.994479	9.205400	10.794600	53
8	9.200666	9.994459	9.206207	10.793793	52
9	9.201451	9.994438	9.207013	10.792987	51
10	9.202234	9.994418	9.207817	10.792183	50
11	9.203017	9.994398	9.208619	10.791381	49
12	9.203797	9 994377	9.209420	10.790580	48
13	9.204577	9.994357	9.210220	10.789780	47
14	9.205354	9.994336	9.211018	10.788982	46
15	9.206131	9.994316	9.211815	10.788185	45
16	9.206906	9.994295	9.212611	10.787389	44
17	9.207679	9.994274	9.213405	10.786595	43
18	9.208452	9.994254	9.214198	10.785802	42
19	9.209222	9.994233	9.214989	10.785011	41
20	9.209992	9.994212	9.215780	10.784220	40
21	9.210760	9.994191	9.216568	10.783432	39
22	9.211526	9.994171	9.217356	10.782644	38
23	9.212291	9.994150	9.218142	10.781858	37
24	9.213055	9.994129	9.218926	10.781074	36
25	9.213818	9.994108	9.219710	10.780290	35
26	9.214579	9.994087	9.220491	10.779508	34
27	9.215338	9.994066	9.221272	10.778728	33
28	9.216097	9.994044	9.222052	10.777948	32
29	9.216854	9.994024	9.222830	10.777170	31
30	9.217609	9.994003	9.223607	10.776393	30
	Co-ſine	Sine.	*Co-tang.*	Tangent.	M

Degree 80.

The Table of Sines and Tangents.

Degree 9.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
30	9.217609	9.994003		9.223607	10.776393	30
31	9.218363	9.993982		9.224382	10.775618	29
32	9.219116	9.993960		9.225156	10.774844	28
33	9.219868	9.993939		9.225929	10.774071	27
34	9.220618	9.993918		9.226700	10.773300	26
35	9.221367	9.993897		9.227471	10.772529	25
36	9.222115	9.993875		9.228240	10.771760	24
37	9.222861	9.993854		9.229007	10.770993	23
38	9.223606	9.993832		9.229774	20.770226	22
39	9.224349	9.993811		9.230539	10.769461	21
40	9.225092	9.993789		9.231302	10.768698	20
41	9.225833	9.993768		9.232065	10.767935	19
42	9.226573	9.993746		9.232826	10.767174	18
43	9.227311	9.993725		9.233586	10.766414	17
44	9.228048	9.993703		9.234345	10.765655	16
45	9.228784	9.993681		9.235103	10.764897	15
46	9.239518	9.993660		9.235859	10.764141	14
47	9.230252	9.993638		9.236614	10.763386	13
48	9.230984	9.993616		9.237368	10.762632	12
49	9.231715	9.993594		9.238120	10.761880	11
50	9.232444	9.993572		9.238872	10.761128	10
51	9.233173	9.993550		9.239622	10.760378	9
52	9.233899	9.993528		9.240371	10.759629	8
53	9.234625	9.993506		9.241118	10.758882	7
54	9.235349	9.993484		9.241865	10.758135	6
55	9.236073	9.993462		9.242610	10.757390	5
56	9.236795	9.993440		9.243354	10.756646	4
57	9.237515	9.993418		9.244097	10.755903	3
58	9.238235	9.993396		9.244839	10.755161	2
59	9.238952	9.993374		9.245579	10.754421	1
60	9.239670	9.993351		9.246319	10.753681	0
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 80.

The Table of Sines and Tangents.

Degree 10.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.239670	9.993351	9.246319	10.753681	60
1	9.240386	9.993329	9.247057	10.752943	59
2	9.241101	9.993307	9.247794	10.752206	58
3	9.241814	9.993284	9.248530	10.751470	57
4	9.242526	9.993262	9.249264	10.750736	56
5	9.243237	9.993240	9.249998	10.750002	55
6	9.243947	9.993117	9.250730	10.749270	54
7	9.244656	9.993195	9.251461	10.748539	53
8	9.245363	9.993172	9.252191	10.747809	52
9	9.246070	9.993149	9.252920	10.747080	51
10	9.246775	9.993127	9.253648	10.746352	50
11	9.247478	9.993104	9.254374	10.745626	49
12	9.248181	9.993081	9.255100	10.744900	48
13	9.248883	9.993059	9.255824	10.744176	47
14	9.249583	9.993036	9.256547	10.743453	46
15	9.250282	9.993013	9.257269	10.742731	45
16	9.250980	9.992990	9.257990	10.742010	44
17	9.251677	9.992967	9.258710	10.741290	43
18	9.252373	9.992944	9.259429	10.740571	42
19	9.253067	9.992921	9.260146	10.739854	41
20	9.253761	9.992898	9.260863	10.739137	40
21	9.254453	9.992875	9.261578	10.738422	39
22	9.255144	9.992852	9.262292	10.737708	38
23	9.255834	9.992829	9.263005	10.736995	37
24	9.256523	9.992806	9.263717	10.736283	36
25	9.257211	9.992783	9.264428	10.735572	35
26	9.257898	9.992759	9.265138	10.734862	34
27	9.258583	9.992736	9.265847	10.734153	33
28	9.259268	9.992713	9.266555	10.733445	32
29	9.259951	9.992690	9.267261	10.732739	31
30	9.260633	9.992666	9.267967	10.732033	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 79.

The Table of Sines and Tangents.

Degree 10.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.260633	9.992666	9.267967	10.732033	30
31	9.261314	9.992643	9.268671	10.731329	29
32	9.261994	9.992619	9.269375	10.730625	28
33	9.262673	9.992596	9 270077	10.729923	27
34	9.263351	9.992572	9.270779	10.729221	26
35	9.264027	9.992549	9.271479	10.728521	25
36	9.264703	9.992525	9.272178	10.727822	24
37	9.265378	9.992501	9.272876	10.727124	23
38	9.266051	9.992478	9.273573	10.726427	22
39	9.266723	9.992454	9.274269	10.725731	21
40	9.267395	9.992430	9.274964	10.725036	20
41	9.268065	9.992406	9.275658	10.724342	19
42	9.268734	9.992382	9.276351	10.723649	18
43	9.269402	9.992359	9.277043	10.722957	17
44	9.270069	9.992335	9.277734	10.722266	16
45	9.270735	9.992311	9.278424	10.721576	15
46	9.271400	9.992287	9.279113	10.720887	14
47	9.272063	9.992263	9.279801	10.720199	13
48	9.272726	9.992239	9.280488	10.719512	12
49	9.273388	9.992214	9.281174	10.718826	11
50	9.274049	9.992190	9.281858	10.718142	10
51	9.274708	9.992166	9.282542	10.717458	9
52	9.275367	9.992142	9.283225	10.716775	8
53	9.276025	9.992118	9.283907	10.716093	7
54	9.276681	9.992093	9.284588	10.715412	6
55	9.277337	9.992069	9.285268	10.714732	5
56	9.277991	9.992045	9.285946	10.714053	4
57	9.278645	9.992020	9.286624	10.713376	3
58	9.279297	9.991996	9.287301	10.712699	2
59	9.279948	9.991971	9.287977	10.712023	1
60	9.280599	9.991947	9.288652	10.711348	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 79.

The Table of Sines and Tangents.

Degree 11.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.280599	9.991947	9.288652	10.711348	60
1	9.281249	9.991922	9.289326	10.710674	59
2	9.281897	9.991897	9.289999	10.710001	58
3	9.282544	9.991873	9.290671	10.709329	57
4	9.283190	9.991848	9.291342	10.708658	56
5	9.283836	9.991823	9.292013	10.707987	55
6	9.284480	9.991799	9.292682	10.707318	54
7	9.285124	9.991774	9.293350	10.706650	53
8	9.285766	9.991749	9.294017	10.705983	52
9	9.286408	9.991724	9.294684	10.705316	51
10	9.287048	9.991699	9.295349	10.704651	50
11	9.287688	9.991674	9.296013	10.703987	49
12	9.288326	9.991649	9.296677	10.703323	48
13	9.288964	9.991624	9.297339	10.702661	47
14	9.289600	9.991599	9.298001	10.701999	46
15	9.290236	9.991574	9.298662	10.701338	45
16	9.290870	9.991549	9.299322	10.700678	44
17	9.291504	9.991524	9.299980	10.700020	43
18	9.292137	9.991498	9 300638	10.699362	42
19	9.292768	9.991473	9.301295	10.698705	41
20	9.293399	9.991448	9.301951	10.698049	40
21	9.294029	9.991422	9.302607	10.697393	39
22	9.294658	9.991397	9.303261	10.696739	38
23	9.295286	9.991372	9.303914	10.696086	37
24	9.295913	9.991346	9.304567	10.695433	36
25	9.296539	9.991321	9.305218	10.694782	35
26	9.297164	9.991295	9.305869	10.694131	34
27	9.297788	9.991270	9.306519	10.693481	33
28	9.298412	9.991244	9.307168	10.692832	32
29	9.299034	9.991218	9.307816	10.692184	31
30	9.299655	9.991193	9.308463	10.691537	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 78.

The Table of Sines and Tangents.

Degree 11.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.299655	9.991193	9.308463	10.691537	30
31	9.300276	9.991167	9.309109	10.690891	29
32	9.300895	9.991141	9.309754	10.690246	28
33	9.301514	9.991115	9.310399	10.689601	27
34	9.302132	9.991090	9.311042	10.688958	26
35	9.302749	9.991064	9.311685	10.688315	25
36	9.303364	9.991038	9.312327	10.687673	24
37	9.303979	9.991012	9.312968	10.687032	23
38	9.304593	9.990986	9.313608	10.686392	22
39	9.305207	9.990960	9.314247	10.685753	21
40	9.305819	9.990934	9.314885	10.685115	20
41	9.306430	9.990908	9.315523	10.684477	19
42	9.307041	9.990882	9.316159	10.683841	18
43	9.307650	9.990855	9.316795	10.683205	17
44	9.308259	9.990829	9.317430	10.682570	16
45	9.308867	9.990803	9.318064	10.681936	15
46	9.309474	9.990777	9.318697	10.681303	14
47	9.310080	9.990750	9.319330	10.680670	13
48	9.310685	9.990724	9.319961	10.680039	12
49	9.311289	9.990697	9.320592	10.679408	11
50	9.311893	9.990671	9.321222	10.678778	10
51	9.312495	9.990645	9.321851	10.678149	9
52	9.313097	9.990618	9.322479	10.677521	8
53	9.313698	9.990591	9.323106	10.676894	7
54	9.314297	9.990565	9.323733	10.676267	6
55	9.314897	9.990538	9.324358	10.675642	5
56	9.315495	9.990512	9.324983	10.675017	4
57	9.316092	9.990485	9.325607	10.674393	3
58	9.316689	9.990458	9.326231	10.573769	2
59	9.317284	9.990431	9.326853	10.673147	1
60	9.317879	9.990404	9.327475	10.672525	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 78.

The Table of Sines and Tangent.

Degree 12.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9·317879	9·990404	9·327475	10.672525	60
1	9·318473	9·990377	9·328095	10.671905	59
2	9·319066	9·990351	9·328715	10.671285	58
3	9·319658	9·990324	9·329334	10.670666	57
4	9·320250	9·990297	9·329953	10.670047	56
5	9·320840	9·990270	9·320570	10.669430	55
6	9·321430	9·990242	9·331187	10.668813	54
7	9·322019	9·990215	9·331803	10.668197	53
8	9·322607	9·990188	9·332418	10.667582	52
9	9·323194	9·990161	9·333033	10.666967	51
10	9·323780	9·990134	9·333646	10.666354	50
11	9·324366	9·990107	9·334259	10.665741	49
12	9·324950	9·990079	9·334871	10.665129	48
13	9·325534	9·990052	9·335482	10.664518	47
14	9·326117	9·990025	9·336093	10.663907	46
15	9·326699	9·989997	9·336702	10.663298	45
16	9·327281	9·989970	9·337311	10.662689	44
17	9·327862	9·989942	9·337919	10.662081	43
18	9·328441	9·989915	9·338527	10.661473	42
19	9·329020	9·989887	9·339133	10.660867	41
20	9·329599	9·989860	9·339739	10.660261	40
21	9·330176	9·989832	9·340344	10.659656	39
22	9·330753	9·989804	9·340948	10.659052	38
23	9·331328	9·989777	9·341552	10.658448	37
24	9·331903	9·989749	9·342155	10.657845	36
25	9·332478	9·989721	9·342757	10.657243	35
26	9·333051	9·989693	9·343358	10.656642	34
27	9·333624	9·989665	9·343958	10.656042	33
28	9·334195	9·989637	9·344558	10.655442	32
29	9·334766	9·989609	9·345157	10.654843	31
30	9·335337	9·989581	9·345755	10.654245	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 77.

The Table of Sines and Tangents.

Degree 12.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
30	9·335337	9·989581		9·345755	10.654245	30
31	9·335906	9·989553		9·346353	10.653647	29
32	9·336475	9·989525		9·346949	10.653051	28
33	9·337043	9·989597		9·347545	10.652455	27
34	9·337610	9·989469		9·348141	10.651859	26
35	9·338176	9·989441		9·348735	10.651265	25
36	9·338742	9·989413		9·349329	10.650671	24
37	9·339306	9·989384		9·349922	10.650078	23
38	9·339870	9·989356		9·350514	10.649486	22
39	9·340434	9·989328		9·351106	10.648894	21
40	9·340996	9·989299		9·351697	10.648303	20
41	9·341558	9·989271		9·352287	10.647713	19
42	9·342119	9·989243		9·352876	10.647124	18
43	9·342679	9·989214		9·353465	10.646535	17
44	9·343239	9·989186		9·354053	10.645947	16
45	9·343797	9·989157		9·354640	10.645360	15
46	9·344355	9·989128		9·355227	10.644773	14
47	9·344912	9·989100		9·355812	10.644187	13
48	9·345469	9·989071		9·356398	10.643602	12
49	9·346024	9·989042		9·356982	10.643018	11
50	9·346579	9·989014		9·357566	10.642434	10
51	9·347134	9·988985		9·358149	10.641851	9
52	9·347687	9·988956		9·358731	10.641269	8
53	9·348240	9·988927		9·359313	10.640687	7
54	9·348792	9·988898		9·359893	10.640107	6
55	9·349343	9·988869		9·360474	10.639526	5
56	9·349893	9·988840		9·361053	10.638947	4
57	9·350443	9·988811		9·361632	10.638369	3
58	9·350992	9·988782		9·362210	10.637790	2
59	9·351540	9·988754		9·362787	10.637213	1
60	9·392088	9,988724		9·363364	10.636636	0
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 77.

The Table of Sines and Tangents.

Degree 13.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	9.352088	9.988724	9.363364	10.636636	60
1	9.352635	9.988695	9.363940	10.636060	59
2	9.353181	9.988666	9.364515	10.635485	58
3	9.353726	9.988636	9.365090	10.634910	57
4	9.354271	9.988607	9.365664	10.634336	56
5	9.354815	9.988578	9.366237	10.633763	55
6	9.355358	9.988548	9.366810	10.633190	54
7	9.355901	9.988519	9.367382	10.632618	53
8	9.356443	9.988489	9.367953	10.632047	52
9	9.356984	9.988460	9.368524	10.631476	51
10	9.357524	9.988430	9.369094	10.630906	50
11	9.358064	9.988401	9.369663	10.630337	49
12	9.358603	9.988371	9.370232	10.629768	48
13	9.359141	9.988341	9.370799	10.629201	47
14	9.359679	9.988312	9.371367	10.628633	46
15	9.350215	9.988282	9.371933	10.628067	45
16	9.360752	9.988252	9.372499	10.627501	44
17	9.361287	9.988223	9.373064	10.626936	43
18	9.361822	9.988193	9.373629	10.626371	42
19	9.362356	9.988163	9.374193	10.625807	41
20	9.362889	9.988133	9.374756	10.625244	40
21	9.363432	9.988103	9.375319	10.624681	39
22	9.363954	9.988073	9.375881	10.624119	38
23	9.364485	9.988043	9.376442	10.623558	37
24	9.365016	9.988013	9.377003	10.622997	36
25	9.365546	9.987983	9.377563	10.622437	35
26	9.366075	9.987953	9.378122	10.621878	34
27	9.366604	9.987922	9.378681	10.621319	33
28	9.367132	9.987892	9.379239	10.620761	32
29	9.367659	9.987862	9.379797	10.620203	31
30	9.368185	9.987832	9.380354	10.619646	30
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 76.

The Table of Sines and Tangents.

Degree 13.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.368185	9.987832	9.380354	10.619646	30
31	9.368711	9.987801	9.380910	10.619090	29
32	9.369236	9.987771	9.381466	10.618534	28
33	9.369761	9.987740	9.382021	10.617980	27
34	9.370285	9.987710	9.382575	10.617425	26
35	9.370808	9.987679	9.383129	10.616871	25
36	9.371330	9.987649	9.383682	10.616318	24
37	9.371852	9.987618	9.384234	10.615766	23
38	9.372373	9.987588	9.384786	10 615214	22
39	9.372894	9.987557	9.385337	10.614663	21
40	9.373414	9.987526	9.385888	10.614112	20
41	9 373933	9.987496	9.386438	10.613562	19
42	9.374452	9.987465	9.386987	10.613013	18
43	9.374970	9.987434	9.387536	10.612464	17
44	9.375487	9.987403	9.388084	10.611916	16
45	9.376003	9.987372	9.388631	10.611369	15
46	9.376519	9.987341	9.389178	10.610822	14
47	9.377035	9.987310	9.389724	10.610276	13
48	9.377549	9.987279	9 390270	10.609730	12
49	9.378063	9.987248	9.390815	10.609185	11
50	9.378577	9.987217	9.391360	10.608640	10
51	9.379089	9.987186	9.391903	10.608097	9
52	9.379601	9.987155	9.392447	10.607553	8
53	9.380113	9.987124	9.392989	10.607011	7
54	9.380624	9.987092	9.393531	10.606469	6
55	9.381134	9.987061	9.394073	10.605927	5
56	9.381643	9.987030	9.394614	10.605386	4
57	9.382152	9.986998	9.395154	10.604846	3
58	9.382661	9.986967	9.395694	10.604306	2
59	9.383168	9.986936	9,396233	10.603767	1
60	9.383675	9.986904	9.396771	10.603229	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 76.

Degree 14.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
0	9.383675	9.986904		9.396771	10.603229	60
1	9.384181	9.986872		9.397309	10.602691	59
2	9.384687	9.986841		9.397846	10.602154	58
3	9.385192	9.986809		9.398383	10.601617	57
4	9.385697	9.986778		9.398919	10.601081	56
5	9.386201	9.986746		9.399455	10.600545	55
6	9.386704	9.986714		9.399990	10.600010	54
7	9.387207	9.986683		9.400524	10.599476	53
8	9.387709	9.986651		9.401058	10.598942	52
9	9.388210	9.986619		9.401591	10.598409	51
10	9.388711	9.986587		9.402124	10.597876	50
11	9.389211	9.986555		9.402656	10.597344	49
12	9.389711	9.986523		9.403187	10.596813	48
13	9.390210	9.986491		9.403718	10.596282	47
14	9.390708	9.986459		9.404249	10.595751	46
15	9.391206	9.986427		9.404778	10.595222	45
16	9.391703	9.986395		9.405308	10.594692	44
17	9.392199	9.986363		9.405836	10.594164	43
18	9.392695	9.986331		9.406364	10.593636	42
19	9.393190	9.986299		9.406892	10.593108	41
20	9.393685	9.986266		9.407419	10.592581	40
21	9.394179	9.986234		9.407945	10.592055	39
22	9.394673	9.986201		9.408471	10.591529	38
23	9.395166	9.986169		9.408996	10.591004	37
24	9.395658	9.986137		9.409521	10.590479	36
25	9.396150	9.986104		9.410045	10.589954	35
26	9.396641	9.986072		9.410569	10.589431	34
27	9.397131	9.986039		9.411092	10.588908	33
28	9.397621	9.986007		9.411615	10.588385	32
29	9.398111	9.985974		9.412137	10.587863	31
30	9.398600	9.985942		9.412658	10.587342	30
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 75.

The Table of Sines and Tangents.

Degree 14.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.398600	9.985942	9.412658	10.587342	30
31	9.399087	9.985909	9.413179	10.586821	29
32	9.399575	9.985876	9.413699	10.586301	28
33	9.400062	9.985843	9.414219	10.585781	27
34	9.400549	9.985811	9.414738	10.585262	26
35	9.401035	9.985778	9.415257	10.584743	25
36	9.401520	9.985745	9.415775	10.584225	24
37	9 402005	9.985712	9.416293	10.583707	23
38	9.402489	9.985679	9.416810	10.583190	22
39	9.402973	9.985643	9.417326	10.582674	21
40	9.403455	9.985613	9.417842	10.582157	20
41	9.403938	9.985580	9.418358	10.581642	19
42	9.404420	9.985547	9.418873	10.581127	18
43	9.404901	9.985513	9.419387	10.580613	17
44	9.405382	9.985480	9.419901	10.580099	16
45	9.405862	9.985447	9.420415	10.579585	15
46	9.406341	9.985414	9.420927	10.579072	14
47	9.406820	9.985380	9.421440	10.578560	13
48	9.407299	9.985347	9.421951	10.578047	12
49	9.407777	9.985314	9.422463	10.577537	11
50	9.408254	9.985280	9.422973	10.577026	10
51	9.408731	9.985247	9.423484	10.576516	9
52	9.409207	9.985213	9.423993	10.576007	8
53	9.409682	9.985180	9.424503	10.575497	7
54	9.410157	9.985146	9.425011	10.574989	6
55	9 410632	9.985112	9.425519	10.574481	5
56	9.411106	9.985079	9.426027	10.573973	4
57	9.411579	9.985045	9.426534	10.573466	3
58	9.412052	9.985011	9.427041	10.572959	2
59	9.412524	9.984977	9.427547	10.572453	1
60	9.412996	9.984943	9.428052	10.571947	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 75.

The Table of Sines and Tangents.

Degree 15.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.412996	9.984944	9.428052	10.571947	60
1	9.413467	9.984910	9.428557	10.571442	59
2	9.413938	9.984876	9.429062	10.570938	58
3	9.414408	9.984842	9.429566	10.570434	57
4	9.414878	9.984808	9.420070	10.579930	56
5	9.415347	9.984774	9.420573	10.579427	55
6	9.415815	9.984740	9.431075	10.568925	54
7	9.416283	9.984706	9.431577	10.568423	53
8	9.416751	9.984672	9.432079	10.567921	52
9	9.417217	9.984637	9.432580	10.567420	51
10	9.417684	9.984603	9.433080	10.566920	50
11	9.418149	9.984569	9.433580	10.566419	49
12	9.418615	9.984535	9.434080	10.565920	48
13	9.419079	9.984500	9.434579	10.565421	47
14	9.419544	9.984466	9.435078	10.564922	46
15	9.420007	9.984431	9.435576	10.564424	45
16	9.420470	9.984397	9.436073	10.563927	44
17	9.420933	9.984363	9.436570	10.563430	43
18	9.421395	9.984328	9.437067	10.562933	42
19	9.421857	9.984293	9.437563	10.562437	41
20	9.422318	9.984259	9.438059	10.561941	40
21	9.422778	9.984224	9.438554	10.561446	39
22	9.423238	9.984189	9.439048	10.560952	38
23	9.423697	9.984155	9.439543	10.560457	37
24	9.424156	9.984120	9.440036	10.559964	36
25	9.424615	9.984085	9.440529	10.559471	35
26	9.425073	9.984050	9.441022	10.558978	34
27	9.425530	9.984015	9.441514	10.558486	33
28	9.425987	9.983980	9.442006	10.557994	32
29	9.426443	9.983945	9.442497	10.557503	31
30	9.426899	9.983910	9.442988	10.557011	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 74.

The Table of Sines and Tangents.

Degree 15.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.426899	9.983910	9.442988	10.557011	30
31	9.427354	9.983875	9.443479	10.556521	29
32	9.427809	9.983840	9.443968	10.556031	28
33	9.428263	9.983805	9.444458	10.555542	27
34	9.428717	9.983770	9.444947	10.555053	26
35	9.429170	9.983735	9.445435	10.554565	25
36	9.429623	9.983699	9.445923	10.554077	24
37	9.430075	9.983664	9.446411	10.553589	23
38	9.430507	9.983629	9.446898	10.553102	22
39	9.430987	9.983593	9.447384	10.552616	21
40	9.431429	9.983558	9.447870	10.552129	20
41	9.431879	9.983523	9.448356	10.551644	19
42	9.432328	9.983487	9.448841	10.551159	18
43	9.432778	9.983452	9.449326	10.550674	17
44	9.433226	9.983416	9.449810	10.550190	16
45	9.433674	9.983380	9.450294	10.559706	15
46	9.434122	9.983345	9.450777	10.549223	14
47	9.434569	9.983309	9.451260	10.548740	13
48	9.435016	9.983273	9.451743	10.548257	12
49	9.435462	9.983238	9.452225	10.547775	11
50	9.435918	9.983202	9.452706	10.547294	10
51	9.436353	9.983166	9.453187	10.546813	9
52	9.436798	9.983130	9.453668	10.546332	8
53	9.437242	9.983094	9.454148	10.545852	7
54	9.437686	9.983058	9.454627	10.545372	6
55	9.438129	9.983022	9.455107	10.544893	5
56	9.438572	9.982986	9.455586	10.544414	4
57	9.439014	9.982950	9.456064	10.543936	3
58	9.439456	9.982914	9.456542	10.543458	2
59	9.439897	9.982878	9.457019	10.542980	1
60	9.440338	9.982842	9.457496	10.542503	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 74.

The Table of Sines and Tangents.

Degree 16.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	9.440338	9.982842	9.457496	10.542503	60
1	9.440778	9.982805	9.457973	10.542027	59
2	9.441218	9.982769	9.458449	10.541551	58
3	9.441658	9.982733	9.458925	10.541075	57
4	9.442096	9.982696	9.459400	10.540600	56
5	9.442535	9.982660	9.459875	10.540125	55
6	9.442973	9.982623	9.460349	10.539651	54
7	9.443410	9.982587	9.460823	10.539177	53
8	9.443847	9.982550	9.461297	10.538703	52
9	9.444284	9.982514	9.461770	10.538230	51
10	9.444720	9.982477	9.462242	10.537758	50
11	9.445155	9.982441	9.462714	10.537285	49
12	9.445590	9.982404	9.463186	10.536814	48
13	9.446025	9.982367	9.463658	10.536342	47
14	9.446459	9.982330	9.464129	10.535871	46
15	9.446893	9.982294	9.464599	10.535401	45
16	9.447326	9.982257	9.465069	10.534931	44
17	9.447759	9.982220	9.465539	10.534461	43
18	9.448191	9.982183	9.466008	10.533992	42
19	9.448623	9.982146	9.466476	10.533523	41
20	9.449054	9.982109	9.466945	10.533055	40
21	9.449485	9.982072	9.467413	10.532587	39
22	9.449915	9.982035	9.467880	10.532120	38
23	9.450345	9.981998	9.468347	10.531653	37
24	9.450775	9.981961	9.468814	10.531186	36
25	9.451204	9.981923	9.469280	10.530720	35
26	9.451632	9.981886	9.469746	10.530254	34
27	9.452060	9.981849	9.470211	10.529789	33
28	9.452488	9.981812	9.470676	10.529324	32
29	9.452915	9.981774	9.471141	10.528859	31
30	9.453342	9.981737	9.471605	10.528395	30
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 73.

Degree 16.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.453342	9.981737	9.471605	10.528395	30
31	9.453768	9.981699	9.472068	10.527931	29
32	9.454194	9.981662	9.472532	10.527468	28
33	9.454619	9.981624	9.472995	10.527005	27
34	9.455044	9.981587	9.473457	10.526543	26
35	9.455469	9.981549	9.473919	10.526081	25
36	9.455892	9.981512	9.474381	10.525619	24
37	9.456316	9.981474	9.474842	10.525158	23
38	9.456739	9.981436	9.475303	10.524697	22
39	9.457162	9.981398	9.475763	10.524237	21
40	9.457584	9.981361	9.476223	10.523777	20
41	9.458006	9.981323	9.476683	10.523317	19
42	9.458427	9.981285	9.477142	10.522858	18
43	9.458848	9.981247	9.477601	10.522399	17
44	9.459268	9.981209	9.478059	10.521941	16
45	9.459688	9.981171	9.478517	10.521483	15
46	9.460108	9.981133	9.478975	10.521025	14
47	9.460527	9.981095	9.479432	10.520568	13
48	9.460946	9.981057	9.479889	10.520111	12
49	9.461364	9.981019	9.480345	10.519655	11
50	9.461782	9.980980	9.480801	10.519199	10
51	9.462199	9.980942	9.481257	10.518743	9
52	9.462616	9.980904	9.481712	10.518288	8
53	9.463032	9.980866	9.482167	10.517833	7
54	9.463448	9.980827	9.482621	10.517379	6
55	9.463864	9.980789	9.483075	10.516925	5
56	9.464279	9.980750	9.483528	10.516471	4
57	9.464694	9.980712	9.483982	10.516018	3
58	9.465108	9.980674	9.484434	10.515565	2
59	9.465522	9.980635	9.484887	10.515113	1
60	9.465935	9.980596	9.485339	10.514661	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 73.

The Table of Sines and Tangent.

Degree 17.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.465935	9.980596	9.485339	10.514661	60
1	9.466348	9.980558	9.485791	10.514209	59
2	9.466761	9.980519	9.486242	10.513758	58
3	9.467173	9.980480	9.486693	10.513307	57
4	9.467585	9.980441	9.487143	10.512857	56
5	9.457996	9.980403	9.487593	10.512407	55
6	9.468407	9.980364	9.488043	10.511957	54
7	9.468817	9.980325	9.488492	10.511507	53
8	9.469227	9.980286	9.488941	10.511059	52
9	9.469637	9.980247	9.489390	10.510610	51
10	9.470046	9.980208	9.489838	10.510162	50
11	9.470455	9.980169	9.490286	10.509714	49
12	9.470863	9.980130	9.490733	10.509267	48
13	9.471271	9.980091	9.491180	10.508820	47
14	9.471678	9.980052	9.491627	10.508373	46
15	9.472086	9.980012	9.492073	10.507927	45
16	9.472492	9.979973	9.492519	10.507481	44
17	9.472898	9.979934	9.492964	10.507035	43
18	9.473304	9.979894	9.493410	10.506590	42
19	9.473710	9.979855	9.493854	10.506145	41
20	9.474115	9.979816	9.494299	10.505701	40
21	9.474519	9.979776	9.494743	10.505257	39
22	9.474923	9.979737	9.495186	10.504813	38
23	9.475327	9.979697	9.495630	10.504370	37
24	9.475730	9.979658	9.496073	10.503927	36
25	9.476133	9.979618	9.496515	10.503485	35
26	9.476536	9.979578	9.496957	10.503043	34
27	9.476938	9.979539	9.497399	10.402601	33
28	9.477340	9.979499	9.497840	10.502160	32
29	9.477741	9.979459	9.498282	10.501718	31
30	9.478142	9.979419	9.498722	10.501278	30
	Co-ſine.	Sinc.	*Co-tang.*	Tangent.	M

Degree 72.

The Table of Sines and Tangents.

Degree 17.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
30	9.478142	9.979419	9.498722	10.501278	30
31	9.478542	9.979380	9.499163	10.500837	29
32	9.478942	9.979340	9.499602	10.500398	28
33	9.479342	9.979300	9.500042	10.499958	27
34	9.479741	9.979260	9.500481	10.499519	26
35	9.480140	9.979220	9.500920	10.499080	25
36	9.480538	9.979180	9.501359	10.498641	24
37	9.480937	9.979140	9.501797	10.498203	23
38	9.481334	9.979099	9.502235	10.497765	22
39	9.481731	9.979059	9.502672	10.497328	21
40	9.482128	9.979019	9.503109	10.496891	20
41	9.482525	9.978980	9.503546	10.496454	19
42	9.482921	9.978939	9.503982	10.496018	18
43	9.483316	9.978898	9.504418	10.495582	17
44	9.483711	9.978858	9.504854	10.495146	16
45	9.484106	9.978817	9.505289	10.494711	15
46	9.484501	9.978777	9.505724	10.494276	14
47	9.484895	9.978736	9.506158	10.493841	13
48	9.485289	9.978696	9.506593	10.493407	12
49	9.485682	9.978655	9.507026	10.492973	11
50	9.486075	9.978615	9.507459	10.492540	10
51	9.486467	9.978574	9.507892	10.492107	[illegible]
52	9.486859	9.978533	9.508326	10.491674	8
53	9.487251	9.978493	9.508759	10.491241	7
54	9.487642	9.978452	9.509191	10.490809	6
55	9.488033	9.978411	9.509622	10.490377	5
56	9.488424	9.978370	9.510054	10.489946	4
57	9.488814	9.978329	9.510485	10.489515	3
58	9.489204	9.978288	9.510916	10.489084	2
59	9.489593	9.978247	9.511346	10.488654	1
60	9.489982	9.968206	9.511776	10 488225	0
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 72.

The Table of Sines and Tangents.

Degree 18.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
0	9.489982	9.978206		9.511776	10.488224	60
1	9.490371	9.978165		9.512206	10.487794	59
2	9.490759	9.978124		9.512635	10.487365	58
3	9.491147	9.978083		9.513064	10.486936	57
4	9.791534	9.978042		9.513493	10.486507	56
5	9.491922	9.978001		9.513921	10.486079	55
6	9.492308	9.977959		9.514349	10.485651	54
7	9.492695	9.977918		9.514777	10.485223	53
8	9.493080	9.977877		9.515204	10.484796	52
9	9.493466	9.977835		9.515631	10.484369	51
10	9.493851	9.977794		9.516057	10.483942	50
11	9.494236	9.977752		9.516484	10.483516	49
12	9.494620	9.977711		9.516910	10.483090	48
13	9.495005	9.977669		9.517335	10.482665	47
14	9.495388	9.977628		9.517761	10.482239	46
15	9.495771	9.977586		9.518185	10.481814	45
16	9.496154	9.977544		9.518610	10.481390	44
17	9.496537	9.977503		9.519034	10.480966	43
18	9.496919	9.977461		9.519458	10.480542	42
19	9.497301	9.977419		9.519882	10.480118	41
20	9.497682	9.977377		9.520305	10.489695	40
21	9.498063	9.977335		9.520728	10.479272	39
22	9.498444	9.977293		9.521151	10.478849	38
23	9.498824	9.977251		9.521573	10.478427	37
24	9.499204	9.977209		9.521995	10.478005	36
25	9.499584	9.977167		9.522417	10.477583	35
26	9.499963	9.977125		9.522838	10.477162	34
27	9.500342	9.977083		9.523259	10.476741	33
28	9.500720	9.977041		9.523679	10.476320	32
29	9.501099	9.976999		9.524100	10.475900	31
30	9.501476	9.976957		9.524520	10.475480	30
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 71.

Degree 18.

M	Sine.	*Co-sine.*	Tangent.	*Co-tang.*	
30	9.501476	9.976956	9.524520	10.475480	30
31	9.501854	9.976914	9.524939	10.475060	29
32	9.502231	9.976872	9.525359	10.474641	28
33	9.502607	9.976830	9.525778	10.474222	27
34	9.502984	9.976787	9.526197	10.473803	26
35	9.503360	9.976745	9.526615	10.473385	25
36	9.503735	9.976702	9.527033	10.472967	24
37	9.504110	9.976660	9.527451	10.472549	23
38	9.504485	9.976617	9.527868	10.472132	22
39	9.504860	9.976574	9.528285	10.471715	21
40	9.505234	9.976532	9.528702	10.471298	20
41	9 505608	9.976489	9.529118	10.470881	19
42	9.505981	9.976446	9.529535	10.470465	18
43	9.506354	9.976404	9.529950	10.470049	17
44	9.506727	9.976361	9.530366	10.469634	16
45	9.507099	9.976318	9.530781	10.469219	15
46	9.507471	9.976275	9.531196	10.468804	14
47	9.507843	9.976232	9.531611	10.468389	13
48	9.508214	9.976189	9.532025	10.467975	12
49	9.508585	9.976146	9.532439	10.467561	11
50	9.508955	9.976103	9.532853	10.467147	10
51	9.509326	9.976060	9.533266	10.466734	9
52	9.509696	9.976017	9.533679	10.466321	8
53	9.510065	9.975973	9.534092	10.465908	7
54	9.510434	9.975930	9.534504	10.465496	6
55	9.510803	9.975887	9.534916	10.465084	5
56	9.511171	9.975844	9.535328	10.464672	4
57	9.511540	9.975800	9.535739	10.464261	3
58	9.511907	9.975757	9.536150	10.463849	2
59	9.512275	9.975713	9.536561	10.463439	1
60	9.512642	9.975670	9.536972	10.463028	0
	Co-sine.	Sine.	*Co-tang.*	Tangent.	M

Degree 71.

The Table of Sines and Tangents.

Degree 19.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.512642	9.975670	9.536978	10.463028	60
1	9.513009	9.975626	9.537382	10.462618	59
2	9.513375	9.975583	9.537792	10.462208	58
3	9.513741	9.975539	9.538202	10.461798	57
4	9.514107	9.975496	9.538611	10.461389	56
5	9.514472	9.975452	9.539020	10.460980	55
6	9.514837	9.975408	9.539429	10.460571	54
7	9.515202	9.975364	9.539837	10.460163	53
8	9.515566	9.975321	9.540245	10.459755	52
9	9.515930	9.975277	9.540653	10.459347	51
10	9.516294	9.975233	9.541061	10.458939	50
11	9.516657	9.975189	9.541468	10.458532	49
12	9.517020	9.975145	9.541875	10.458125	48
13	9.517382	9.975101	9.542281	10.457719	47
14	9.517745	9.975057	9.542688	10.457312	46
15	9.518107	9.975013	9.543094	10.456906	45
16	9.518468	9.974969	9.543499	10.456501	44
17	9.518829	9.974925	9.543905	10.456095	43
18	9.519190	9.974880	9.544310	10.455690	42
19	9.519551	9.974836	9.544715	10.455285	41
20	9.519911	9.974792	9.545119	10.454881	40
21	9.520271	9.974747	9.545524	10.454476	39
22	9.520631	9.974703	9.545927	10.454072	38
23	9.520990	9.974659	9.546331	10.453669	37
24	9.521349	9.974614	9.546735	10.453265	36
25	9.521707	9.977570	9.547138	10.452862	35
26	9.522066	9.974525	9.547540	10.452459	34
27	9.522423	9.974480	9.547943	10.452057	33
28	9.522781	9.974436	9.548345	10.451655	32
29	9.523138	9.974391	9.548747	10.451253	31
30	9.523495	9.974346	9.549149	10.450851	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 70.

The Table of Sines and Tangents.

Degree 19.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.523495	9.974346	9.549149	10.450851	30
31	9.523852	9.974302	9.549550	10.450450	29
32	9.524208	9.974257	9.549951	10.450049	28
33	9.524564	9.974212	9.550352	10.449648	27
34	9.524920	9.974167	9.550752	10.449248	26
35	9.525275	9.974122	9.551152	10.448848	25
36	9.525630	9.974077	9.551552	10.448448	24
37	9.525984	9.974032	9.551952	10.448048	23
38	9.526339	9.973987	9.552351	10.447649	22
39	9.526693	9.973942	9.552750	10.447250	21
40	9.527046	9.973897	9.553149	10.446851	20
41	9.527400	9.973852	9.553548	10.446452	19
42	9.527753	9.973807	9.553946	10.446054	18
43	9.528105	9.973761	9.554344	10.445656	17
44	9.528458	9.973716	9.554741	10.445259	16
45	9.528810	9.973671	9.555139	10.444861	15
46	9.529161	9.973625	9.555536	10.444464	14
47	9.529513	9.973580	9.555932	10.444068	13
48	9.529864	9.973535	9.556329	10.443671	12
49	9.530214	9.973489	9.556725	10.443275	11
50	9.530565	9.973443	9.557121	10.442879	10
51	9.530915	9.973398	9.557517	10.442483	9
52	9.531265	9.973352	9.557912	10.442088	8
53	9.531614	9.973307	9.558308	10.441698	7
54	9.531963	9.973261	9.558702	10.441298	6
55	9.532312	9.973215	9.559097	10.440903	5
56	9.532661	9.973169	9.559491	10.440509	4
57	9.533009	9.973123	9.559885	10.440115	3
58	9.533357	9.973078	9.560279	10.439721	2
59	9.533704	9.973032	9.560673	10.439327	1
60	9.534052	9.972986	9.561066	10.438934	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 70.

Degree 20.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.534052	9.972986	9.561066	10.438934	60
1	9.534399	9.972940	9.561459	10.438541	59
2	9.534745	9.972894	9.561851	10.438148	58
3	9.535091	9.972848	9.562244	10.437756	57
4	9.535437	9.972801	9.562636	10.437364	56
5	9.535783	9.972755	9.563028	10.436972	55
6	9.536129	9.972709	9.563419	10.436580	54
7	9.536474	9.972663	9.563811	10.436189	53
8	9.536818	9.972617	9.564202	10.435798	52
9	9.537163	9.972570	9.564592	10.435407	51
10	9.537507	9.972524	9.564983	10.435017	50
11	9.537851	9.972477	9.565373	10.434627	49
12	9.538194	9.972431	9.565763	10.434237	48
13	9.538537	9.972384	9.566153	10.433847	47
14	9.538880	9.972338	9.566542	10.433457	46
15	9.539223	9.972291	9.566932	10.433068	45
16	9.539565	9.972245	9.567320	10.432679	44
17	9.539907	9.972198	9.567709	10.432291	43
18	9.940249	9.972151	9.568097	10.431902	42
19	9.540590	9.972105	9.568486	10.431514	41
20	9.540931	9.972058	9.568873	10.431126	40
21	9.541272	9.972011	9.569261	10.430739	39
22	9.541612	9.971964	9.569648	10.430351	38
23	9.541953	9.971917	9.560035	10.429964	37
24	9.542292	9.971870	9.560422	10.429578	36
25	9.542632	9.971823	9.560809	10.429191	35
26	9.542971	9.971776	9.571195	10.428805	34
27	9.543310	9.971729	9.571581	10.428419	33
28	9.543649	9.971682	9.571967	10.428033	32
29	9.543987	9.971635	9.572352	10.427648	31
30	9.544325	9.971588	9.572738	10.427262	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 69.

The Table of Sines and Tangents.

Degree 20.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.544325	9.971588	9.572738	10.427262	30
31	9.544663	9.971540	9.573123	10.426877	29
32	9.545000	9.971493	9.573507	10.426492	28
33	9.545338	9.971446	9.573892	10.426108	27
34	9.545674	9.971398	9.574276	10.425724	26
35	9.546011	9.971351	9.574660	10.425340	25
36	9.546347	9.971303	9.575044	10.424956	24
37	9.546683	9.971256	9.575427	10.424573	23
38	9.547019	9.971208	9.575810	10.424189	22
39	9.547354	9.971161	9.576193	10.423807	21
40	9.547689	9.971113	9.576576	10.423424	20
41	9.548024	9.971065	9.576958	10.423041	19
42	9.548358	9.971018	9.577341	10.422659	18
43	9.548693	9.970970	9.577723	10.422277	17
44	9.549027	9.970922	9.578104	10.421896	16
45	9.549360	9.970874	9.578486	10.421514	15
46	9.549693	9.970826	9.578867	10.421133	14
47	9.550026	9.970779	9.579248	10.420752	13
48	9.550359	9.970731	9.579628	10.420371	12
49	9.550692	9.970683	9.580009	10.419991	11
50	9.551024	9.970634	9.580389	10.419611	10
51	9.551355	9.970586	9.580769	10.419231	9
52	9.551687	9.970538	9.581149	10.418851	8
53	9.552018	9.970490	9.581528	10.418472	7
54	9.552349	9.970442	9.581907	10.418092	6
55	9.552680	9.970394	9.582286	10.417713	5
56	9.553010	9.970345	9.582665	10.417335	4
57	9.553340	9.970297	9.583043	10.416956	3
58	9.553670	9.970249	9.583422	10.416578	2
59	9.554000	9.970200	9.583800	10.416200	1
60	9.554329	9.970152	9.584177	10.415823	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 69.

The Table of Sines and Tangents.

Degree 21.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.554329	9.970152	9.584177	10.415822	60
1	9.554658	9.970103	9.584555	10.415445	59
2	9.554987	9.970055	9.584932	10.415068	58
3	9.555315	9.970006	9.585309	10.414691	57
4	9.555643	9.969957	9.585686	10.414314	56
5	9.555971	9.969909	9.586062	10.413938	55
6	9.556299	9.969860	9.586439	10.413561	54
7	9.556626	9.969811	9.586815	10.413185	53
8	9.556953	9.969762	9.587190	10.412810	52
9	9.557279	9.969713	9.587566	10.412434	51
10	9.557606	9.969665	9.587941	10.412059	50
11	9.557932	9.969616	9.588316	10.411684	49
12	9.558258	9.969567	9.588691	10.411309	48
13	9.558583	9.969518	9.589066	10.410934	47
14	9.558909	9.969469	9.589440	10.410560	46
15	9.559234	9.969419	9.589814	10.410185	45
16	9.559558	9.969370	9.590188	10.409812	44
17	9.559883	9.969321	9.590561	10.409438	43
18	9.560207	9.969272	9.590935	10,409065	42
19	9.560531	9.969223	9.591308	10.408692	41
20	9.560855	9.969173	9.591681	10.408319	40
21	9.561178	9.969124	9.592054	10.407946	39
22	9.561501	9.969075	9.592426	10.407574	38
23	9.561824	9.969025	9.592798	10.407201	37
24	9.562146	9.968976	9.593170	10.406829	36
25	9.562468	9.968926	9.593542	10.406457	35
26	9.562790	9.968877	9.593914	10.406086	34
27	9.563112	9.968827	9.594285	10.405715	33
28	9.563433	9.968777	9.594656	10.405344	32
29	9.563754	9.968728	9.595027	10.405073	31
30	9.564075	9.968678	9.595397	10.404602	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 68.

The Table of Sines and Tangents.

Degree 21.

M	Sine.	Co-sine.	Tangent.	Co-tang.	
30	9.564075	9.968678	9.595397	10.404602	30
31	9.564396	9.968628	9.595768	10.404232	29
32	9.564716	9.968578	9.596138	10.403862	28
33	9.565036	9.968528	9.596508	10.403492	27
34	9.565356	9.968478	9.596878	10.403122	26
35	9.565675	9.968428	9.597247	10.402753	25
36	9.565995	9.968378	9.597616	10.402384	24
37	9.566314	9.968328	9.597985	10.402015	23
38	9.566632	9.968278	9.598354	10.401646	22
39	9.566951	9.968228	9.598722	10.401277	21
40	9.567269	9.968178	9.599091	10.400909	20
41	9.567587	9.968128	9.599459	10.400541	19
42	9.567904	9.968078	9.599827	10.400173	18
43	9.568222	9.968027	9.600194	10.399806	17
44	9.568539	9.967977	9.600562	10.399438	16
45	9.568855	9.967927	9.600929	10.399071	15
46	9.569172	9.967876	9.601296	10.398704	14
47	9.569488	9.967826	9.601662	10.398337	13
48	9.569804	9.967775	9.602029	10.397971	12
49	9.570120	9.967725	9.602395	10.397605	11
50	9.570435	9.967674	9.602761	10.397239	10
51	9.570751	9.967623	9.603127	10.396873	9
52	9.571065	9.967573	9.603493	10.396507	8
53	9.571380	9.967522	9.603858	10.396142	7
54	9.571695	9.967471	9.604223	10.395777	6
55	9.572009	9.967420	9.604588	10.395412	5
56	9.572322	9.967370	9.604953	10.395047	4
57	9.572636	9.967319	9.605317	10.394683	3
58	9.572949	9.967268	9.605682	10.394318	2
59	9.573263	9.967217	9.606046	10.393954	1
60	9.573575	9.967166	9.606410	10.393590	0
	Co-sine.	Sine.	*Co-tang.*	Tangent.	M

Degree 68.

The Table of Sines and Tangents.

Degree 22.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.573575	9.967166	9.606409	10.393590	60
1	9.573888	9.967115	9.606773	10.393227	59
2	9.574200	9.967064	9.607136	10.392863	58
3	9.574512	9.967012	9.607500	10.392500	57
4	9.574824	9.966961	9.607862	10.392137	56
5	9.575135	9.966910	9.608225	10.391774	55
6	9.575447	9.966859	9.608588	10.391412	54
7	9.575758	9.966807	9.608950	10.391050	53
8	9.576068	9.966756	9.609312	10.390688	52
9	9.576379	9.966705	9.609674	10.390326	51
10	9.576689	9.966653	9.600036	10.399964	50
11	9.576999	9.966602	9.610397	10.389603	49
12	9.577309	9.966550	9.610758	10.389241	48
13	9.577618	9.966499	9.611119	10.388880	47
14	9.577927	9.966447	9.611480	10.388520	46
15	9.578236	9.966395	9.611841	10.288159	45
16	9.578545	9.966344	9.612201	10.387799	44
17	9.578853	9.966292	9.612561	10.387438	43
18	9.579161	9.966240	9.612921	10.387078	42
19	9.579469	9.966188	9.613281	10.386719	41
20	9.579777	9.966136	9.613641	10.386359	40
21	9.580084	9.966084	9.614000	10.386000	39
22	9.580392	9.966032	9.614359	10.385641	38
23	9.580698	9.965980	9.614718	10.385282	37
24	9.581005	9.965928	9.615077	10.384923	36
25	9.581311	9.965876	9.615435	10.384565	35
26	9.581618	9.965824	9.615793	10.384207	34
27	9.581923	9.965772	9.616151	10.383848	33
28	9.582229	9.965720	9.616509	10.383491	32
29	9.582534	9.965668	9.616867	10.383133	31
30	9.582840	9.965615	9.617224	10.382776	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 67.

The Table of Sines and Tangents.

Degree 22.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.582840	9.965615	9.617224	10.382776	30
31	9.583144	9.965563	9.617581	10.382418	29
32	9.583449	9.965511	9.617938	10.382061	28
33	9.583753	9.965458	9.618295	10.381705	27
34	9.584058	9.965406	9.618652	10.381348	26
35	9.584361	9.965353	9.619008	10.380992	25
36	9.584665	9.965301	9.619364	10.380635	24
37	9.584968	9.965248	9.619720	10.380279	23
38	9.585271	9.965195	9.620076	10.379924	22
39	9.585574	9.965143	9.620432	10.379568	21
40	9.585877	9.965090	9.620787	10.379213	20
41	9.586179	9.965037	9.621142	10.378858	19
42	9.586481	9.964984	9.621497	10.378503	18
43	9.586783	9.964931	9.621852	10.378148	17
44	9.587085	9.964878	9.622206	10.377793	16
45	9.587386	9.964825	9.622561	10.377439	15
46	9.587687	9.964772	9.622915	10.377085	14
47	9.587988	9.964719	9.623269	10.376731	13
48	9.588289	9.964666	9.623623	10.376377	12
49	9.588589	9.964613	9.623976	10.376024	11
50	9.588890	9.964560	9.624330	10.375670	10
51	9.589190	9.964507	9.624683	10.375317	9
52	9.589489	9.964454	9.625036	10.374964	8
53	9.589789	9.964400	9.625388	10.374612	7
54	9.590088	9.964347	9.625741	10.374259	6
55	9.590387	9.964294	9.626093	10.373907	5
56	9.590686	9.964240	9.626445	10.373555	4
57	9.590984	9.964187	9.626797	10.373203	3
58	9.591282	9.964133	9.627149	10.372850	2
59	9.591580	9.964080	9.627501	10.372499	1
60	9.591878	9.964026	9.627852	10.372148	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 67.

Degree 23.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.591878	9.964026	9.627852	10.372148	60
1	9.592175	9.963972	9.628203	10.371797	59
2	9.592473	9.963919	9.628554	10.371446	58
3	9.592770	9.963865	9.628905	10.371095	57
4	9.593067	9.963811	9.629255	10.370744	56
5	9.593363	9.963757	9.629606	10.370394	55
6	9.593659	9.963703	9.629956	10.370044	54
7	9.593955	9.963650	9.630306	10.369694	53
8	9.594251	9.963596	9.630655	10.369344	52
9	9.594547	9.963542	9.631005	10.368995	51
10	9.594842	9.963488	9.631354	10.368645	50
11	9.595137	9.963433	9.631704	10.368296	49
12	9.595432	9.963379	9.632053	10.367947	48
13	9.595727	9.963325	9.632401	10.367599	47
14	9.596021	9.963271	9.632750	10.367250	46
15	9.596315	9.963217	9.633098	10.366901	45
16	9.596610	9.963102	9.633447	10.366553	44
17	9.596903	9.963108	9.633795	10.366205	43
18	9.597196	9.963054	9.634143	10.365857	42
19	9.597490	9.962999	9.634490	10.365510	41
20	9.597783	9.962945	9.634838	10.365162	40
21	9.598075	9.962890	9.635185	10.364815	39
22	9.598368	9.962836	9.635532	10.364468	38
23	9.598660	9.962781	9.635879	10.364121	37
24	9.598952	9.962726	9.636226	10.363774	36
25	9.599244	9.962672	9.636572	10.363428	35
26	9.599536	9.962617	9.636918	10.363081	34
27	9.599827	9.962562	9.637265	10.362735	33
28	9.600118	9.962507	9.637610	10.362389	32
29	9.600409	9.962453	9.637956	10.362044	31
30	9.600700	9.962398	9.638302	10.361698	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 66.

Degree 23.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.600700	9.962398	9.638302	10.361698	30
31	9.600990	9.962343	9.638647	10.361353	29
32	9.601280	9.962288	9.638992	10.361007	28
33	9.601570	9.962233	9.639337	10.360662	27
34	9.601860	9.962178	9.639682	10.360318	26
35	9.602149	9.962122	9.640027	10.359973	25
36	9.602439	9.962067	9.640371	10.359629	24
37	9.602728	9.962012	9.640716	10.359284	23
38	9.603017	9.961957	9.641060	10.358940	22
39	9.603305	9.961902	9.641404	10.358596	21
40	9.603594	9.961846	9.641747	10.358253	20
41	9.603882	9.961791	9.642091	10.357909	19
42	9.604170	9.961735	9.642434	10.357566	18
43	9.604457	9.961680	9.642777	10.357223	17
44	9.604745	9.961624	9.643120	10.356980	16
45	9.605032	9.961569	9.643463	10.356537	15
46	9.605319	9.961513	9.643806	10.356194	14
47	9.605606	9.961458	9.644148	10.355852	13
48	9.605892	9.961402	9.644490	10.355510	12
49	9.606179	9.961346	9.644832	10.355168	11
50	9.606465	9.961290	9.645174	10.354826	10
51	9.606750	9.961235	9.645516	10.354484	9
52	9.607036	9.961179	9.645857	10.354142	8
53	9.607322	9.961123	9.646199	10.353801	7
54	9.607607	9.961067	9.646540	10.353466	6
55	9.607892	9.961011	9.646881	10.353119	5
56	9.608176	9.960955	9.647222	10.352778	4
57	9.608461	9.960899	9.647563	10.352438	3
58	9.608745	9.960842	9.647903	10.352097	2
59	9.609029	9.960786	9.648242	10.351757	1
60	9.609313	9.960730	9.648583	10.351417	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 66.

Degree 24.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	9.609313	9.960730	9.648583	10.351417	60
1	9.609597	9.960674	9.648923	10.351077	59
2	9.609880	9.960617	9.649263	10.350737	58
3	9.610163	9.960561	9.649602	10.350398	57
4	9.610446	9.960505	9.649942	10.350058	56
5	9.610729	9.960448	9.650281	10.349719	55
6	9.611012	9.960392	9.650620	10.349380	54
7	9.611294	9.960335	9.650959	10.349041	53
8	9.611576	9.960279	9.651297	10.348703	52
9	9.611858	9.960222	9.651636	10.348364	51
10	9.612140	9.960165	9.651974	10.348026	50
11	9.612421	9.960109	9.652312	10.347688	49
12	9.612702	9.960052	9.652650	10.347350	48
13	9.612983	9.959995	9.652988	10.347012	47
14	9.613264	9.959938	9.653326	10.346674	46
15	9.613545	9.959881	9.653663	10.346337	45
16	9.613825	9.959824	9.654000	10.345999	44
17	9.614105	9.959768	9.654337	10.345662	43
18	9.614385	9.959710	9.654674	10.345325	42
19	9.614665	9.959653	9.655011	10.344989	41
20	9.614944	9.959596	9.655348	10.344652	40
21	9.615223	9.959539	9.655684	10.344316	39
22	9.615502	9.959482	9.656020	10.343980	38
23	9.615781	9.959425	9.656356	10.343644	37
24	9.616060	9.959367	9.656692	10.343308	36
25	9.616338	9.959310	9.657028	10.342972	35
26	9.616616	9.959253	9.657363	10.342636	34
27	9.616894	9.959195	9.657699	10.342301	33
28	9.617172	9.959138	9.658034	10.341966	32
29	9.617450	9.959080	9.658369	10.341531	31
30	9.617727	9.959023	9.658704	10.341296	30
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 65.

Degree 24.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.617727	9.959023	9.658704	10.341296	30
31	9.618004	9.958965	9.659039	10.340961	29
32	9.618281	9.958908	9.659373	10.340627	28
33	9.618558	9.958850	9.659708	10.340292	27
34	9.618834	9.958792	9.660042	10.339958	26
35	9.619110	9.958734	9.660376	10.339624	25
36	9.619386	9.958677	9.660710	10.339290	24
37	9.619662	9.958619	9.661043	10.338957	23
38	9.619938	9.958561	9.661377	10.338623	22
39	9.620213	9.958503	9.661710	10.338290	21
40	9.620488	9.958445	9.662043	10.337956	20
41	9.620763	9.958387	9.662376	10.337623	19
42	9.621038	9.958329	9.662709	10.337291	18
43	9.621313	9.958271	9.663042	10.336958	17
44	9.621587	9.958212	9.663374	10.336625	16
45	9.621861	9.958154	9.663707	10.336293	15
46	9.622135	9.958096	9.664039	10.335961	14
47	9.622409	9.958038	9.664371	10.335629	13
48	9.622682	9.957979	9.664703	10.335297	12
49	9.622956	9.957921	9.665035	10.334965	11
50	9.623229	9.957862	9.665366	10.334634	10
51	9.623502	9.957804	9.665697	10.334302	9
52	9.623774	9.957745	9.666029	10.333971	8
53	9.624047	9.957687	9.666360	10.333640	7
54	9.624319	9.957628	9.666691	10.333309	6
55	9.624591	9.957570	9.667021	10.332979	5
56	9.624863	9.957511	9.667352	10.332648	4
57	9.625134	9.957452	9.667682	10.332318	3
58	9.625406	9.957393	9.668012	10.331987	2
59	9.625677	9.957334	9.668343	10.331657	1
60	9.625948	9.957276	9.668672	10.331327	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 65.

The Table of Sines and Tangents.

Degree 25.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.625948	9.957276	9.668672	10.331327	60
1	9.626219	9.957217	9.669002	10.330998	59
2	9.626490	9.957158	9.669332	10.330668	58
3	9.626760	9.957099	9.669661	10.330339	57
4	9.627030	9.957040	9.669990	10.330009	56
5	9.627300	9.956981	9.670320	10.329680	55
6	9.627570	9.956922	9.670649	10.329351	54
7	9.627840	9.956862	9.670977	10.329022	53
8	9.628109	9.956803	9.671306	10.328694	52
9	9.628378	9.956744	9.671634	10.328365	51
10	9.628647	9.956684	9.671963	10.328037	50
11	9.628916	9.956625	9.672291	10.327709	49
12	9.629184	9.956565	9.672619	10.327381	48
13	9.629453	9.956506	9.672947	10.327053	47
14	9.629721	9.956446	9.673274	10.326725	46
15	9.629989	9.956387	9.673602	10.326398	45
16	9.630257	9.956327	9.673929	10.326071	44
17	9.630524	9.956267	9.674257	10.325743	43
18	9.630792	9.956208	9.674584	10.325416	42
19	9.631059	9.956148	9.674910	10.325089	41
20	9.631326	9.956088	9.675237	10.324763	40
21	9.631592	9.956029	9.675564	10.324436	39
22	9.631859	9.955969	9.675890	10.324110	38
23	9.632125	9.955909	9.676216	10.323783	37
24	9.632392	9.955849	9.676543	10.323457	36
25	9.632657	9.955789	9.676869	10.323131	35
26	9.632923	9.955729	9.677194	10.322805	34
27	9.633189	9.955669	9.677520	10.322480	33
28	9.633454	9.955609	9.677845	10.322154	32
29	9.633719	9.955548	9.678171	10.321829	31
30	9.633984	9.955488	9.678496	10.321504	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 64.

Degree 25.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.633984	9.955488	9 678496	10.321504	30
31	9.634249	9.955428	9.678821	10.321179	29
32	9.634514	9.955367	9.679146	10.320854	28
33	9.634778	9.955307	9.679471	10.320529	27
34	9.635042	9.955246	9.679795	10.320205	26
35	9.635306	9.955186	9.680120	10.319880	25
36	9.635570	9.955125	9.680444	10.319556	24
37	9.635833	9.955065	9.680768	10.319232	23
38	9.636097	9.955004	9.681092	10.318908	22
39	9.636360	9.954944	9.681416	10.318584	21
40	9.636623	9.954883	9.681740	10.318260	20
41	9.636886	9.954823	9.682063	10.317937	19
42	9.637148	9.954762	9.682386	10.317613	18
43	9.637411	9.954701	9.682710	10.317290	17
44	9.637673	9.954640	9.683033	10.316967	16
45	9.637935	9.954579	9.683356	10.316644	15
46	9.638197	9.954518	9.683678	10.316321	14
47	9.638458	9.954457	9.684001	10.315999	13
48	9.638719	9.954396	9.684324	10.315676	12
49	9.638981	9.954335	9.684646	10.315354	11
50	9.639242	9.954274	9.684968	10.315032	10
51	9.639503	9.954213	9.685290	10.314710	9
52	9.639764	9.954152	9.685612	10.314388	8
53	9.640024	9.954090	9.685934	10.314066	7
54	9.640284	9.954029	9.686255	10.313745	6
55	9.640544	9.953968	9.686577	10.313423	5
56	9.640804	9.953906	9.686898	10.313102	4
57	9.641064	9.953845	9.687219	10.312781	3
58	9.641323	9.953783	9.687540	10.312460	2
59	9.641583	9.953722	9.687861	10.312138	1
60	9.641842	9.953660	9.688182	10.311818	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 64.

The Table of Sines and Tangents.

Degree 26.

M	Sine.	*Co-sine.*	Tangent.	*Co-tang.*	
0	9.641842	9.953660	9.688182	10.311818	60
1	9.642101	9.953598	9.688502	10.311498	59
2	9.642360	9.953537	9.688823	10.311177	58
3	9.642618	9.953475	9 689143	10.310857	57
4	9.642876	9.953413	9.689463	10.310537	56
5	9.643135	9.953351	9.689783	10.310217	55
6	9.643393	9.953290	9.690103	10.309897	54
7	9.643650	9.953228	9.690423	10.309577	53
8	9.643908	9.953166	9.690742	10.309258	52
9	9.644165	9.953104	9.691061	10.308938	51
10	9.644423	9.953042	9.691381	10.308619	50
11	9.644680	9.952980	9.691700	10.308300	49
12	9.644936	9.952917	9.692019	10.307981	48
13	9.645193	9.952855	9.692338	10.307662	47
14	9.645449	9.952793	9.692656	10.307343	46
15	9.645706	9.952731	9.692975	10.307025	45
16	9.645962	9.952668	9.693293	10.306706	44
17	9.646218	9.952606	9.693612	10.306388	43
18	9.646473	9.952544	9.693930	10.306070	42
19	9.646729	9.952481	9.694248	10.305752	41
20	9.646984	9.952419	9.694566	10.305434	40
21	9.647239	9.952356	9.694883	10.305117	39
22	9.647494	9.952294	9.695201	10.304799	38
23	9.647749	9.952231	9.695518	10.304482	37
24	9.648004	9.952168	9.695835	10.304164	36
25	9.648258	9.952105	9.696153	10.303847	35
26	9.648512	9.952043	9.696470	10.303530	34
27	9.648766	9.951980	9.696786	10.303213	33
28	9.649020	9.951917	9.697103	10.302897	32
29	9.649274	9.951854	9.697420	10.302580	31
30	9.649527	9.951791	9.697736	10.302264	30
	Co-sine.	Sine.	*Co-tang.*	Tangent.	M

Degree 63.

Degree 26.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.649527	9.951791	9.697736	10.302264	30
31	9.649781	9.951728	9.698053	10.301947	29
32	9.650034	9.951665	9.698369	10.301631	28
33	9.650287	9.951602	9.698685	10.301315	27
34	9.650519	9.951539	9.699001	10.300999	26
35	9.650798	9.951476	9.699316	10.300684	25
36	9.651044	9.951412	9.699632	10.300368	24
37	9.651296	9.951349	9.699947	10.300052	23
38	9.651648	9.951286	9.700263	10.299737	22
39	9.651800	9.951222	9.700578	10.299422	21
40	9.652052	9.951159	9.700893	10.299107	20
41	9.652303	9.951095	9.701208	10.298792	19
42	9.652555	9.951032	9.701522	10.298477	18
43	9.652806	9.950968	9.701837	10.298163	17
44	9.653057	9.950905	9.702152	10.297848	16
45	9.653307	9.950841	9.702466	10.297534	15
46	9.653558	9.950777	9.702780	10.297219	14
47	9.653808	9.950714	9.703095	10.296905	13
48	9.654059	9.950650	9.703409	10.296591	12
49	9.654309	9.950586	9.303722	10.296277	11
50	9.654558	9.950522	9.704036	10.295964	10
51	9.654808	9.950458	9.704350	10.295650	9
52	9.655057	9.950394	9.704663	10.295337	8
53	9.655307	9.950330	9.704976	10.295023	7
54	9.655556	9.950266	9.705290	10.294710	6
55	9.655805	9.950202	9.705603	10.294397	5
56	9.656053	9.950138	9.705916	10.294084	4
57	9.656302	9.950074	9.706228	10.293771	3
58	9.656550	9.950009	9.706541	10.293459	2
59	9.656799	9.949945	9.706853	10.293146	1
60	9.657047	9.949881	9.707166	10.292834	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 63.

The Table of Sines and Tangents.

Degree 27.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	9.657047	9.949880	9.707166	10.292834	60
1	9.657295	9.949816	9.707478	10.292522	59
2	9.657542	9.949752	9.707790	10.292210	58
3	9.657790	9.949687	9.708102	10.291898	57
4	9.658037	9.949623	9.708414	10.291586	56
5	9.658284	9.949558	9.708726	10.291274	55
6	9.658531	9.949494	9.709037	10.290963	54
7	9.658777	9.949429	9.709349	10.290651	53
8	9.659024	9.949364	9.709660	10.290340	52
9	9.659271	9.949300	9.709971	10.290029	51
10	9.659517	9.949235	9.710282	10.289718	50
11	9.659763	9.949170	9.710593	10.289407	49
12	9.660009	9.949105	9.710904	10.289096	48
13	9.660255	9.949040	9.711214	10.288785	47
14	9.660500	9.948976	9.711525	10.288475	46
15	9.660746	9.948910	9.711836	10.288164	45
16	9.660991	9.948845	9.712146	10.287854	44
17	9.661236	9.948780	9.712456	10.287544	43
18	9.661481	9.948715	9.712766	10.287234	42
19	9.661726	9.948650	9.713076	10.286924	41
20	9.661970	9.948584	9.713386	10.286614	40
21	9.662214	9.948519	9.713695	10.286305	39
22	9.662459	9.948453	9.714005	10.285995	38
23	9.662702	9.948388	9.714314	10.285686	37
24	9.662946	9.948323	9.714624	10.285376	36
25	9.663190	9.948257	9.714933	10.285067	35
26	9.663433	9.948191	9.715241	10.284758	34
27	9.663677	9.948126	9.715550	10.284449	33
28	9.663920	9.948060	9.715859	10.284149	32
29	9.664163	9.947995	9.716168	10.283832	31
30	9.664406	9.947929	9.716477	10.283523	30
	Co-ſine.	Sime.	Co-tang.	Tangent.	M

Degree 62.

The Table of Sines and Tangents.

Degree 27.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.664406	9.947929	9.716477	10.283523	30
31	9.664648	9.947863	9.716785	10.283215	29
32	9.664891	9.947797	9.717093	10.282907	28
33	9.665133	9.947731	9.717401	10.282598	27
34	9.665375	9.947665	9.717709	10.282291	26
35	9.665617	9.947599	9.718017	10.281983	25
36	9.665858	9.947533	9.718325	10.281675	24
37	9.666100	9 947467	9.718633	10.281367	23
38	9.666341	9.947401	9.718940	10.281060	22
39	9.666583	9.947335	9.719248	10.280752	21
40	9.666824	9.947269	9.719555	10.280445	20
41	9.667065	9.947203	9.719862	10.280138	19
42	9.667305	9.947136	9.720169	10.279831	18
43	9.667546	9.947070	9.720476	10.279524	17
44	9.667786	9.947004	9.720783	10.279217	16
45	9.668026	9.946937	9.721089	10.278911	15
46	9.668266	9.946871	9.721395	10.278604	14
47	9.668506	9.946804	9.721702	10.278298	13
48	9.668746	9.946738	9.722008	10.277991	12
49	9.668986	9.946671	9.722315	10.277685	11
50	9.669225	9.946604	9.722621	10.277379	10
51	9.669464	9.946537	9.722927	10.277073	9
52	9.669703	9.946471	9.723232	10.276768	8
53	9.669942	9.946404	9.723538	10.276462	7
54	9.670181	9.946337	9.723843	10.276156	6
55	9.670419	9.946270	9.724149	10.275851	5
56	9.670657	9.946203	9.724454	10.275546	4
57	9.670896	9.946136	9.724759	10.275240	3
58	9.671134	9.946069	9.725065	10.274935	2
59	9.671372	9.946002	9.725369	10.274630	1
60	9.671609	9.945935	9.725674	10.274326	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 62.

The Table of Sines and Tangents.

Degree 28.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.671609	9.945935	9.725674	10.274326	60
1	9.671847	9.945868	9.725979	10.274021	59
2	9.672084	9.945800	9.726284	10.273716	58
3	9.672321	9.945733	9.726588	10.273412	57
4	9.672558	9.945666	9.726892	10.273107	56
5	9.672795	9.945598	9.727197	10.272803	55
6	9.673032	9.945531	9.727501	10.272499	54
7	9.673269	9.945463	9.727805	10.272195	53
8	9.673505	9.945396	9.728109	10.271891	52
9	9.673741	9.945328	9.728412	10.271587	51
10	9.673977	9.945261	9.728716	10.271284	50
11	9.674213	9.945193	9.729020	10.270980	49
12	9.674449	9.945125	9.729323	10.270677	48
13	9.674684	9.945058	9.729626	10.270374	47
14	9.674919	9.944990	9.729929	10.270070	46
15	9.675154	9.944922	9.730232	10.269767	45
16	9.675389	9.944854	9.730535	10.269464	44
17	9.675624	9.944786	9.730838	10.269162	43
18	9.675859	9.944718	9.731141	10.268859	42
19	9.676094	9.944650	9.731443	10.268556	41
20	9.676328	9.944582	9.731746	10.268254	40
21	9.676562	9.944514	9.732048	10.267952	39
22	9.676796	9.944446	9.732351	10.267649	38
23	9.677030	9.944377	9.732653	10.267347	37
24	9.677264	9.944309	9.732955	10.367045	36
25	9.677497	9.944241	9.733257	10.266743	35
26	9.677731	9.944172	9.733558	10.266441	34
27	9.677964	9.944104	9.733860	10.266140	33
28	9.678197	9.944036	9.734162	10.265838	32
29	9.678430	9.943967	9.734463	10.265537	31
30	9.678663	9.943898	9.734764	10.265236	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 61.

Degree 28.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.678663	9.943898	9.734764	10.265236	30
31	9.678895	9.943830	9.735066	10.264934	29
32	9.679128	9.943761	9.735367	10.264633	28
33	9.679360	9.943692	9.735668	10.264332	27
34	9.679592	9.943624	9.735968	10.264031	26
35	9.679824	9.943555	9.736269	10.263731	25
36	9.680056	9.943486	9.736570	10.263430	24
37	9.680288	9.943417	9.736870	10.263130	23
38	9.680519	9.943348	9.737171	10.262829	22
39	9.680750	9.943279	9.737471	10.262529	21
40	9.680982	9.943210	9.737771	10.262229	20
41	9.681213	9.943141	9.738071	10.261929	19
42	9.681443	9.943072	9.738371	10.261629	18
43	9.681674	9.943003	9.738671	10.261329	17
44	9.681904	9.942933	9.738971	10.261029	16
45	9.682135	9.942864	9.739271	10.260729	15
46	9.682365	9.942795	9.739570	10.260430	14
47	9.682595	9.942725	9.739870	10.260130	13
48	9.682825	9.942656	9.740169	10.259831	12
49	9.683055	9.942587	9.740468	10.259532	11
50	9.683284	9.942517	9.740767	10.259233	10
51	9.683514	9.942448	9.741066	10.258934	9
52	9.683743	9.942378	9.741365	10.258635	8
53	9.683972	9.942308	9.741664	10.258336	7
54	9.684201	9.942239	9.741962	10.258038	6
55	9.684430	9.942169	9.742261	10.257739	5
56	9.684658	9.942099	9.742559	10.257441	4
57	9.684887	9.942029	9.742858	10.257142	3
58	9.685115	9.941959	9.743156	10.256844	2
59	9.685343	9.941889	9.743454	10.256546	1
60	9.685571	9.941819	9.743751	10.256248	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 61.

The Table of Sines and Tangents.

Degree 29.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.685571	9.941819	9.743752	10.256248	60
1	9.685799	9.941749	9.744050	10.255950	59
2	9.686027	9.941679	9.744348	10.255652	58
3	9.686254	9.941609	9.744645	10.255355	57
4	9.686482	9.941539	9.744943	10.255057	56
5	9.686709	9.941468	9.745240	10.254760	55
6	9.686936	9.941398	9.745538	10.254462	54
7	9.687163	9.941328	9.745835	10.254165	53
8	9.687389	9.941257	9.746132	10.253868	52
9	9.687616	9.941187	9.746429	10.253571	51
10	9.687842	9.941116	9.746726	10.253274	50
11	9.688069	9.941046	9.747023	10.252977	49
12	9.688295	9.940975	9.747319	10.252680	48
13	9.688521	9.940905	9.747616	10.252384	47
14	9.688747	9.940834	9.747912	10.252087	46
15	9.688972	9.940763	9.748209	10.251791	45
16	9.689198	9.940693	9.748505	10.251495	44
17	9.689423	9.940622	9.748801	10.251199	43
18	9.689648	9.940551	9.749097	10.250903	42
19	9.689873	9.940480	9.749393	10.250607	41
20	9.690098	9.940409	9.749689	10.250311	40
21	9.690323	9.940338	9.749985	10.250015	39
22	9.690548	9.940267	9.750281	10.249719	38
23	9.690772	9.940196	9.750576	10.249424	37
24	9.690996	9.940125	9.750872	10.249128	36
25	9.691220	9.940053	9.751167	10.248833	35
26	9.691444	9.939982	9.751462	10.248538	34
27	9.691668	9.939911	9.751757	10.248243	33
28	9.691892	9.939840	9.752052	10.247948	32
29	9.692115	9.939768	9.752347	10.247653	31
30	9.692339	9.939697	9.752642	10.247358	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 60.

The Table of Sines and Tangents.

Degree 29.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
30	9.692339	9.939697		9.752642	10.247358	30
31	9.692562	9.939625		9.752937	10.247063	29
32	9.692785	9.939554		9.753231	10.246769	28
33	9.693008	9.939482		9.753526	10.246474	27
34	9.693231	9.939410		9.753820	10.246180	26
35	9.693453	9.939339		9.754115	10.245885	25
36	9.693676	9.939267		9.754409	10.245591	24
37	9.693898	9.939195		9.754703	10.245297	23
38	9.694120	9.939123		9.754997	10.245003	22
39	9.694342	9.939051		9.755291	10.244709	21
40	9.694564	9.938980		9.755584	10.244415	20
41	9.694786	9.938908		9.755878	10.244122	19
42	9.695007	9.938835		9.756172	10.243828	18
43	9.695229	9.938763		9.756465	10.243535	17
44	9.695450	9.938691		9.756759	10.243241	16
45	9.695671	9.938619		9.757052	10.242948	15
46	9.695892	9.938547		9.757345	10.242655	14
47	9.696113	9.938475		9.757638	10.242362	13
48	9.696334	9.938402		9.757931	10.242069	12
49	9.696554	9.938330		9.758224	10.241776	11
50	9.696774	9.938257		9.758517	10.241483	10
51	9.696995	9.938185		9.758810	10.241190	9
52	9.697215	9.938112		9.759102	10.240898	8
53	9.697435	9.938040		9.759395	10.240605	7
54	9.697654	9.937967		9.759687	10.240313	6
55	9.697874	9.937895		9.759979	10.240021	5
56	9.698093	9.937822		9.760271	10.239728	4
57	9.698313	9.937749		9.760564	10.239436	3
58	9.698532	9.937676		9.760856	10.239144	2
59	9.698751	9.937603		9.761147	10.238852	1
60	9.698970	9.937531		9.761439	10.238561	0
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 60.

The Table of Sines and Tangents.

Degree 30.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.698970	9.937531	9.761439	10.238561	60
1	9.699189	9.937458	9.761731	10.228269	59
2	9.699407	9.937385	9.762023	10.237977	58
3	9.699626	9.937312	9.762314	10.237686	57
4	9.699844	9.937238	9.762606	10.237394	56
5	9.700062	9.937165	9.762897	10.237103	55
6	9.700280	9.937092	9.763188	10.236812	54
7	9.700498	9.937019	9.763479	10.236521	53
8	9.700716	9.936945	9.763770	10.236230	52
9	9.700933	9.936872	9.764061	10.235939	51
10	9.701151	9.936799	9.764352	10.235648	50
11	9.701368	9.936725	9.764643	10.235357	49
12	9.701585	9.936652	9.764933	10.235067	48
13	9.701802	9.936578	9.765224	10.234776	47
14	9.702019	9.936505	9.765514	10.234486	46
15	9.702236	9.936431	9.765805	10.234195	45
16	9.702452	9.936357	9.766095	10.233905	44
17	9.702669	9.936284	9.766385	10.233615	43
18	9.703885	9.936210	9.766675	10.233325	42
19	9.703101	9.936136	9.766965	10.233035	41
20	9.703317	9.936062	9.767255	10.232745	40
21	9.703533	9.935988	9.767545	10.232455	39
22	9.703748	9.935914	9.767834	10.232166	38
23	9.703964	9.935840	9.768124	10.231876	37
24	9.704179	9.935766	9.768413	10.231587	36
25	9.704395	9.935692	9.768703	10.231297	35
26	9.704610	9.935618	9.768992	10.231008	34
27	9.704825	9.935543	9.769281	10.230719	33
28	9.705040	9.935469	9.769570	10.230430	32
29	9.705254	9.935395	9.769859	10.230141	31
30	9.705469	9.935320	9.770148	10.229852	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 59.

The Table of Sines and Tangents.

Degree 30.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9·705469	9·935320	9·770148	10.229852	30
31	9·705683	9·935246	9·770437	10.229563	29
32	9·705897	9·935171	9·770726	10.229274	28
33	9·706112	9·935097	9·771015	10.228985	27
34	9·706326	9·935022	9·771303	10.228697	26
35	9·706539	9·934948	9·771592	10.228408	25
36	9·706753	9·934873	9·771880	10.228120	24
37	9·706967	9·934798	9·772168	10.227832	23
38	9·707180	9·934723	9·772456	10.227543	22
39	9·707393	9·934649	9·772745	10.227255	21
40	9·707606	9·934574	9·773033	10.226967	20
41	9·707819	9·934499	9·773321	10.226679	19
42	9·708032	9·934424	9·773608	10.226391	18
43	9·708245	9·934349	9·773896	10.226104	17
44	9·708457	9·934274	9·774184	10.225816	16
45	9·708670	9·934199	9·774471	10.225529	15
46	9·708882	9·934123	9·774759	10.225241	14
47	9·709094	9·934048	9·775046	10.224954	13
48	9·709306	9·933973	9·775333	10.224666	12
49	9·709518	9·933897	9·775621	10.224379	11
50	9·709730	9·933822	9·775908	10.224092	10
51	9·709941	9·933747	9·776195	10.223805	9
52	9·710153	9·933671	9·776482	10.223518	8
53	9·710364	9·933596	9·776768	10.223232	7
54	9·710575	9·933520	9·777055	10.222945	6
55	9·710786	9·933444	9·777342	10.222658	5
56	9·710997	9·933369	9·777628	10.222372	4
57	9·711208	9·933293	9·777915	10.222085	3
58	9·711418	9·933217	9·778201	10.221799	2
59	9·711629	9·933141	9·778487	10.221513	1
60	9·711839	9·933066	9·778774	10.221226	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 59.

The Table of Sines and Tangents.

Degree 31.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	9.711839	9.933066	9.778774	10.221226	60
1	9.712049	9.932990	9.779060	10.220940	59
2	9.712259	9.932914	9.779346	10.220654	58
3	9.712469	9.932838	9.779632	10.220368	57
4	9.712679	9.932761	9.779918	10.220082	56
5	9.712889	9.932685	9.780203	10.219796	55
6	9.713098	9.932609	9.780489	10.219511	54
7	9.713308	9.932533	9.780775	10.219225	53
8	9.713517	9.932457	9.781060	10.218940	52
9	9.713726	9.932380	9.781346	10.218654	51
10	9.713935	9.932304	9.781631	10.218369	50
11	9.714144	9.932227	9.781916	10.218084	49
12	9.714352	9.932151	9.782202	10.217799	48
13	9.714561	9.932074	9.782486	10.217514	47
14	9.714769	9.931998	9.782771	10.217229	46
15	9.714977	9.931921	9.783056	10.216944	45
16	9.715186	9.931845	9.783341	10.216659	44
17	9.715394	9.931768	9.783626	10.216374	43
18	9.715601	9.931691	9.783910	10.216090	42
19	9.715809	9.931614	9.784195	10.215805	41
20	9.716017	9.931537	9.784479	10.215520	40
21	9.716224	9.931460	9.784764	10.215236	39
22	9.716431	9.931383	9.785048	10.214952	38
23	9.716639	9.931306	9.785332	10.214668	37
24	9.716846	9.931229	9.785616	10.214384	36
25	9.717053	9.931152	9.785900	10.214099	35
26	9.717259	9.931075	9.786184	10.213816	34
27	9.717466	9.930998	9.786468	10.213632	33
28	9.717672	9.930920	9.786752	10.213248	32
29	9.717879	9.930843	9.787036	10.212974	31
30	9.718085	9.930766	9.787319	10.212681	30
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 58.

The Table of Sines and Tangents.

Degree 31.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.718085	9.930766	9.787319	10.212681	30
31	9.718291	9.930688	9.787603	10.212397	29
32	9.718497	9.930611	9.787886	10.212114	28
33	9.718703	9.930533	9.788170	10.211830	27
34	9.718909	9.930456	9.788453	10.211547	26
35	9.719114	9.930378	9.788736	10.211264	25
36	9.719320	9.930300	9.789019	10.210981	24
37	9.719525	9.930223	9.789302	10.210698	23
38	9.719730	9.930145	9.789585	10.210415	22
39	9.719935	9.930067	9.789868	10.210132	21
40	9.720140	9.929989	9.790151	10.209849	20
41	9.720345	9.929911	9.790433	10.209566	19
42	9.720549	9.929833	9.790716	10.209284	18
43	9.720754	9.929755	9.790999	10.209001	17
44	9.720958	9.929677	9.791281	10.208719	16
45	9.721162	9.929599	9.791563	10.208436	15
46	9.721366	9.929521	9.791846	10.208154	14
47	9.721570	9.929442	9.792128	10.207872	13
48	9.721774	9.929364	9.792410	10.207590	12
49	9.721978	9.929286	9.792692	10.207308	11
50	9.722181	9.929207	9.792974	10.207026	10
51	9.722385	9.929129	9.793256	10.206744	9
52	9.722588	9.929050	9.793538	10.206462	8
53	9.722791	9.928972	9.793819	10.206180	7
54	9.722994	9.928893	9.794101	10.205899	6
55	9.723197	9.928814	9.794383	10.205617	5
56	9.723400	9.928736	9.794664	10.205336	4
57	9.723603	9.928657	9.794945	10.205054	3
58	9.723805	9.928578	9.795227	10.204773	2
59	9.724007	9.928499	9.795508	10.204492	1
60	9.724210	9.928420	9.795789	10.204211	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 58.

The Table of Sines and Tangents.

Degree 32.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.724210	9.928420	9.795789	10.204211	60
1	9.724412	9.928341	9.796070	10.203930	59
2	9.724614	9.928262	9.796351	10.203649	58
3	9.724816	9.928183	9.796632	10.203368	57
4	9.725017	9.928104	9.796913	10.203087	56
5	9.725219	9.928025	9.797194	10.202806	55
6	9.725420	9.927946	9.797474	10.202526	54
7	9.725622	9.927867	9.797755	10.202245	53
8	9.725823	9.927787	9.798036	10.201964	52
9	9.726024	9.927708	9.798316	10.201684	51
10	9.726225	9.927628	9.798596	10.201404	50
11	9.726426	9.927549	9.798877	10.201123	49
12	9.726626	9.927469	9.799157	10.200843	48
13	9.726827	9.927390	9.799437	10.200563	47
14	9.727027	9.927310	9.799717	10.200283	46
15	9.727228	9.927231	9.799997	10.200003	45
16	9.727428	9.927151	9.800277	10.199723	44
17	9.727628	9.927071	9.800557	10.199443	43
18	9.727828	9.926991	9.800836	10.199163	42
19	9.728027	9.926911	9.801116	10.198884	41
20	9.728227	9.926831	9.801396	10.198604	40
21	9.728427	9.926751	9.801675	10.198325	39
22	9.728626	9.926671	9.801955	10.198045	38
23	9.728825	9.926591	9.802234	10.197766	37
24	9.729024	9.926511	9.802513	10.197487	36
25	9.729223	9.926431	9.802792	10.197207	35
26	9.729422	9.926351	9.803072	10.196928	34
27	9.729621	9.926270	9.803351	10.196649	33
28	9.729820	9.926190	9.803630	10.196370	32
29	9.730018	9.926110	9.803908	10.196091	31
30	9.730216	9.926029	9.804187	10.195813	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 57.

The Table of Sines and Tangents.

Degree 32.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9·730216	9·926029	9.804187	10.195813	30
31	9·730415	9·925949	9.804466	10.195534	29
32	9·730613	9·925868	9.804745	10.195255	28
33	9·730811	9·925787	9.805023	10.194977	27
34	9·731009	9·925707	9.805302	10.194698	26
35	9·731206	9·925626	9.805580	10.194420	25
36	9·731404	9·925545	9.805859	10.194141	24
37	9·731601	9·925464	9.806137	10.193863	23
38	9·731799	9·925384	9.806415	10.193585	22
39	9·731996	9·925303	9.806693	10.193307	21
40	9·732193	9·925222	9.806971	10.193029	20
41	9·732390	9·925141	9.807249	10.192751	19
42	9·732587	9·925060	9.807527	10.192433	18
43	9·732784	9·924978	9.807805	10.192195	17
44	9·732980	9·924897	9.808083	10.191917	16
45	9·733177	9·924816	9.808361	10.191639	15
46	9·733373	9·924735	9.808638	10.191362	14
47	9·733569	9·924653	9.808916	10.191084	13
48	9·733765	9·924572	9 809193	10.190807	12
49	9·733961	9·924491	9.809471	10.190529	11
50	9·734157	9·924409	9.809748	10.190252	10
51	9·734353	9·924328	9.810025	10.189975	9
52	9·734548	9·924246	9.810302	10.189697	8
53	9·754744	9·924164	9.810580	10.189420	7
54	9·734939	9·924083	9.810857	10.189143	6
55	9·735134	9·924001	9.811134	10.188866	5
56	9·735330	9·923919	9.811410	10.188589	4
57	9·735525	9·923837	9.811687	10.188313	3
58	9·735719	9·923755	9.811964	10.188036	2
59	9·735914	9·923673	9.812241	10.187759	1
60	9·736109	9·923591	9.812517	10.187483	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 57.

The Table of Sines and Tangents.

Degree 33.

M	Sine.	Co-sine.	Tangent.	Co-tang.	
0	9.736109	9.923591	9.812517	10.187483	60
1	9.736303	9.923509	9.812794	10.187206	59
2	9.736497	9.923427	9.813070	10.186930	58
3	9.736692	9.923345	9.813347	10.186653	57
4	9.736886	9.923263	9.813623	10.186377	56
5	9.737080	9.923180	9.813899	10.186101	55
6	9.737274	9.923098	9.814175	10.185824	54
7	9.737467	9.923016	9.814452	10.185548	53
8	9.737661	9.922933	9.814728	10.185272	52
9	9.737854	9.922851	9.815004	10.184996	51
10	9.738048	9.922768	9.815279	10.184720	50
11	9.738241	9.922686	9.815555	10.184445	49
12	9.738434	9.922603	9.815831	10.184169	48
13	9.738627	9.922520	9.816107	10.183893	47
14	9.738820	9.922438	9.816382	10.183617	46
15	9.739013	9.922355	9.816658	10.183342	45
16	9.739206	9.922272	9.816933	10.183066	44
17	9.739398	9.922189	9.817209	10.182791	43
18	9.739590	9.922106	9.817484	10.182516	42
19	9.739783	9.922023	9.817759	10.182240	41
20	9.739975	9.921940	9.818035	10.181965	40
21	9.740167	9.921857	9.818310	10.181690	39
22	9.740359	9.921774	9.818585	10.181415	38
23	9.740550	9.921691	9.818860	10.181140	37
24	9.740742	9.921607	9.819135	10.180865	36
25	9.740934	9.921524	9.819410	10.180590	35
26	9.741125	9.921441	9.819684	10.180315	34
27	9.741316	9.921357	9.819959	10.180041	33
28	9.741507	9.921274	9.820234	10.179766	32
29	9.741698	9.921190	9.820508	10.179492	31
30	9.781889	9.921107	9.820783	10.179217	30
	Co-sine.	Sine.	*Co-tang.*	Tangent.	M

Degree 56.

The Table of Sines and Tangents.

Degree 33.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
30	9.741889	9.921107	9.820783	10.179217	30
31	9.742080	9.921023	9.821057	10.178943	29
32	9.742271	9.920939	9.821332	10.178668	28
33	9.742461	9.920855	9.821606	10.178394	27
34	9.742652	9.920772	9.821880	10.178120	26
35	9.742842	9.920688	9.822154	10.177846	25
36	9.743032	9.920604	9.822429	10.177571	24
37	9.743223	9.920520	9.822703	10.177297	23
38	9.743412	9.920436	9.822977	10.177023	22
39	9.743602	9.920352	9.823250	10.176749	21
40	9.743792	9.920268	9.823524	10.176476	20
41	9.743982	9.920184	9.823798	10.176202	19
42	9.744171	9.920099	9.824072	10.175928	18
43	9.744361	9.920015	9.824345	10.175655	17
44	9.744550	9.919931	9.824619	10.175381	16
45	9.744739	9.919846	9.824892	10.175108	15
46	9.744928	9.919762	9.825166	10.174834	14
47	9.745117	9.919677	9.825439	10.174560	13
48	9.745306	9.919593	9.825713	10.174287	12
49	9.745494	9.919508	9.825986	10.174014	11
50	9.745683	9.919424	9.826259	10.173741	10
51	9.745871	9.919339	9.826532	10.173468	9
52	9.746059	9.919254	9.826805	10.173195	8
53	9.746248	9.919169	9.827078	10.172922	7
54	9.746436	9.919084	9.827351	10.172649	6
55	9.746624	9.918999	9.827624	10.172376	5
56	9.746811	9.918915	9.827897	10.172103	4
57	9.746999	9.918830	9.828170	10.171830	3
58	9.747187	9.918744	9.828442	10.171558	2
59	9.747374	9.918659	9.828715	10.171285	1
60	9.747562	9.918574	9.828987	10.171012	0
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 56.

The Table of Sines and Tangents.

Degree 34.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.747562	9.918574	9.828987	10.171012	60
1	9.747749	9.918489	9.829260	10.170740	59
2	9.747936	9.918404	9.829532	10.170468	58
3	9.748123	9.918318	9.829805	10.170195	57
4	9.748310	9.918233	9.830077	10.169923	56
5	9.748497	9.918147	9.830349	10.169651	55
6	9.748683	9.918062	9.830621	10.169379	54
7	9.748870	9.917976	9.830893	10.169106	53
8	9.749056	9.917891	9.831165	10.168834	52
9	9.749242	9.917805	9.831437	10.168563	51
10	9.749429	9.917719	9.831709	10.128291	50
11	9.749615	9.917634	9.831981	10.168019	49
12	9.749801	9.917548	9.832253	10.167747	48
13	9.749986	9.917462	9.832525	10.167475	47
14	9.750172	9.917376	9.832796	10.167204	46
15	9.750358	9.917290	9.833068	10.166932	45
16	9.750543	9.917204	9.833339	10.166660	44
17	9.750729	9.917118	9.833611	10.166389	43
18	9.750914	9.917032	9.833882	10.166118	42
19	9.751099	9.916945	9.834154	10.165846	41
20	9.751284	9.916859	9.834425	10.165575	40
21	9.751469	9.916773	9.834696	10.165304	39
22	9.751654	9.916686	9.834967	10.165033	38
23	9.751838	9.916600	9.835238	10.124762	37
24	9.752023	9.916514	9.835509	10.164491	36
25	9.752207	9.916427	9.835780	10.164220	35
26	9.752392	9.916340	9.836051	10.163949	34
27	9.752576	9.916254	9.836322	10.163678	33
28	9.752760	9.916167	9.836593	10.163407	32
29	9.752944	9.916080	9.836864	10.163136	31
30	9.753128	9.915994	9.837134	10.162866	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 55.

The Table of Sines and Tangents.

Degree 34.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.753128	9.915994	9.837134	10.162866	30
31	9.753312	9.915907	9.837405	10.162595	29
32	9.753495	9.915820	9.837675	10.162325	28
33	9.753679	9.915733	9.837946	10.162054	27
34	9.753862	9.915646	9.838216	10.161784	26
35	9.754046	9.915559	9.838487	10.161513	25
36	9.754229	9.915472	9.838757	10.161243	24
37	9.754412	9.915385	9.839027	10.160973	23
38	9.754595	9.915297	9.839297	10.160702	22
39	9.754778	9.915210	9.839568	10.160432	21
40	9.754960	9.915123	9.839838	10.160161	20
41	9.755143	9.915035	9.840108	10.159892	19
42	9.755325	9.914948	9.840378	10.159622	18
43	9.755508	9.914860	9.840647	10.159352	17
44	9.755690	9.914773	9.840917	10.159083	16
45	9.755872	9.914685	9.841187	10.158813	15
46	9.756054	9.914597	9.841457	10.158543	14
47	9.756236	9.914510	9.841726	10.158273	13
48	9.756418	9.914422	9.841996	10.158004	12
49	9.756600	9.914334	9.842266	10.157734	11
50	9.756781	9.914246	9.842535	10.157465	10
51	9.756963	9.914158	9.842804	10.157195	9
52	9.757144	9.914070	9.843074	10.156926	8
53	9.757326	9.913982	9.843343	10.156657	7
54	9.757507	9.913894	9.843612	10.156387	6
55	9.757688	9.913806	9.843882	10.156118	5
56	9.757869	9.913718	9.844151	10.155849	4
57	9.758049	9.913630	9.844420	10.155580	3
58	9.758230	9.913541	9.844689	10.155311	2
59	9.758411	9.913453	9.844958	10.155042	1
60	9.758591	9.913364	9.845227	10.154774	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 55.

Degree 35.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.758591	9.913364	9.845227	10.154774	60
1	9.758772	9.913276	9.845496	10.154504	59
2	9.758952	9.913187	9.845764	10.154235	58
3	9.759132	9.913099	9.846033	10.153967	57
4	9.759312	9.913010	9.846302	10.153698	56
5	9.759492	9.912921	9.846570	10.153429	55
6	9.759672	9.912833	9.846839	10.153161	54
7	9.759851	9.912744	9.847107	10.152892	53
8	9.760031	9.912655	9.847376	10.152624	52
9	9.760210	9.912566	9.847644	10.152356	51
10	9.760390	9.912477	9.847913	10.152087	50
11	9.760569	9.912388	9.848181	10.151819	49
12	9.760748	9.912299	9.848449	10.151551	48
13	9.760927	9.912210	9.848717	10.151283	47
14	9.761106	9.912121	9.848985	10.151015	46
15	9.761285	9.912031	9.849254	10.150746	45
16	9.761464	9.911942	9.849522	10.150478	44
17	9.761642	9.911853	9.849789	10.150210	43
18	9.761821	9.911763	9.850057	10.149943	42
19	9.761999	9.911674	9.850325	10.149675	41
20	9.762177	9.911584	9.850593	10.149407	40
21	9.762356	9.911495	9.850861	10.149139	39
22	9.762534	9.911405	9.851128	10.148872	38
23	9.762712	9.911315	9.851396	10.148604	37
24	9.762889	9.911226	9.851664	10.148336	36
25	9.763067	9.911136	9.851931	10.148069	35
26	9.763245	9.911046	9.852199	10.147801	34
27	9.763422	9.910956	9.852466	10.147534	33
28	9.763599	9.910866	9.852734	10.147267	32
29	9.763777	9.910776	9.853001	10.146999	31
30	9.763954	9.910686	9.853268	10.146732	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 54.

The Table of Sines and Tangents.

Degree 35.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.763954	9.910686	9.853268	10.146732	30
31	9.764131	9.910596	9.853535	10.146465	29
32	9.764308	9.910506	9.853802	10.146198	28
33	9.764485	9.910415	9.854069	10.145930	27
34	9.764662	9.910325	9.854336	10.145664	26
35	9.764838	9.910235	9.854603	10.145397	25
36	9.765015	9.910144	9.854870	10.145130	24
37	9.765191	9.910054	9.855137	10.144863	23
38	9.765367	9.909963	9.855404	10.144596	22
39	9.765544	9.909873	9.855671	10.144329	21
40	9.765720	9.909782	9.855937	10.144063	20
41	9.765896	9.909691	9.856204	10.143796	19
42	9.766071	9.909601	9.856471	10.143529	18
43	9.766247	9.909510	9.856737	10.143263	17
44	9.766423	9.909419	9.857004	10.142996	16
45	9.766598	9.909328	9.857270	10.142730	15
46	9.766774	9.909237	9.857537	10.142463	14
47	9.766949	9.909146	9.857803	10.142197	13
48	9.767124	9.909055	9.858069	10.141931	12
49	9.767299	9.908964	9.858336	10.141664	11
50	9.767474	9.908873	9.858602	10.141398	10
51	9.767649	9.908781	9.858868	10.141132	9
52	9.767824	9.908690	9.859134	10.140866	8
53	9.767999	9.908599	9.859400	10.140600	7
54	9.768173	9.908507	9.859666	10.140334	6
55	9.768348	9.908416	9.859932	10.140068	5
56	9.768522	9.908324	9.860198	10.139802	4
57	9.768696	9.908233	9.860464	10.139536	3
58	9.768871	9.908141	9.860730	10.139270	2
59	9.769045	9.908049	9.860995	10.139005	1
60	9.769219	9.907958	9.861261	10.138739	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 54.

Degree 36.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	9.769219	9.907958	9.861261	10.138739	60
1	9.769392	9.907866	9.861527	10.138473	59
2	9.769566	9.907774	9.861792	10.138208	58
3	9.769740	9.907682	9.862058	10.137942	57
4	9.769913	9.907590	9.862323	10.137677	56
5	9.770087	9.907498	9.862589	10.137411	55
6	9.770260	9.907406	9.862854	10.137146	54
7	9.770433	9.907314	9.863119	10.136880	53
8	9.770606	9.907221	9.863385	10.136615	52
9	9.770779	9.907129	9.863650	10.136350	51
10	9.770952	9.907037	9.863915	10.136085	50
11	9.771125	9.906945	9.864180	10.135820	49
12	9.771298	9.906852	9.864445	10.135554	48
13	9.771470	9.906760	9.864710	10.135289	47
14	9.771643	9.906667	9.864975	10.135924	46
15	9.771815	9.906574	9.865240	10.134759	45
16	9.771987	9.906482	9.865505	10.134495	44
17	9.772159	9.906389	9.865770	10.134230	43
18	9.772331	9.906296	9.866035	10.133965	42
19	9.772503	9.906203	9.866300	10.133700	41
20	9.772675	9.906111	9.866564	10.133436	40
21	9.772847	9.906018	9.866829	10.133171	39
22	9.773018	9.905925	9.867094	10.132906	38
23	9.773190	9.905832	9.867358	10.132642	37
24	9.773361	9.905738	9.867623	10.132377	36
25	9.773533	9.905645	9.867887	10.132113	35
26	9.773704	9.905552	9.868152	10.131848	34
27	9.773875	9.905459	9.868416	10.131584	33
28	9.774046	9.905365	9.868680	10.131320	32
29	9.774217	9.905272	9.868945	10.131055	31
30	9.774388	9.905179	9.869209	10.130791	30
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 53.

The Table of Sines and Tangents.

Degree 36.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.774388	9.905179	9.869209	10.130791	30
31	9.774558	9.905085	9.869473	10.130527	29
32	9.774729	9.904992	9.869737	10.130263	28
33	9.774899	9.904898	9.870001	10.129999	27
34	9.775070	9.904804	9.870265	10.129735	26
35	9.775240	9.904711	9.870529	10.129471	25
36	9.775410	9.904617	9.870793	10.129207	24
37	9.775580	9.904523	9.871057	10.128943	23
38	9.775750	9.904429	9.871321	10.128679	22
39	9.775920	9.904335	9.871585	10.128415	21
40	9.776090	9.904241	9.871849	10.128151	20
41	9.776259	9.904147	9.872112	10.127888	19
42	9.776429	9.904053	9.872376	10.127624	18
43	9.776598	9.903959	9.872640	10.127360	17
44	9.776768	9.903864	9.872903	10.127097	16
45	9.776937	9.903770	9.873167	10.126833	15
46	9.777106	9.903676	9.873430	10.126570	14
47	9.777275	9.903581	9.873694	10.126306	13
48	9.777444	9.903486	9.873957	10.126043	12
49	9.777613	9.903392	9.874220	10.125780	11
50	9.777781	9.903298	9.874484	10.125516	10
51	9.777950	9.903203	9.874747	10.125253	9
52	9.778119	9.903108	9.875010	10.124990	8
53	9.778287	9.903013	9.875273	10.124727	7
54	9.778455	9.902919	9.875536	10.124464	6
55	9.778623	9.902824	9.875799	10.124201	5
56	9.778792	9.902729	9.876063	10.123937	4
57	9.778960	9.902634	9.876326	10.123674	3
58	9.779127	9.902539	9.876589	10.123411	2
59	9.779295	9.902444	9.876851	10.123149	1
60	9.779463	9.902349	9.877114	10.122886	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 53.

Degree 37.

M	Sine.	*Co-sine.*	Tangent.	*Co-tang.*	
0	9.779463	9.902349	9.877114	10.122885	60
1	9.779631	9.902253	9.877377	10.122623	59
2	9.779798	9.902158	9.877640	10.122360	58
3	9.779965	9.902063	9.877903	10.122097	57
4	9.780133	9.901967	9.878165	10.121834	56
5	9.780300	9.901872	9.878428	10.121572	55
6	9.780467	9.901776	9.878691	10.121309	54
7	9.780634	9.901681	9.878953	10.121047	53
8	9.780801	9.901585	9.879216	10.120784	52
9	9.780968	9.901489	9.879478	10.120522	51
10	9.781134	9.901394	9.879741	10.120259	50
11	9.781301	9.901298	9.880003	10.119997	49
12	9.781467	9.901202	9.880265	10.119734	48
13	9.781634	9.901106	9.880528	10.119472	47
14	9.781800	9.901010	9.880790	10.119210	46
15	9.781966	9.900914	9.881052	10.118948	45
16	9.782132	9.900818	9.881314	10.118686	44
17	9.782298	9.900722	9.881576	10.118424	43
18	9.782464	9.900626	9.881839	10.118161	42
19	9.782630	9.900529	9.882101	10.117899	41
20	9.782796	9.900433	9.882363	10.117637	40
21	9.782961	9.900337	9.882625	10.117375	39
22	9.783127	9.900240	9.882886	10.117114	38
23	9.783292	9.900144	9.883148	10.116852	37
24	9.783457	9.900047	9.883410	10.116590	36
25	9.783623	9.899951	9.883672	10.116328	35
26	9.783788	9.899854	9.883934	10.116066	34
27	9.783953	9.899757	9.884195	10.115805	33
28	9.784118	9.899660	9.884457	10.115543	32
29	9.784282	9.899563	9.884719	10.115281	31
30	9.784447	9.899467	9.884980	10.115020	30
	Co-sine.	Sine.	*Co-tang.*	Tangent.	M

Degree 52.

The Table of Sines and Tangents.

Degree 37.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.784447	9.899467	9.884980	10.115020	30
31	9.784612	9.899370	9.885242	10.114758	29
32	9.784776	9.899273	9.885503	10.114497	28
33	9.784941	9.899175	9.885765	10.114235	27
34	9.785105	9.899078	9.886026	10.113974	26
35	9.785269	9.898981	9.886288	10.113712	25
36	9.785433	9.898884	9.886549	10.113451	24
37	9.785597	9.898787	9.886810	10.113190	23
38	9.785761	9.898689	9.887072	10.112928	22
39	9.785925	9.898592	9.887333	10.112667	21
40	9.786088	9.898494	9.887594	10.112406	20
41	9.786252	9.898397	9.887855	10.112145	19
42	9.786416	9.898299	9.888116	10.111884	18
43	9.786579	9.898201	9.888377	10.111623	17
44	9.786742	9.898104	9.888638	10.111362	16
45	9.786906	9.898006	9.888899	10.111101	15
46	9.787069	9.897908	9.889160	10.110840	14
47	9.787232	9.897810	9.889421	10.110579	13
48	9.787395	9.897712	9.889682	10.110318	12
49	9.787557	9.897614	9.889943	10.110057	11
50	9.787720	9.897516	9.890204	10.109796	10
51	9.787883	9.897418	9.890465	10.109535	9
52	9.788045	9.897320	9.890725	10.109275	8
53	9.788208	9.897222	9.890986	10.109014	7
54	9.788370	9.897123	9.891247	10.108753	6
55	9.788532	9.897025	9.891507	10.108493	5
56	9.788694	9.896926	9.891768	10.108232	4
57	9.788856	9.896828	9.892028	10.107972	3
58	9.789018	9.896729	9.892289	10.107711	2
59	9.789180	9.896631	9.892549	10.107451	1
60	9.789342	9.896532	9.892810	10.107190	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 52.

The Table of Sines and Tangents.

Degree 38.

M	Sine.	Co-ſine.	Tangent.	Co-tang.	
0	9.789342	9.896532	9.892810	10.107190	60
1	9.789504	9.896433	9.893070	10.106930	59
2	9.789665	9.896335	9.893330	10.106669	58
3	9.789827	9.896236	9.893591	10.106409	57
4	9.789988	9.896137	9.893851	10.106149	56
5	9.790149	9.896038	9.894111	10.105889	55
6	9.790310	9.895939	9.894371	10.105628	54
7	9.790471	9.895840	9.894632	10.105368	53
8	9.790632	9.895741	9.894892	10.105108	52
9	9.790793	9.895641	9.895152	10.104848	51
10	9.790954	9.895542	9.895412	10.104588	50
11	9.791115	9.895443	9.895672	10.104328	49
12	9.791275	9.895343	9.895932	10.104068	48
13	9.791436	9.895244	9.896192	10.103808	47
14	9.791596	9.895144	9.896452	10.103548	46
15	9.791756	9.895045	9.896712	10.103288	45
16	9.791917	9.894945	9.896971	10.103028	44
17	9.792077	9.894846	9.897231	10.102769	43
18	9.792237	9.894746	9.897491	10.102509	42
19	9.792397	9.894646	9.897751	10.102249	41
20	9.792557	9.894546	9.898010	10.101990	40
21	9.792716	9.894446	9.898270	10.101730	39
22	9.792876	9.894346	9.898530	10.101470	38
23	9.793035	9.894246	9.898789	10.101211	37
24	9.793195	9.894146	9.899049	10.100951	36
25	9.793354	9.894046	9.899308	10.100692	35
26	9.793513	9.893946	9.899568	10.100432	34
27	9.793673	9.893845	9.899827	10.100173	33
28	9.793832	9.893745	9.900086	10.099913	32
29	9.793991	9.893645	9.900346	10.099654	31
30	9.794149	9.893544	9.900605	10.099395	30
	Co-ſine.	Sine.	Co-tang.	Tangent.	M

Degree 51.

Degree 38.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	M
30	9.794149	9.893444	9.900605	10.099395	30
31	9.794308	9.893444	9.900864	10.099135	29
32	9.794467	9.893343	9.901124	10.098876	28
33	9.794626	9.893243	9.901383	10.098617	27
34	9.794784	9.893142	9.901642	10.098358	26
35	9.794942	9.893041	9.901901	10.098099	25
36	9.795101	9.892940	9.902160	10.097839	24
37	9.795259	9.892839	9.902419	10.097580	23
38	9.795417	9.892738	9.902678	10.097321	22
39	9.795575	9.892637	9.902937	10.097062	21
40	9.795733	9.892536	9.903196	10.096803	20
41	9.795891	9.892435	9.903455	10.096544	19
42	9.796049	9.892334	9.903714	10.096285	18
43	9.796206	9.892233	9.903973	10.096027	17
44	9.796364	9.892132	9.904232	10.095768	16
45	9.796521	9.892030	9.904491	10.095509	15
46	9.796678	9.891929	9.904750	10.095250	14
47	9.796836	9.891827	9.905008	10.094991	13
48	9.796993	9.891726	9.905267	10.094733	12
49	9.797150	9.891624	9.905526	10.094474	11
50	9.797307	9.891522	9.905784	10.094215	10
51	9.797464	9.891421	9.906043	10.093957	9
52	9.797621	9.891319	9.906302	10.093698	8
53	9.797777	9.891217	9.906560	10.093440	7
54	9.797934	9.891115	9.906819	10.093181	6
55	9.798091	9.891013	9.907077	10.092923	5
56	9.798247	9.890911	9.907336	10.092664	4
57	9.798403	9.890809	9.907594	10.092406	3
58	9.798560	9.890707	9.907852	10.092147	2
59	9.798716	9.890605	9.908111	10.091889	1
60	9.798872	9.890503	9.908369	10.091631	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 51.

The Table of Sines and Tangents.

Degree 39.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.798872	9.890503	9.908369	10.091631	60
1	9.799028	9.890400	9.908627	10.091373	59
2	9.799184	9.890298	9.908886	10.091114	58
3	9.799339	9.890195	9.909144	10.090856	57
4	9.799495	9.890093	9.909402	10.090598	56
5	9.799651	9.889990	9.909660	10.090340	55
6	9.799806	9.889888	9.909918	10.090081	54
7	9.799961	9.889785	9.910176	10.089823	53
8	9.800117	9.889682	9.910435	10.089565	52
9	9.800272	9.889579	9.910693	10.089307	51
10	9.800427	9.889476	9.910951	10.089049	50
11	9.800582	9.889374	9.911209	10.088791	49
12	9.800737	9.889271	9.911467	10.088533	48
13	9.800892	9.889167	9.911724	10.088275	47
14	9.801047	9.889064	9.911982	10.088017	46
15	9.801201	9.888961	9.912240	10.087760	45
16	9.801356	9.888858	9.912498	10.087502	44
17	9.801510	9.888755	9.912756	10.087244	43
18	9.801665	9.888651	9.913014	10.086986	42
19	9.801819	9.888548	9.913271	10.086729	41
20	9.801973	9.888444	9.913529	10.086471	40
21	9.802127	9.888341	9.913787	10.086213	39
22	9.802281	9.888237	9.914044	10.085956	38
23	9.802435	9.888133	9.914302	10.085698	37
24	9.802589	9.888030	9.914560	10.085440	36
25	9.802743	9.887926	9.914817	10.085183	35
26	9.802897	9.887822	9.915075	10.084925	34
27	9.803050	9.887718	9.915332	10.084668	33
28	9.803204	9.887614	9.915590	10.084410	32
29	9.803357	9.887510	9.915847	10.084153	31
30	9.803510	9.887406	9.916104	10.083895	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 50.

The Table of Sines and Tangents.

Degree 39.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.803510	9.887406	9.916104	10.083895	30
31	9.803664	9.887302	9.916362	10.083638	29
32	9.803817	9.887198	9.916619	10.083381	28
33	9.803970	9.887093	9.916876	10.083123	27
34	9.804123	9.886989	9.917134	10.082866	26
35	9.804276	9.886884	9.917391	10.082609	25
36	9.804428	9.886780	9.917648	10.082352	24
37	9.804581	9.886675	9.917905	10.082094	23
38	9.804734	9.886571	9.918162	10.081837	22
39	9.804886	9.886466	9.918420	10.081580	21
40	9.805038	9.886361	9.918677	10.081323	20
41	9.805191	9.886257	9.918934	10.081066	19
42	9.805343	9.886152	9.919191	10.080809	18
43	9.805495	9.886047	9.919448	10.080552	17
44	9.805647	9.885942	9.919705	10.080295	16
45	9.805799	9.885837	9.919962	10.080038	15
46	9.805951	9.885732	9.920219	10.079781	14
47	9.806103	9.885627	9.920476	10.079524	13
48	9.806254	9.885521	9.920733	10.079267	12
49	9.806406	9.885416	9.920990	10.079010	11
50	9.806557	9.885311	9.921247	10.078753	10
51	9.806709	9.885205	9.921503	10.078496	9
52	9.806860	9.885100	9 921760	10.078240	8
53	9.807011	9.884994	9.922017	10.077983	7
54	9.807162	9.884889	9.922274	10.077726	6
55	9.807314	9.884783	9.922530	10.077469	5
56	9.807464	9.884677	9.922787	10.077213	4
57	9.807615	9.884572	9.923044	10.076956	3
58	9.807766	9.884466	9.923300	10.076699	2
59	9.807917	9.884360	9.923557	10.076443	1
60	9.808067	9.884254	9.923813	10.076186	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 50.

The Table of Sines and Tangents.

Degree 40.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.808067	9.884254	9.923813	10.076186	60
1	9.808218	9.884148	9.924070	10.075930	59
2	9.808368	9.884042	9.924327	10.075673	58
3	9.808519	9.883936	9.924583	10.075417	57
4	9.808669	9.883829	9.924839	10.075160	56
5	9.808819	9.883723	9.925096	10.074904	55
6	9.808969	9.883617	9.925352	10.074647	54
7	9.809119	9.883510	9.925609	10.074391	53
8	9.809269	9.883404	9.925865	10.074135	52
9	9.809419	9.883297	9.926121	10.073878	51
10	9.809569	9.883191	9.926378	10.073622	50
11	9.809718	9.883084	9.926634	10.073366	49
12	9.809868	9.882977	9.926890	10.073110	48
13	9.810017	9.882871	9.927147	10.072853	47
14	9.810166	9.882764	9.927403	10.072597	46
15	9.810316	9.882657	9.927659	10.072341	45
16	9.810465	9.882550	9.927915	10.072085	44
17	9.810614	9.882443	9.928171	10.071829	43
18	9.810763	9.882336	9.928427	10.071573	42
19	9.810912	9.882228	9.928683	10.071317	41
20	9.810061	9.882121	9.928940	10.071060	40
21	9.811210	9.882014	9.929196	10.070804	39
22	9.811358	9.881907	9.929452	10.070548	38
23	9.811506	9.881799	9.929708	10.070292	37
24	9.811655	9.881692	9.929964	10.070036	36
25	9.811804	9.881584	9.930219	10.069781	35
26	9.811952	9.881477	9.930475	10.069525	34
27	9.812100	9.881369	9.930731	10.069269	33
28	9.812248	9.881261	9.930987	10.069013	32
29	9.812396	9.881153	9.931243	10.068757	31
30	9.812544	9.881045	9.931499	10.068501	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 49.

Degree 40.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.812544	9.881045	9.931499	10.068501	30
31	9.812692	9.880937	9.931755	10.068245	29
32	9.812840	9.880829	9.932010	10.067989	28
33	9.812988	9.880721	9.932266	10.067734	27
34	9.813135	9.880613	9.932522	10.067478	26
35	9.813283	9.880505	9.932778	10.067222	25
36	9.813430	9.880397	9.933033	10.066967	24
37	9.813578	9.880289	9.933289	10.066711	23
38	9.813725	9.880180	9.933545	10.066455	22
39	9.813872	9.880072	9.933800	10.066200	21
40	9.814019	9.879963	9.934056	10.065944	20
41	9.814166	9.879855	9.934311	10.065688	19
42	9.814313	9.879746	9.934567	10.065433	18
43	9.814460	9.879637	9.934822	10.065177	17
44	9.814607	9.879529	9.935078	10.064922	16
45	9.814753	9.879420	9.935333	10.064666	15
46	9.814900	9.879311	9.935589	10.064411	14
47	9.815046	9.879202	9.935844	10.064156	13
48	9.815193	9.879093	9.936100	10.063900	12
49	9.815339	9.878984	9.936355	10.063645	11
50	9.815485	9.878875	9.936610	10.063389	10
51	9.815631	9.878766	9.936866	10.063134	9
52	9.815777	9.878656	9.937121	10.062879	8
53	9.815923	9.878547	9.937376	10.062623	7
54	9.816069	9.878438	9.937632	10.062368	6
55	9.816215	9.878328	9.937887	10.062113	5
56	9.816361	9.878219	9.938142	10.061858	4
57	9.816506	9.878109	9.938397	10.061602	3
58	9.816652	9.877999	9.938653	10.061347	2
59	9.816797	9.877890	9.938908	10.061092	1
60	9.816943	9.877780	9.939163	10.060837	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 49.

The Table of Sines and Tangents.

Degree 41.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.816943	9.877780	9.939163	10.060837	60
1	9.817088	9.877670	9.939418	10.060582	59
2	9.817233	9.877560	9.939673	10.060327	58
3	9.817378	9.877450	9.939928	10.060072	57
4	9.817523	9.877340	9.940183	10.059817	56
5	9.817668	9.877230	9.940438	10.059562	55
6	9.817813	9.877120	9.940693	10.059307	54
7	9.817958	9.877009	9.940948	10.059052	53
8	9.818103	9.876899	9.941203	10.058797	52
9	9.818247	9.876789	9.941458	10.058542	51
10	9.819392	9.876678	9.941713	10.058287	50
11	9.818536	9.876568	9.941968	10.058032	49
12	9.818681	9.876457	9.942223	10.057777	48
13	9.818825	9.876347	9.942478	10.057522	47
14	9.818969	9.876236	9.942733	10.057267	46
15	9.819113	9.876125	9.942988	10.057012	45
16	9.819257	9.876014	9.943243	10.056757	44
17	9.819401	9.875904	9.943498	10.056502	43
18	9.819545	9.875793	9.943752	10.056248	42
19	9.819689	9.875682	9.944007	10.055993	41
20	9.819832	9.875571	9.944262	10.055738	40
21	9.819976	9.875459	9.944517	10.055483	39
22	9.820119	9.875348	9.944771	10.055229	38
23	9.820263	9.875237	9.945026	10.054974	37
24	9.820406	9.875125	9.945281	10.054719	36
25	9.820549	9.875014	9.945535	10.054464	35
26	9.820693	9.874903	9.945790	10.054210	34
27	9.820836	9.874791	9.946045	10.053955	33
28	9.820979	9.874679	9.946299	10.053701	32
29	9.821122	9.874568	9.946554	10.053446	31
30	9.821264	9.874456	9.946808	10.053192	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 48.

Degree 41.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.821264	9.874456	9.946808	10.053192	30
31	9.821407	9.874344	9.947063	10.052937	29
32	9.821550	9.874232	9.947317	10.052682	28
33	9.821692	9.874120	9.947572	10.052428	27
34	9.821835	9.874008	9.947826	10.052173	26
35	9.821977	9.873896	9.948081	10.051919	25
36	9.822120	9.873784	9.948335	10.051664	24
37	9.822262	9.873672	9.948590	10.051410	23
38	9.822404	9.873560	9.948844	10.051156	22
39	9.822546	9.873447	9.949099	10.050901	21
40	9.822688	9.873335	9.949353	10.050647	20
41	9.822830	9.873223	9.949607	10.050393	19
42	9.822972	9.873110	9.949862	10.050138	18
43	9.823114	9.872998	9.950116	10.049884	17
44	9.823255	9.872885	9.950370	10.049630	16
45	9.823397	9.872772	9.950625	10.049375	15
46	9.823538	9.872659	9.950879	10.049121	14
47	9.823680	9.872546	9.951133	10.048867	13
48	9.823821	9.872434	9.951388	10.048612	12
49	9.823962	9.872321	9.951642	10.048358	11
50	9.824104	9.872208	9.951896	10.048104	10
51	9.824245	9.872094	9.952150	10.047850	9
52	9.824386	9.871981	9.952404	10.047595	8
53	9.824527	9.871868	9.952659	10.047341	7
54	9.824667	9.871755	9.952913	10.047087	6
55	9.824808	9.871641	9.953167	10.046833	5
56	9.824949	9.871528	9.953421	10.046579	4
57	9.825090	9.871414	9.953675	10.046325	3
58	9.825230	9.871301	9.953929	10.046071	2
59	9.825370	9.871187	9.954183	10.045817	1
60	9.825511	9.871073	9.954437	10.045562	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 48.

The Table of Sines and Tangents.

Degree 42.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.825511	9.871073	9.954437	10.045562	60.
1	9.825651	9.870960	9.954691	10.045308	59
2	9.825791	9.870846	9.954945	10.045054	58
3	9.825931	9.870732	9.955199	10.044800	57
4	9.826071	9.870618	9.955453	10.044546	56
5	9.826211	9.870504	9.955707	10.044292	55
6	9.826351	9.870390	9.955961	10.044038	54
7	9.826491	9.870275	9.956215	10.043784	53
8	9.826631	9.870161	9.956469	10.043531	52
9	9.826770	9.870047	9.956723	10.043277	51
10	9.826910	9.869933	9.956977	10.043023	50
11	9.827049	9.869818	9.957231	10.042769	49
12	9.827189	9.869704	9.957485	10.042515	48
13	9.827328	9.869589	9.957739	10.042261	47
14	9.827467	9.869474	9.957993	10.042007	46
15	9.827606	9.869360	9.958246	10.041753	45
16	9.827745	9.869245	9.958500	10.041500	44
17	9.827884	9.869130	9.958754	10.041246	43
18	9.828023	9.869015	9.959008	10.040992	42
19	9.828162	9.868900	9.959262	10.040738	41
20	9.828301	9.868785	9.959515	10.040485	40
21	9.828439	9.868670	9.959769	10.040231	39
22	9.828578	9.868555	9.960023	10.039977	38
23	9.828716	9.868439	9.960277	10.039723	37
24	9.828855	9.868324	9.960530	10.039469	36
25	9.828993	9.868209	9.960784	10.039216	35
26	9.829131	9.868093	9.961038	10.038962	34
27	9.829269	9.867978	9.961291	10.038608	33
28	9.829407	9.867862	9.961545	10.038455	32
29	9.829545	9.867747	9.961799	10.038201	31
30	9.829683	9.867631	9.962052	10.037947	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 47.

The Table of Sines and Tangents.

Degree 42.

M	Sine.	*Co-ſine.*		Tangent.	*Co-tang.*	
30	9.829683	9.867631		9.962052	10.037947	30
31	9.829821	9.867515		9.962306	10.037694	29
32	9.829959	9.867399		9.962560	10.037440	28
33	9.830096	9.867283		9 962813	10.037187	27
34	9.830234	9.867167		9.963067	10.036933	26
35	9.830372	9.867051		9.963320	10.036680	25
36	9.830509	9.866935		9.963574	10.036426	24
37	9.830646	9.866819		9.963827	10.036173	23
38	9.830784	9.866703		9.964081	10.035919	22
39	9.830921	9.866586		9.964335	10.035665	21
40	9.831058	9.866470		9.964588	10.035412	20
41	9.831195	9.866353		9.964842	10.035158	19
42	9.831332	9.866237		9.965095	10.034905	18
43	9.831469	9.866120		9.965348	10.034652	17
44	9.831606	9.866004		9.965602	10.034398	16
45	9.831742	9.865887		9.965855	10.034144	15
46	9.831879	9.865770		9.966109	10.033891	14
47	9.832015	9.865653		9.966362	10.033638	13
48	9.832152	9.865536		9.966616	10.033384	12
49	9.832288	9.865419		9.966869	10.033131	11
50	9.832425	9.865302		9.967128	10.032878	10
51	9.832561	9.865185		9.967376	10.032624	9
52	9.832697	9.865068		9.967629	10.032371	8
53	9:832833	9.864950		9.967883	10.032117	7
54	9.832969	9.864833		9.968136	10.031864	6
55	9.833105	9.864716		9.998389	10.031611	5
56	9.833241	9.864598		9.968643	10.031357	4
57	9.833376	9.864480		9:968896	10.031104	3
58	9.833512	9.864363		9.969149	10.030851	2
59	9.833648	9.864245		9.969403	10.030597	1
60	9.833783	9.864127		9.969656	10.030344	0
	Co-ſine.	Sine.		*Co-tang.*	Tangent.	M

Degree 47.

Degree 43.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.833783	9.864127	9.969656	10.030344	60
1	9.833919	9.864010	9.969909	10.030091	59
2	9.834054	9.863892	9.970162	10.029838	58
3	9.834189	9.863774	9.970416	10.029584	57
4	9.834324	9.863656	9.970669	10.029331	56
5	9.834460	9.863537	9.970922	10.029078	55
6	9.834595	9.863419	9.971175	10.028825	54
7	9.834730	9.863301	9.971428	10.028571	53
8	9.834865	9.863183	9.971682	10.028318	52
9	9.834999	9.863064	9.971935	10.028065	51
10	9.835134	9.862946	9.972188	10.027812	50
11	9.835269	9.862827	9.972441	10.027559	49
12	9.835403	9.862709	9.972694	10.027306	48
13	9.835538	9.862590	9.972948	10.027052	47
14	9.835672	9.862471	9.973201	10.026799	46
15	9.835806	9.862353	9.973454	10.026546	45
16	9.835941	9.862234	9.973707	10.026293	44
17	9.836075	9.862115	9.973960	10.026040	43
18	9.836209	9.861996	9.974213	10.025787	42
19	9.836343	9.861877	9.974466	10.025533	41
20	9.836477	9.861757	9.974719	10.025280	40
21	9.836611	9.861638	9.974973	10.025027	39
22	9.836745	9.861519	9.975226	10.024774	38
23	9.836878	9.861399	9.975479	10.024521	37
24	9.837012	9.861280	9.975732	10.024268	36
25	9.837146	9.861161	9.975985	10.024015	35
26	9.837279	9.861041	9.976238	10.023762	34
27	9.837412	9.860921	9.976491	10.023505	33
28	9.837546	9 860802	9.976744	10.023256	32
29	9.837679	9.860682	9.976997	10.023003	31
30	9.837812	9.860562	9.977250	10.022750	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 46.

Degree 43.

M	Sine.	Co-ſine.		Tangent.	Co-tang.	
30	9.837812	9.860562		9.977250	10.022750	30
31	9.837945	9.860442		9.977503	10.022497	29
32	9.838078	9.860322		9.977756	10.022244	28
33	9.838211	9.860202		9.978009	10.021991	27
34	9.838344	9.860082		9.978262	10.021738	26
35	9.838477	9.859962		9.978515	10.021485	25
36	9.838609	9.859842		9.978768	10.021232	24
37	9.838742	9.859721		9.979021	10.020979	23
38	9.838875	9.859601		9.979274	10.020726	22
39	9.839007	9.859480		9.979527	10.020473	21
40	9.839140	9.859360		9.979780	10.020220	20
41	9.839272	9.859239		9.980033	10.019967	19
42	9.839404	9.859119		9.980285	10.019714	18
43	9.839536	9.858998		9.980538	10.019461	17
44	9.839668	9.858877		9.980791	10.019209	16
45	9.839800	9.858756		9.981044	10.018956	15
46	9.839932	9.858635		9.981297	10.018703	14
47	9.840064	9.858514		9.981550	10.018450	13
48	9.840196	9.858393		9.981803	10.018197	12
49	9.840428	9.858272		9.982056	10.017944	11
50	9.840459	9.858150		9.982309	10.017691	10
51	9.840591	9.858029		9.982562	10.017438	9
52	9.840722	9.857908		9.982814	10.017185	8
53	9.840854	9.857786		9.983067	10.016933	7
54	9.840985	9.857665		9.983320	10.016680	6
55	9.841116	9.857543		9.983573	10.016427	5
56	9.841247	9.857421		9.983826	10.016174	4
57	9.841378	9.857300		9.984079	10.015921	3
58	9.841509	9.857178		9.984331	10.015668	2
59	9.841640	9.857056		9.984584	10.015416	1
60	9.841771	9.856934		9.984837	10.015163	0
	Co-ſine.	Sine.		Co-tang.	Tangent.	M

Degree 46.

Degree 44.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
0	9.841771	9.856934	9.984837	10.015162	60
1	9.841902	9.856812	9.985090	10.014910	59
2	9.842033	9.856690	9.985343	10.014657	58
3	9.842163	9.856568	9.985596	10.014404	57
4	9.842294	9.856445	9.985848	10.014151	56
5	9.842424	9.856323	9.986101	10.013899	55
6	9.842555	9.856201	9.986354	10.013646	54
7	9.842685	9.856078	9.986607	10.013393	53
8	9.842815	9.855956	9.986859	10.013140	52
9	9.842945	9.855833	9.987112	10.012888	51
10	9.843076	9.855710	9.987365	10.012635	50
11	9.843206	9.855588	9.987618	10.012382	49
12	9.843336	9.855465	9.987871	10.012129	48
13	9.843465	9.855342	9.988123	10.011877	47
14	9.843595	9.855219	9.988376	10.011624	46
15	9.843725	9.855096	9.988629	10.011371	45
16	9.843855	9.854973	9.988882	10.011118	44
17	9.843984	9.854850	9.989134	10.010866	43
18	9.844114	9.854727	9.989387	10.010613	42
19	9.844243	9.854603	9.989640	10.010360	41
20	9.844372	9.854480	9 989893	10.010107	40
21	9.844502	9.854356	9.990145	10.009855	39
22	9.844631	9.854233	9.990398	10.009602	38
23	9.844760	9.854109	9.990651	10.009349	37
24	9.844889	9.853986	9.990903	10.009096	36
25	9.845018	9.853862	9.991156	10.008844	35
26	9.845147	9.853738	9.991409	10.008598	34
27	9.845276	9.853614	9.991662	10.008331	33
28	9.845404	9.853490	9.991914	10.008086	32
29	9.845533	9.853366	9.992167	10.007833	31
30	9.845662	9.853242	9.992420	10.007580	30
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 45.

Degree 44.

M	Sine.	*Co-ſine.*	Tangent.	*Co-tang.*	
30	9.845662	9.853242	9.992420	10.007580	30
31	9.845790	9.853118	9.992672	10.007328	29
32	9.845919	9.852994	9.992925	10.007075	28
33	9.846047	9.852869	9.993178	10.006822	27
34	9.846175	9.852745	9.993430	10.006569	26
35	9.846304	9.852620	9.993683	10.006317	25
36	9.846432	9.852496	9.993936	10.006064	24
37	9.846560	9.852371	9.994189	10.005811	23
38	9.846688	9.852246	9.994441	10.005559	22
39	9.846816	9.852122	9.994694	10.005306	21
40	9.846944	9.851997	9.994947	10.005053	20
41	9.847071	9.851872	9.995199	10.004801	19
42	9.847199	9.851747	9.995452	10.004548	18
43	9.847327	9.851622	9.995705	10.004295	17
44	9.847454	9.851497	9.995957	10.004043	16
45	9.847582	9.851372	9.996210	10.003790	15
46	9.847709	9.851246	9.996463	10.003537	14
47	9.847836	9.851121	9.996715	10.003285	13
48	9.847964	9.850996	9.996968	10.003032	12
49	9.848091	9.850870	9.997220	10.002779	11
50	9.848218	9.850745	9.997473	10.002527	10
51	9.848345	9.850619	9.997726	10.002274	9
52	9.848472	9.850493	9.997979	10.002021	8
53	9.848599	9.850367	9.998231	10.001769	7
54	9.848726	9.850242	9.998484	10.001516	6
55	9.848852	9.850116	9.998737	10.001263	5
56	9.848979	9.849990	9.998989	10.001011	4
57	9.849106	9.849864	9.999242	10.001758	3
58	9.849232	9.849737	9.999495	10.000505	2
59	9.849359	9.849611	9.999747	10.000253	1
60	9.849485	9.849485	10.000000	10.000000	0
	Co-ſine.	Sine.	*Co-tang.*	Tangent.	M

Degree 45.